IISER Kolkata **End Semester Examination**

MA 3104: Linear Algebra II

Date: November 26, 2018 **Duration: 3 hours**

Maximum marks 50

1. Let A be an $n \times n$ matrix with real entries such that $A^2 + I = 0$.
   (i) Prove that n is even.
   (ii) If $n = 2k$ and I is the $k \times k$ identity matrix, is A similar over the field of real numbers to a matrix of the block form

$$\begin{bmatrix} 0 & -I \\ I & 0 \end{bmatrix}?$$

   Justify your answer by proving your assertion, or, by giving a counterexample.

1+4

3. Prove, or, disprove by counterexample, no credit for identifying a false statement without a counterexample.
   (i) Any two 2×2 matrices are similar over the field $\mathbb{C}$ if they have the same set of eigenvalues.
   (ii) Any two 2×2 matrices are similar over the field $\mathbb{C}$ if they have the same minimal polynomial.
   (iii) Any two 3×3 matrices are similar over the field $\mathbb{C}$ if they have the same minimal polynomial.
   (iv) Any two 4×4 matrices are similar over the field $\mathbb{C}$ if they have the same characteristic polynomial and the same minimal polynomial.
   (v) Any two 6×6 nilpotent matrices over the field F are similar if they have the same minimal polynomial and the same kernel dimension.
   (vi) Any two 7×7 nilpotent matrices over the field F are similar if they have the same minimal polynomial and the same kernel dimension.
   (vii) A linear operator T on a finite dimensional complex inner product space is non-negative if each eigenvalue of T is non-negative.
   (viii) A positive unitary operator on a finite dimensional inner product space is the identity operator.
   (ix) There exists a 4×4 matrix A such that A^3 is normal but A is not normal.
   (x) A linear operator T on a finite dimensional complex inner product space is unitary if each eigenvalue of T has absolute value 1.

2+2+3+3+4+4+4+5+5+4

4. Let n be a positive integer, $n \geq 2$, and let N be an $n \times n$ matrix over the field F such that $N^n = 0$ but $N^{n-1} \neq 0$. Prove that N has no square root, that is, there is no $n \times n$ matrix A such that $A^2 = N$. 4

5. Consider $\mathbb{C}^n$ as an inner product space with the standard inner product $\langle \cdot, \cdot \rangle$. For $v_1, \ldots, v_n \in \mathbb{C}^n$, define the $n \times n$ matrix $A = \left(\!\left(\langle v_j, v_i \rangle\right)\!\right)_{i,j=1}^{n}$.

(i) Prove that A is non-negative linear operator on $\mathbb{C}^n$.

(ii) Does the converse hold? Justify your answer by proving your assertion, or, by giving a counterexample.

3+5

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