PH4102 Introduction to Astrophysics

5/12/2018.

End-Semester Exam

Wednessday, December 5, 2018 at $t = t_0$.

Duration: 3 Hours

Calculators are not allowed, all calculations have to be done by hand

- Mass of sun=$M_\odot = 3 \times 10^{30} Kg$
- 1 Year = 365 days
- 1AU = $1.49 \times 10^{11} M$
- $G = 6.67 \times 10^{-11}$ MKS units
- $c = 3 \times 10^8 M/s$
- $Light\, year = c \times 1\, Year$
- $1pc = 3 \times 10^{16}$ M
- radiation constant $\alpha = 7.5 \times 10^{-16} J\, M^{-3} K^{-4}$
- $k_B = 1.38064852 \times 10^{-23} M^2 Kg\, S^{-2} K^{-1}$
- Hubble constant ~ 70 KM/s/Mpc
- 1 eV = 1.60218×10^{-19} Joules

1. Estimate mass of the central object with a planet at distance $R = 9.5826$ AU from it, has an orbital period 29.4571 years. What would be its's orbital period in years if $R = 1$AU.

2. The Friedman equationt is given by

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho .$$

Relevant fluid equation is given by:

$$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) = 0 .$$

(a) Assume Universe is filled with radiation only and flat. Obtain solution for $a(t)$ and $\rho(t)$
(b) If current temperature of the Universe is $2.7^o K$, what is radiation energy density coming from blackbody radiation and what is Ω_{rad}?
(c) Show that $T \propto \frac{1}{a}$, where T is temperature of the universe and a is the scale factor.
(d) What is the age of Universe with respect to t_0, when $T = 3000^o K$. The temperature at the core of the Sun is around $10^7 K$. How old was the Universe when it was this hot?

3. The full Friedmann equation is

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} .$$

Consider the case $k > 0$, with a Universe containing only matter ($p = 0$) so that $\rho = \rho_0/a^3$. Demonstrate that the parametric solution

$$a(\theta) = \frac{4\pi G\rho_0}{3k}\left(1 - \cos\theta\right) ,$$
$$t(\theta) = \frac{4\pi G\rho_0}{3k^{3/2}}\left(\theta - \sin\theta\right) .$$

solves this equation, where θ is a variable which runs from 0 to 2π. Sketch $a$and t as function of θ

4. The galaxy's age can be estimated by radioactive decay of Uranium. Uranium is produced as an r-process element in supernovae (don't worry if you don't know what that is!), and on this basis the initial abundances of the two isotopes U^{235} and U^{238} are expected to be in the ratio

$$\left.\frac{U^{235}}{U^{238}}\right|_{\text{initial}} \simeq 1.65$$

The decaay rates of the isotopes are

$$\lambda(U^{235}) = 0.97 \times 10^{-9}\text{yr}^{-1}$$
$$\lambda(U^{238}) = 0.15 \times 10^{-9}\text{yr}^{-1}$$

Finally, the present abundance ratio is

$$\left.\frac{U^{235}}{U^{238}}\right|_{\text{initial}} \simeq 0.0072$$

Use the decay law

$$U(t) = U(0)\exp(-\lambda t)\,,$$

to estimate the age of the galaxy.

5. Consider a spherical elliptical galaxy of radius R that has uniform density and which consists of N stars each of mass m having typical velocities v. Assume the system is under virial equilibrium. Only measurable quantities are, v, R, derive expression form total mass M.

   (a) Compute M for an typical elliptical galaxy having a velocity $v = 350km/sec = 3.5{\times}10^5 m/s$, and a radius $R = 10kpc = 3.1 \times 10^{20}m$.

6. Derive the Earth-Sun distance D assuming that both are perfect blackbodies. Use $R_\odot = 7 \times 10^8$ M, $T_\odot = 5780^o$K, and $T_\oplus = 288^o$K

7. The Lane–Emden equation can be written as

$$\frac{d^2\theta(x)}{dx^2} + \frac{2}{x}\frac{d\theta(x)}{dx} + \theta^n(x) = 0\,,$$

The boundary condition for the equation is given by $\left.\frac{d\theta(x)}{dx}\right|_{x=0} = 0$ and $\theta(0) = 1$. Show that for $n = 1$the solution is given by $\theta(x) = \frac{\sin x}{x}$.
$x = \frac{r}{\alpha}$, where $\alpha^2 = \frac{(n+1)P_c}{4\pi G\rho_c^2}$, For Sun $P_c = 2.7 \times 10^{14}N/m^2$, $\rho_c = 1.62 \times 10^5 Kg/m^3$, calculate radius of sun as per this model

8. Obtain the expression for hydrostatic equilibrium of a spherical distribution of a self-gravitating system.

9. Do not answers these questions:

   (a) What is Luminosity of a star ?
   (b) What is Chandrashekar Limit ?
   (c) What are Pulsars ?
   (d) Name atleast one person who discovered radio pulsar ?
   (e) Can spam messages be sent using gravitational waves ?
   (f) What was before bigbang ?