Mid-Sem Exam 20 - 04 - 2018

PH4101 **Total Marks: 20** **Time: 60 mins**

ANSWER ANY TWO

1. Consider a 2D square lattice formed with identical atoms of mass M, lattice parameter a. The force constant between nearest neighbours is C. The atoms are forced to move perpendicular to the lattice plane (Z-mode).
   a. Write down the equation of motion relative to displacement u_{lm} of an atom belonging to l^{th} column and m^{th} row.
   b. Starting from a solution in the form of running waves with wave vector $\vec{q} = \vec{q_x} + \vec{q_y}$, find the dispersion relation for the z-mode vibrations, $\omega = f(q)$.
   c. Sketch the corresponding curves in the [10] and [11] directions in the first Brillouin zone and find out the frequency values at the zone boundary along the above two directions. **[Marks:2+6+2]**
2. Consider a sample (3D) with N non-interacting electrons at ground state, in which each electron behave as a free particle having mass m, energy $\varepsilon = \frac{h^2k^2}{2m}$. Considering Sommerfeld model of metals and periodic boundary condition for an infinite lattice show that density of states at Fermi energy, $g(\varepsilon_F)$ is constant, where ε_F is Fermi energy. **[Marks:10]**
3. In Drude model the probability of an electron suffering a collision in any infinitesimal interval dt is just dt/τ, where τ is the mean free time.
   a. Show that an electron picked at random at a given moment had not suffered a collision in the preceding t-seconds with probability $e^{-t/\tau}$.
   b. Show as a consequence of (a) that at any moment the mean time back to the last collision averaged over all electrons is τ. **[Marks:5+5]**
4. Primitive basis vectors of a conventional lattice are given by: $\vec{t1} = \frac{a}{2}(-\hat{x} + \hat{y} + \hat{z}), \vec{t2} = \frac{a}{2}(\hat{x} - \hat{y} + \hat{z}), \vec{t3} = \frac{a}{2}(\hat{x} + \hat{y} - \hat{z})$.
   a. What is the Bravais lattice?
   b. What will be the reciprocal lattice of the above Bravais lattice?
   c. If one carried out X-ray diffraction measurements, reflections from which $d(h,k,l)$ planes will be observed?
   d. Draw and describe a 2D honeycomb hexagonal lattice. Show the primitive vectors and basis vectors. **[Marks:2+2+3+3]**