Statistical Methods

Problem Sheet 4

1. **Covariance Matrix and χ^2:**

   (a) Dowload the file `covariance.C` from the website. It generates two uniform random numbers `(x[0], y[0])`, convertes them in to two Gaussian distributed random numbers `(x[1], y[1])` using Box-Muller method. Then a linear transformation give `(w1,w2)`. All these joint distributions are plotted using the file. Run it and try to understand the transformations.

   (b) Modify the code to generate the marginalized distributions for `x[1], y[1], w1` and `w2`.

   (c) Find sample mean of `w1` and `w2`.

   (d) Find the sample covariance matrix for variables `(w1,w2)` and invert it.

   (e) Using

   $$\chi^2 = (\vec{w} - \vec{\mu})^T \; V^{-1} \; (\vec{w} - \vec{\mu})$$

   plot the $\chi^2 = 1$ ellipse for the data sample.

2. **Hypothesis testing:**

   (a) One theorist belives that the life-time of μ^+ should be $2.0\ \mu s$. Construct the pdf of $\hat{\tau}$ using the hypothesis $\tau = 2.0$ (H_0)

   (b) Othe theorist hypothesis the life time to be $2.5\ \mu s$. Construct the pdf of $\hat{\tau}$ using the hypothesis $\tau = 2.5$.

   (c) Downlaod the file `testexpllr.C`. The Log-Likelihood-Ratio (LLR) is defined as:

   $$t_\tau = \log\left(\frac{p(x_\tau|H_1)}{p(x_\tau|H_0)}\right) = \log p(x_\tau|H_1) - \log p(x_\tau|H_0)$$

   for two hypotheses H_0 and H_1 given a data set x_τ. For an ensemble of data set x_τ generate the pdf of t_τ, where the ensemle is generated assuming H_0 to be the correct hypothesis. Repeate the same for ensemble assumming H_1 to be the correct hypothesis. Compare the two distributions.

   (d) Generate a data set x_{exp} assuming $\tau = 2.2\ \mu s$. Calcualte the LLR for this data set. Find the p-values of this data under two hypotheses.