Non linear dynamics(PH4104/PH5104) End Sem Exam

time 3hr

1. Consider the motion of over damped particle:

$\gamma\dot{\theta} = -\sin\theta - \Omega^2 \sin\theta\cos\theta$

a)Find out the fixed points and their stability for different Ω^2. b)Draw the bifurcation diagram with Ω^2 ,find out critical Ω^2 and nature of the bifurcation. (6+3)

2. Population dynamics of two species are described by following equations:

$\dot{x} = x - xy$ and $\dot{y} = -ky + xy$.

a)Find out the fixed points and characterize them according to stability analysis.

b) Draw the trajectories in x-y plane. (6+2).

3. a)Derive the equation for first order correction (ϵ order) of the solution of $\ddot{x} + x = -\epsilon(\dot{x} + \beta x)$ using two time scales (T_0,T_1) (multiscale method).

b) Assuming the solution of the form $x(T_0, T_1) = A(T_1)\cos(T_0 + \phi(T_1))$ find out damping and frequency shift first order in ϵ.

c)From the exact solution of the damped harmonic oscillator equation given in (a) compare A and ϕ. (3+4+3).

4. a)Find out the range of r for which the non zero fixed point of the Logistic map $x_{n+1} = rx_n(1 - x_n)$ is stable. (3)

b)Find out the Lyapunov exponent of map :

$x_{n+1} = rx_n$ for $0 \leq x_n \leq 1/2$ and $x_{n+1} = r(1 - x_n)$ for $1/2 \leq x_n \leq 1$.

Find out the value of r above which the map is chaotic. (2)

c)Show that $f^{(n)}(x)$ of Logistic map has extrema at $x = 1/2$ (2).

d) Show that the map $q_{n+1} = q_n + Tp_n$, $p_{n+1} = p_n - TV'(q_{n+1})$ preserves the phase space area. (3)

e)Consider the map: $x_{n+1} = r\sin(\pi x_n)$. Find out the critical r above which non zero fixed point appears. Write equations to find critical r at which period two cycle appears. (3)

(Project 10)

4.12.18