INDIAN INSTITUTE OF SCIENCE EDUCATION AND RESEARCH KOLKATA

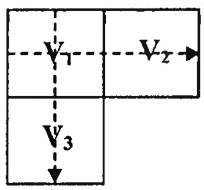
Date: **18/09/2018** Time: **1 Hour** Total Marks: **20**

Autumn **2018** (Mid Semester Exam) Sub. No. **ID4109** Sub. Name: **Inverse Theory**

1. The weighted least squares solution for the following problem is $(G^TWG)^{-1}G^TWd$, where W is the weighted matrix and G is the data kernel matrix. Calculate G^TWG. **(2)**

Zi	1	2	3	4	5
di	2.133	2.987	4.134	4.912	6.087
Error in di	0.1	0.2	0.4	0.8	1.6

2. For the following tomography problem, velocities can be calculated using minimum length method. Calculate the data resolution matrix. **(2)**



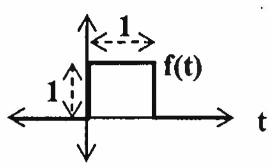
Where V_1, V_2, and V_3 are the velocities in the square boxes (each with 1 km length) 1, 2, and 3, respectively. Travel times from source to receiver (dashed arrow) through the boxes 1 and 2; 1 and 3 are 1.0 and 1.0 sec, respectively.

3. Calculate the solution of the following non-linear inverse problem for one iteration, using starting model, $m_0 = [1\ 1]^T$. **(2)**

$$m_1^2 + m_2 = 1$$

$$2m_1^2 + m_2 = 2$$

4. Calculate f*f (* means convolution) in $0 \le t \le 2$, where f(t) is defined by: **(2)**



5. Calculate amplitude and phase spectrum of a delta function, $\delta(t-t_0)$. **(2)**

6. The output y(n) of an instrument is given by **(3)**

$$y(n) = h(0)x(n) + h(1)x(n-1) + h(2)x(n-2),$$

where x(n) is the input signal and h(n) is the impulse response. Input x(n) can be calculated using damped-least squares method. If h(0) = h(2) = 1, h(1) = 0 and output signal, correspond to the input x(0) = 1.0, x(1) = 1.0, x(2) = 1.0 is y(0) = 1.0, y(1) = 1.5, y(2) = 2.5, respectively. Calculate the damping parameter.

7. Define the following with example(s) (and graph, if necessary) **(3)**

i) a -priori information ii) Transfer function iii) Model resolution matrix

iv) Z-transform v) Deconvolution vi) Impulse response

8. If d_1 = 812.65 km and d_2 = 1105.93 km are the distances of an earthquake from stations ST1 and ST2, respectively and d_{12} = 293.57 km is the interstation distance, calculate Lg attenuation (Q_0) in least squares, using the following relation: **(4)**

$$\ln\left[\frac{3.5}{\pi d_{12}}\ln\left\{\frac{A_1(f)}{A_2(f)}\sqrt{\frac{d_1}{d_2}}\right\}\right] = \ln f - \ln Q_0,$$

where $A_1(f)$ and $A_2(f)$ are the Lg attenuation spectra at frequency f for ST1 and ST2, respectively, are given by

f	0.6	0.8	1.0	1.2	1.4	1.6	1.8
$A_1(f)$	4625	2467	1423	1110	773	373	213
$A_2(f)$	481	201	95	41	29	14	6

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