

Department of Mathematics and Statistics
Indian Institute of Science Education and Research Kolkata
Mid-Autumn Semester Examination 2018

Subject: Analysis-I (MA2101)

Date and Time of Exam: September 24, 2018; 3:00PM

Max. Time: 1 Hour

Full Marks: 20

Instructions: Answer all the questions. The number inside square bracket at the right of each question indicates the marks for that question. The notations have their usual meanings. All intermediate steps must be shown in order to get full credit.

1. Give example for each of the following. [1 × 4 = 4]
 - (a) An infinite collection of open sets whose intersection is not an open set.
 - (b) An infinite collection of open sets whose intersection is an open set.
 - (c) An infinite collection of closed sets whose union is a closed set.
 - (d) An infinite collection of closed sets whose union is not a closed set.
2. Give an example of a set having 1, 2, 3 as the only limit points. [1]
3. Find supremum and infimum of the set $\{3 \sin x + 4 \cos x : -\pi \leq x \leq -\frac{\pi}{2}\}$. [1]
4. For $a, b \in \mathbb{Z}$, define aRb if 7 divides $3a + 4b$. Verify whether R is an equivalence relation. [2]
5. For the function f given by $f(x) = 2 - [x]$, if $\text{Range}(f) = [1, 7] \cap \mathbb{N}$, find the domain of f . [1]
6. Give an example of a bijective mapping $f : A \rightarrow B$ where A is a proper subset of B . [1]
7. For each of the following cases, either prove it or disprove it. [5 × 2 = 10]
 - (a) Let $a * b = a^2 b^2 + b - ab$, for $a, b \in \mathbb{N}$. Then $*$ is a binary ~~operation~~ ^{operation} on \mathbb{N} . 22
 - (b) For two sets $A, B \subseteq \mathbb{R}$, $\overline{A \cap B} = \overline{A} \cap \overline{B}$.

(overleaf)

- (c) For any two ^{statements} ~~sets~~ A, B , $(A \vee \neg(A \wedge B))$ is a tautology.
- (d) Let $f : X \rightarrow Y$ be a function and let $A \subseteq X$, $B \subseteq Y$. Then $f(f^{-1}(B)) = B$.
- (e) If a is a boundary point of A ($\subseteq \mathbb{R}$), then a is a limit point of A , but the converse may not be true.

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