Department of Mathematics and Statistics

Partial Differential Equation(MA5102)

End-Semester Exam

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Thursday
06.12.2018

Full mark 50

Time: **3** Hours.

Answer all questions

1. Classify the following partial differential equations and explain why it is ? [8]

   (a) $e^x \frac{\partial u}{\partial x} - xy\frac{\partial u}{\partial y} = xu^2$

   (b) $\frac{\partial u}{\partial x}(1 + \frac{\partial u}{\partial y}) - z\frac{\partial u}{\partial y} = xy$

   (c) $(x-y)\frac{\partial u}{\partial x} - x\frac{\partial u}{\partial y} = xyu + x$

   (d) $(x^2+u^2)\frac{\partial u}{\partial x} - xy\frac{\partial u}{\partial y} = u^3x + y^2$

2. Write the definition of the Green's function. [2]

3. Solve the following partial differential equation [3]

$$\frac{\partial^2 u}{\partial t^2} - c^2\frac{\partial^2 u}{\partial x^2} = e^{-t}, \qquad -\infty < x < \infty, t > 0$$

   with initial conditions $u(x,0) = \cos x$ and $\frac{\partial u}{\partial t}(x,0) = 1$

4. Solve the following initial value problem for $-\infty < x < \infty$ and $t > 0$ [4]

$$\frac{\partial u}{\partial t} + t\frac{\partial u}{\partial x} = x \quad \text{with } u(x,0) = x^2.$$

5. Reduce the following equation to canonical form. [6]

$$\frac{\partial^2 u}{\partial x^2} + x\frac{\partial^2 u}{\partial y^2} = 0 \quad \text{for } x \neq 0$$

6. Discuss the existence and uniqueness of a solution of the initial value problem. [5]

$$\frac{dy}{dx} = y^{1/3}, \qquad y(0) = 0$$

7. If u is harmonic in a open bounded domain $D \subset \mathbb{R}^2$, Then show that [5]

$$u(\zeta) = \frac{1}{\omega_n r^{n-1}} \int_{\partial B(\zeta,r)} u(x)\, dS_x \quad \text{for every ball } B(\zeta,r) \in D$$

   where ω_n is the surface area of the unit sphere in R^n

8. If $u(x,t)$ satisfies the heat equation on a rectangular region D in xt-space, then prove that the maximum value of $u(x,t)$ is attained either on the initial line or on the boundaries. [5]

9. Show that the following Pfaffian differential equation is integrable and find its integral [5]

$$yzdx + 2xzdy - 3xydz = 0$$

10. Consider $u(r,\theta)$ satisfies the Laplace's equation

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$$

in the region $a \leq r \leq b,\ 0 \leq \theta \leq \pi$. Its value along the boundary $r = a$ is $\theta(\pi - \theta)$ and all other points of the boundary $u = 0$. Then show that

$$u(r,\theta) = \frac{8}{\pi}\sum \frac{b^{2n} - r^{2n}}{b^{2n} - a^{2n}} \frac{a^n}{r^n} \frac{\sin n\theta}{n^3}$$

[7]

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