MA5103: Topics in Networks
End Semester Examination
Autumn Semester 2018

Time : 3h. **Answer all the questions.**

1. Let A be an adjacency matrix of an un-weighted network G. Show that the (i, j)-th element of A^r, (for $r \geq 1$) is the number of walks of length r from node i to node j. [20]

2. Express, (i) clustering coefficient (ii) transitivity (iii) vertex degree centrality (iv) vertex betweenesss centrality (v) edge betweenesss centrality, by an adjacency matrix of a network. [20]

3. If $G(N, p)$ is a E-R random network, then find its transitivity. [15]

4. Let G be a network with n nodes. If the clustering coefficient of G is zero, then show that G can not have more than $n^2/4$ edges. [15]

5. Define cosine similarity between two nodes and show that the cosine similarity between nodes i and j is $\frac{n_{ij}}{\sqrt{d_i d_j}}$, where n_{ij} is the number of common neighbors of i and j and d_i is the degree of i. [15]

6. Let A be the adjacency matrix of a network G (un-weighted and un-directed) with n nodes. Show that $\mathrm{cov}(A_i, A_j) = \sum_k A_{ik}A_{kj} - \frac{d_i d_j}{n}$, where d_i is the degree of the node i, and A_i is the i-th row of A.
[15]

7. Show that the degree assortativity coefficient of a network G with m edges is
$$r = \frac{\sum_{ij}(A_{ij} - d_i d_j/2m)d_i d_j}{\sum_{ij}(d_i \delta_{ij} - d_i d_j/2m)d_i d_j},$$
where d_i is the degree of node i and A is the adjacency matrix of G. Describe the significance when r is equal to $-1, 0$ and 1. [25+10]

8. Let G be an undirected network without any cycle and $n_1, \ldots, n_r$ be the number of nodes of the connected components emerge by deleting a node v of G. Show that the vertex betweeness centrality of v is $n^2 - \sum_{i=1}^{r} n_i^2$, where n is the number of nodes of G. [15]

9. Construct the differential equations for SIRS model. What will be the corresponding differential equations when this model is applied on a network which has adjacency matrix A? [25+25]

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