26/11/2018

Question Paper for END Semester examination, Date 26.11.2018
Biological Physics (PH4108/PH5103); Instructor: Dr. Rumi De; Total Marks: 50

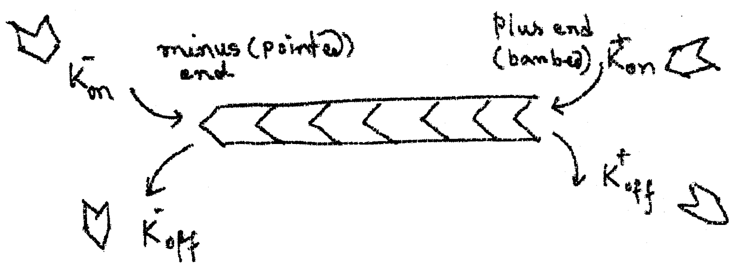
1. (a) (i) Construct the master equation to describe the dynamics of the molecular motors.
(ii) Write down the corresponding drift-diffusion equation and find out what will be the drift velocity of the molecular motors (in terms of the rate constants). **1+3=4**

(b) Suppose a DNA is subjected to an external force **F**, say pulled by the optical tweezers. Consider DNA as a freely jointed chain in 3D (consists of N independent segments connected by hinges), construct the partition function of the chain under external force in 3D. **4**

(c) When bigger colloidal particles are mixed with large no. of smaller particles in solution, they often experience a depletion force that drives the colloidal particles to self assemble together. Show that the depletion force will increase with increase in number and size of the smaller molecules. **4**

(d) Eukaryotic cells absorb signaling molecules diffusing in their surroundings. A cell may be considered as a sphere of radius '**R**'. Assume that the concentration profile of the signaling molecules has spherical symmetry and far field concentration at $r = \infty$ is N_0. Show that in steady state, a perfectly absorbing cell surface results in a rate of intake of signaling molecule is $\mathbf{4\pi R D N_0}$ (**D** is the diffusion coefficient of the signaling molecules). **4**

(e) (i) What is actin treadmilling? **(ii)** The figure shows a polymerization model where each end has its own rate. Write down the rate equations for each end of the filament. **(iii)** State the threshold condition for treadmilling to occur. Also represent it graphically. **1+1+2=4**



2. A particle is undergoing Brownian motion in a fluid in one dimension at temperature **T**. The time evolution of the free Brownian particle is modeled by the Langevin equation,

$$m\frac{dv}{dt} = -\gamma v + \eta(t),$$

where $< \eta(t) > = 0$, and $< \eta(t)\eta(t') > = 2\gamma K_B T\, \delta(t-t')$.

(a) Construct the corresponding Fokker-Planck equation to describe the motion of the Brownian particle. **(b)** Find out the probability distribution P(v) in the limit of $t \to \infty$. **2+3=5**

3. (a) What is the persistence length of a polymer?
(b) Show that the bending rigidity of an elastic filament is proportional to the persistence length and the temperature of the filament (consider gentle curves). **5**

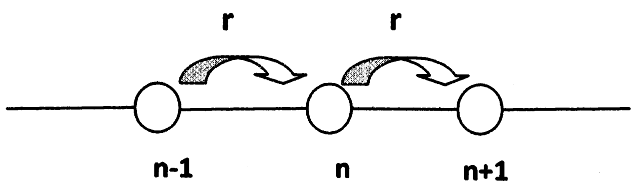
4. Compute the electrostatic energy cost for assembling a spherical shell of charge **Q** and radius $\mathbf{R_1}$ in presence of high concentration of monovalent salt in water,
(a) first, establish the Debye-Huckel equation for spherical symmetric potential.
(b) Calculate the potential at the surface of the charged sphere of radius $\mathbf{R_1}$.
(c) Show how the electrostatic energy varies with the salt concentration. **2+3+2=7**

5. A particle of mass **m** is undergoing Brownian motion in a fluid in one dimension. Assume that it starts at t=0 at position x=x(0) with velocity v=v(0). It's motion could be modeled by the Langevin equation, $m\frac{dv}{dt} = -\gamma v + \eta(t)$, where $\eta(t)$ describes the stochastic force. **(a)** What is Gaussian white noise? Explain why in case of Brownian particle, the stochastic force could be modeled as Gaussian white noise.
(b) Given that the velocity autocorrelation function of the Brownian particle is,

$$< v(t)v(t') > = \left(\frac{K_B T}{m}\right) e^{-\frac{\gamma}{m}|t-t'|};$$

show that the Brownian particle exhibits two distinct characteristics - at short time scale, Ballistic behaviour, and at long time, diffusive behaviour.
(c) Calculate the equilibrium velocity-position equal time correlation function, $< x(t)v(t) >$.
1+4+2=7

6. The states of one step Markov processes are generally represented by a set of non negative integers, **n** and the transitions are allowed only between adjacent states. **(a)** Consider a decay process as shown in the following figure, construct the master equation. Also, solve the master equation and find the probability distribution, **P(n, t)** [initial distribution given as, $P(n, 0) = \delta_{n,0}$].



(b) Consider the following chemical reaction in which it is assumed that the total concentration of A and B is constant. Write the rate equations and construct the corresponding master equation for **P(a, t)**.

$$A \underset{K_2}{\overset{K_1}{\rightleftharpoons}} B$$

3+1+2=6