PH3101 :: Classical Mechanics
Final Examination (November 30, 2018)

Marks: 40, Duration: 3 hour

1. Consider a particle of mass m and electric charge e, moving on a plane x-y in a constant magnetic field B. We shall choose the vector potential of the electromagnetic field in a gauge such that A_x=0 and A_y=Bx; in this gauge the Lagrangian of the system can be written as

$$L = \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2\right) + eBx\dot{y}$$

Write down the Hamilton-Jacobi equation for this problem, solve it by the separation of variables (reduce the problem at least to quadratures). Interpret physically conservation laws that emerged in the process of separating the variables. Solve the resulting equation completely and use the solution to determine the trajectory of the system. [7]

2. Consider the case of a double pendulum where the top pendulum has length L_1 and the bottom length is L_2 and similarly, the bob masses are m_1 and m_2. The motion is restricted to one plane. Find and describe the normal modes and coordinates. Assume small oscillations. [8]

3. A projectile is fired horizontally along earth's surface. Show that to a first approximation the angular deviation from the direction of the fire resulting from the Coriolis effect varies linearly with the time at a rate $\omega\, sin\theta$ where ω is the angular frequency of earth's rotation and θ is the latitude. Show that the direction of the deviation is to the right in the northern hemisphere. [4]

4. The principal moments of inertia of a body at the centre of mass are A, 3A and 6A.The body is so rotated that its angular velocity about the axes are 3n, 2n and n respectively. In the subsequent motion under no force ω_1, ω_2 and ω_3 denote the angular velocity about the principal axes at that time t, show that

$$\omega_1 = 3\omega_3 = \frac{9n}{\sqrt{5}} \operatorname{sech} u \text{ and } \omega_2 = 3n\, tanh\, u$$

where

$$u = 3nt + \frac{1}{2}\log_e 5$$

[7]

5. For the non linear system given by

$$\dot{x} = \sin y \qquad \dot{y} = x(1 - x^2)$$

answer the following questions.

(a) How many fixed points does it have? Determine the fixed points of this system.

(b) Determine the Jacobian matrix for this system for any arbitrary fixed point (x*,y*).

(c) For the fixed points on x = 0 line, determine the type of fixed points.

(d) Draw the phase portrait ONLY around the fixed points lying on x = 0 line.

[8]

6. Show that the angular momentum of the torque-free symmetrical top rotates in the body co-ordinates about the symmetry axis with an angular frequency Ω. Show also that the symmetry axis rotates in space about the fixed direction of the angular momentum with the angular frequency

$$\dot{\phi} = \frac{I_3\omega_3}{I_1 cos\theta}$$

where ϕ is the Euler angle of the line of the nodes with respect to the angular momentum as the space z axis. [6]