

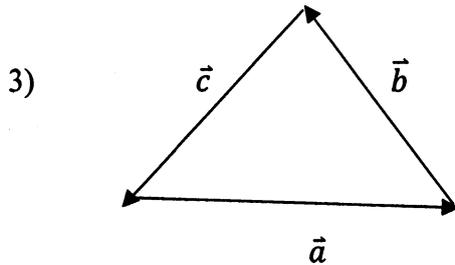
**Indian Institute of Science Education and Research Kolkata**

**End-Semester Examination @ ID4108 (07/12/2018)**

**Time: 3 Hours**

**Full Marks: 50**

- 1) Show that points A(1, 2, 7), B(2, 6, 3) and C(3, 10, -1) are collinear. **2 marks**
- 2) A vector  $\vec{r} = \hat{i}x + \hat{j}y + \hat{k}z$  makes angle  $\alpha$ ,  $\beta$  and  $\gamma$  with  $\hat{i}$ ,  $\hat{j}$  and  $\hat{k}$  direction respectively. Find out a relation between  $\alpha$ ,  $\beta$  and  $\gamma$ . **2 marks**



In the above triangle angle between  $\vec{c}$  and  $\vec{b}$  is  $A$ , angle between  $\vec{c}$  and  $\vec{a}$  is  $B$ , angle between  $\vec{a}$  and  $\vec{b}$  is  $C$ . Show that

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \quad \mathbf{2 \text{ marks}}$$

- 3) When a set of vectors  $\vec{A}_1, \vec{A}_2, \vec{A}_3, \dots, \vec{A}_n$  are said to be linearly independent? **2 marks**
- 4) Show how to convert a point  $P$  at a location in  $(x, y, z)$  in the cartesian coordinate to the spherical polar coordinate. **2 marks**
- 5) Suppose you are walking from RC to your hostel with a velocity 5 km/hour (which is constant). Show that the dot product between the velocity and acceleration is zero. **2 marks**
- 6) Suppose force is given by  $\vec{F}(x, y)\hat{i} = 3xy^2\hat{i} + (3x^2y + 4y^2)\hat{j}$ . Explain whether work done by this force will be path dependent? **2 marks**
- 7) Solve the set of three linear equation with three variables by Cramer's rule:

$$\begin{aligned} x + 2y + 3z &= -5 \\ 3x + y - 3z &= 4 \\ -3x + 4y + 7z &= -7 \end{aligned} \quad \mathbf{2 \text{ marks}}$$

8) Find the inverse of the following matrix

$$\begin{bmatrix} 0 & -3 & -2 \\ 1 & -4 & 2 \\ -3 & 4 & 1 \end{bmatrix}$$

4 marks

9) Write the wave function for Carbon atom in terms of Slater determinant. 3 marks

10) Solve the following 1<sup>st</sup> order linear differential equation: 3 marks

$$\frac{dy}{dx} + \frac{2}{x}y = 10x$$

11) Solve the following 2<sup>nd</sup> order linear differential equation: 3 marks

$$4\frac{d^2y}{dx^2} + 12\frac{dy}{dx} + 9y = 0$$

12) Suppose the non-homogeneous second order linear ordinary differential equation is given by  $\frac{d^2y}{dx^2} + A(x)\frac{dy}{dx} + B(x)y = C(x)$ .  $y_1(x)$  and  $y_2(x)$  are two linear independent solutions for the homogeneous equations. Find out the general solution for the inhomogeneous equation in terms of  $y_1(x)$ ,  $y_2(x)$  and their derivatives. 5 marks

12) Let  $f(x)$  be a periodic function of period  $2\pi$  such that

$$f(x) = \pi^2 - x^2 \text{ for } -\pi \leq x \leq \pi$$

Write down the Fourier series of  $f(x)$ . 5 marks

13) Show that, if a function  $g(x) = \frac{d^2}{dx^2}f(x)$ , then Fourier transformation of  $g(x)$  can be written as below:

$$\tilde{g}(k) = -k^2 \tilde{f}(k) \quad 2 \text{ marks}$$

14) Solve the differential following second order differential equation

$$(x+2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$$

Boundary conditions are:  $y(0) = 1$  and  $y'(0) = -1$  5 marks

15) Find the Fourier transformation of the function

$$f(x) = xe^{-x^2} \quad 4 \text{ marks}$$