Statistical Methods

Problem Sheet 2

1. **Box-Müller method for Gaussian Distribution:**

   (a) Generate two uniformly distributed random numbers r_1 and r_2 between 0 and 1. Then calculate the following:

   $$\begin{aligned} g_1 &= \sqrt{-2\log(r_1)}\ \cos(2\pi\, r_2) \\ g_2 &= \sqrt{-2\log(r_1)}\ \sin(2\pi\, r_2) \end{aligned}$$

   This generates two Gaussian distributed random numbers g_1 and g_2. Write a program to simulate this.

   (b) Using the Jcobian of transformation from r_i to g_i, show that g_1 and g_2 are two independent Gaussian distributed numbers.

2. **Central Limit Theorem:**

   (a) Let r_i be a set of uniformly distributed random numbers. Define

   $$z_n = \frac{1}{n}\sum_{i=1}^{n} r_i \ \text{ for } i = 2, 3, ..., 15$$

   Find out the distribution of z_n for all those values of n. Observe that the distribution approaches Gaussian as n increses from 2 to higher values.

   (b) Explain the triangular shape of the distribution you get for $n = 2$ in the above case.

3. **Linear Congruential Generator:** Uniformaly distributed random integers can be generated from the following mapping:

   $$x_{n+1} = (a\, x_n + c)\ \%\ m$$

   (a) Write a program to generate the sequence and print it for small values of the constants $a,\ c$ and $m < 30$.

   (b) Study the period of the sequence and compare with the condtions discussed in the class.