MA 311 : Analysis in $\mathbb{R}^n$

Problem Set 4

Due date : 11/09/09 before 2 PM

Q 1) A set $E \subseteq \mathbb{R}$ is called nowhere dense if its interior is empty. Prove that E is nowhere dense iff its complement is dense in $\mathbb{R}$.

Q 2) Prove that $\emptyset$ and $\mathbb{R}$ are the only two sets that are both open and closed in $\mathbb{R}$ under the usual metric.

Q 3) Prove that a set $A \subseteq \mathbb{R}$ is totally bounded iff it is bounded.

Q 4) Complete the formal proof of the theorem that "if a function $f : A \subseteq M \to N$ (where (M, d_1) and (N, d_2) are two metric spaces) is continuous on A and A is compact, then f is uniformly continuous on A".