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Indian Institute of Science Education and Research Kolkata
Department of Mathematics and Statistics
Numerical Analysis (MA3105)

Autumn Sem
September 19, 2018

Time: **15:00–16:00** *Mid-Semester Exam* Marks: **20**

1. i. Give an example of a function which has both stable and unstable fixed points.

   ii. Suppose that $g : \mathbb{R} \to \mathbb{R}$ is continuously differentiable and $g(\xi) = \xi$ with $|g'(\xi)| > 1$. Show that the sequence $\{x_k\}$ defined by $x_{k+1} = g(x_k)$, $k \geq 0$, does not converge to ξ from any starting value x_0, $x_0 \neq \xi$, unless the sequence reaches ξ in a finite number of steps

   [2+4=6]

2. Let

$$A = \begin{pmatrix} 0 & 1 & 3 \\ 0 & 2 & 5 \\ 2 & 7 & 9 \end{pmatrix}.$$

   Find a permutation matrix P, a unit lower triangular matrix L, and an upper triangular matrix U such that

$$PA = LU.$$

   [4]

3. Let A be a matrix on $\mathbb{R}^{n\times n}$ and $\|\cdot\|$ be a given norm on $\mathbb{R}^n$. Show that

$$\|A\| = \max\{\|Av\| : \|v\| \leq 1\}.$$

   [5]

4. Let $f : \mathbb{R}^n \to \mathbb{R}$ be a linear function. Show that, for all $w = (w_1, w_2, ..., w_n) \in \mathbb{R}^n$, there exists a positive constant C (independent of w) such that

$$|f(w)|^3 \leq C\big(|w_1|^3 + |w_2|^3 + ... + |w_n|^3\big).$$

   [5]

19/9/18

End of exam