MA 311 : Analysis in $\mathbb{R}^n$

Problem Set 3

Due date : 4/09/09 before 2 PM

Q 1) Prove that in a metric space the limit of a sequence, if it exists, is unique.

Q 2) Let (M, d) be a metric space. Define

$$\delta(x, y) := \frac{d(x, y)}{1 + d(x, y)}, \quad \forall x, y \in M.$$

Show that δ is a metric on M.

Q 3) Let (M, d) be a metric space and $x \in M$. Show that for a fixed $r > 0$, show that the set $\{x \in M : d(x, a) > r\}$ is open in M.

Q 4) For any set A prove that its interior (defined to be the set of interior points of A) is the largest open set contained in A.

Q 5) Let (X, d_1), (Y, d_2) be metric spaces. Show that a map $f : X \to Y$ is continuous iff for every closed set $V \subseteq Y$, its inverse image $f^{-1}(V)$ is open in X.