MA 311 : Analysis in $\mathbb{R}^n$

Problem Set 1

Due date : 21/08/09 before 2 PM

Q 1) Let $A \subseteq \mathbb{R}$ be bounded above and $c \in \mathbb{R}$. Define the sets $c + A$ and cA by

$$\begin{aligned} c + A &= \{c + a \,:\, a \in A\} \\ cA &= \{ca \,:\, a \in A\}. \end{aligned}$$

(a) Show that $\sup(c + A) = c + \sup A$.

(b) If $c \geq 0$, then $\sup(cA) = c \sup A$.

(c) What can you say about $\sup(cA)$ if $c < 0$?

Q 2) Compute, without proofs, $\sup A$ and $\inf A$ for the following sets :

(a) $A = \{n \in \mathbb{N} \,:\, n^2 < 10\}$

(b) $A = \{n/(2n+1) \,:\, n \in \mathbb{N}\}$

Q 3) If $(b_n) \to b$ prove that $(|b_n|) \to |b|$. Is the converse true?

Q 4) Staring from the monotone convergence theorem, prove the nested interval property.

Hint : Look at the proof of NIP that we discussed in class and see whether you can figure out how to use the monotone convergence theorem there.

Q 5) Starting from the nested interval property, prove the monotone convergence theorem.

Hint : (a_n) is bounded by b. Consider $I_0 = [a_0, b_0]$ with $a_0 = a_1 - 1$ and $b_0 = b$. Repeatedly bisect the interval, ensuring that at any stage you choose the half that narrows down to the limit.