IISER Kolkata Mid-semester Examination

MA 3104: Linear Algebra II

Date: September 22, 2018 **Duration: 1 hour**

Maximum marks 20

1. Let A be a 3×3 matrix with real entries. Prove that, if A is not similar over $\mathbb{R}$ to a triangular matrix, then A is similar over $\mathbb{C}$ to a diagonal matrix. 5

2. Let F be a field, n a positive integer, and V be the vector space of $n \times n$ matrices over F. If A is a fixed $n \times n$ matrix over F, let T_A be the linear operator T acting on V defined by
$$T_A(B) = AB - BA,\ B \in V.$$
Consider the family of linear operators
$$\mathcal{F} = \{T_A \mid A \in V \text{ is a diagonal matrix}\}.$$
Show that the operators in $\mathcal{F}$ are simultaneously diagonalizable. 4+2

3. Let T be a linear operator acting on an n-dimensional vector space V, and suppose that T has n distinct eigenvalues. Prove that, if S is an operator on V that commutes with T, then S, T are simultaneously diagonalizable. 6

4. Let T be a linear operator on a finite dimensional vector space over the field of complex numbers. Prove that T is diagonalizable if T is annihilated by some polynomial over $\mathbb{C}$ which has distinct roots. 4

5. Let T be a linear operator on $\mathbb{R}^2$, the matrix of which in the standard ordered basis is
$$\begin{bmatrix} 5 & 1 \\ 0 & 5 \end{bmatrix}.$$
Let W_1 be the subspace of $\mathbb{R}^2$ spanned by the vector $\epsilon_1 = (1, 0)$.
(a) Prove that W_1 is invariant under T.
(b) Prove that there is no subspace W_2 which is invariant under T and which is complementary to W_1:
$$\mathbb{R}^2 = W_1 \oplus W_2.$$
1+4

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