INDIAN INSTITUTE OF SCIENCE EDUCATION AND RESEARCH KOLKATA

Date: **01/12/2018** Time: **3 Hours** Total Marks: **50**
Autumn **2018** (End Semester Exam) Sub. No. **ID4109** Sub. Name: **Inverse Theory**

1. Answer all the questions **(12 × 2 = 24)**

(i) A function f is defined by, f(t) = 1, 0<t<1 and f(t) = 0, elsewhere. Calculate f*f, where * denotes convolution.

(ii) Define aliasing in both time and frequency domain.

(iii) Define cross-correlation and auto-correlation. Using a suitable example, show that auto-correlation of any function has maximum value at zero lag.

(iv) Define Discrete-time Fourier Transform (DTFT) and Inverse Discrete-time Fourier Transform (IDTFT). If $x[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \dots + \delta[n-m]$, calculate DTFT of x[n].

(v) Define high-pass filter and low-pass filter, and calculate their amplitude spectrums.

(vi) Why moving average filter is used in signal processing? Smoothen the following amplitude data by using 3-points moving average method:

Frequency (f)	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
Amplitude (A)	100	200	300	400	50	100	500	800

(vii) Fit a straight line in least squares for the points (1, 0) and (2, 2) with a-priori condition that the line also passes through (0, 0).

(viii) Show that for a least squares (LS) problem, posteriori model covariance matrix [covm] is inversely proportional to curvature of error function E(m) at m_{LS}.

(ix) For a tomography problem, the eigenvalues of G^TG are 4, 2 and 0, and their eigenvectors are $[0.5, 1.0, 0.5]^T$, $[0.5, -1.0, 0.5]^T$ and $[-0.5, 1.0, -0.5]^T$, respectively. If $G^Td = [1.0, 0.5, 1.0]^T$, calculate least squares solution.

(x) What is a Weighted Least Squares problem? For a geoscience problem, depth, temperature and error associated with temperature are given by:

Depth	1	2	3	4	5
Temperature	2.133	2.987	4.134	4.912	6.087
Error	0.1	0.2	0.4	0.8	1.6

Calculate weighted matrix for the problem.

(xi) Let us consider an eigenvalue problem Ax = b, where A is a symmetric matrix. What geometrical interpretation can you make on the basis of eigenvalue problem?

(xii) The output of an instrument is given by: $y(n) = h(0)x(n) + h(1)x(n-1) + \dots + h(N)x(n-N)$, where x(n) and h(n) are input signal and impulse response, respectively. Calculate data kernel matrix.

2. (i) Calculate Fourier series of the following periodic function of period 2π and hence, define Gibb's phenomenon.

$$f(t) = 1, -\pi < t < 0$$
$$= -1, 0 < t < \pi$$ **(4)**

(ii) An earthquake occurred at time t (origin time), in a homogeneous medium of P-wave velocity 5.0 km/s. The following table gives the location (Xi, Yi) of each stations and P-wave arrival times (Ti). Assuming initial guess of the model parameters, $X_0 = 3$, $Y_0 = 4$, $Z_0 = 20$, t = 2 s, calculate data kernel matrix and residual matrix. **(6)**

Xi	-11.0	5.0	-1.0	20.0
Yi	-25.0	-19.0	-11.0	11.0
Ti	5.782	4.409	2.964	5.033

3. (i) Define spline of order k. Fit a quadratic spline through (-1, 0), (0, 1) and (1, 3). **(4)**

(ii) Calculate unit covariance matrix of the following inverse problem **(6)**

$$101a + 200b = 100$$
$$200a + 400b = -100$$

Is the problem stable? Justify your answer.

4. Define the following with suitable example (and graph if necessary) **(4 × 1.5 = 6)**

i) Purely underdetermined and mixed-determined problems
ii) Creeping and jumping methods
iii) Butterworth filter and cosine taper
iv) Model resolution and data resolution matrix

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01/12/2018