INDIAN INSTITUTE OF SCIENCE EDUCATION AND RESEARCH KOLKATA

Department of Mathematics and Statistics

Mid-semestral Examination, Autumn 2018

LINEAR ALGEBRA I (MA 2102)

Date: September 20, 2018 Maximum Marks: 20 Time: 1500 – 1600

Note: You can use well-known theorems taught in the class, but you need to write precise statement of the theorem that you are using.

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(1) Let $W = \{(x_1, x_2, x_3, x_4, x_5, x_6) \in \mathbb{R}^6 : x_1 + x_2 + x_3 = x_2 + x_3 + x_4 = x_5 + x_6 = 0\}$.

(a) Prove that W is a subspace of $\mathbb{R}^6$.

(b) Find a basis of W and extend it to a basis of $\mathbb{R}^6$. [2 + 3 + 3 = 8]

(2) Determine, with justification, if the set $S = \{1, x^2 - x + 5, 4x^3 - x^2 + 5x, 3x + 2\}$ spans the vector space $\mathcal{P}_3(\mathbb{R})$ of polynomials with degree 3 or less. [3]

(3) Let $V = M_{2\times 2}(\mathbb{R})$, W_1= the subspace of matrices in V of the form $\begin{pmatrix} a & b \\ c & a \end{pmatrix}$ and W_2= the subspace of matrices in V of the form $\begin{pmatrix} 0 & e \\ -e & f \end{pmatrix}$.

(a) Find the dimensions of the subspaces W_1, $W_1 \cap W_2$.

(b) Is $W_1 + W_2 = V$? Prove or disprove. [4 + 2 = 6]

(4) Let p, q, r and s be polynomials of degree at most 3. Which, if any, of the following two conditions is sufficient for the conclusion that the polynomials are linearly dependent? Justify your answer.

(a) At 1 each of the polynomials has the value 0.

(b) At 0 each of the polynomials has the value 1. [3 + 3 = 6]

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