MIDSEM Quantum Mechanics -1 CH2102 19.09.2018

$h = 6.627\times10^{-34}$ joule-sec ; $m_e = 9.1\times10^{-31}$ kg ; c = 3×10^{8} meter/sec. ;
1 eV = 1.6 × 10^{-19} joule ; $m_p = 1.67\times10^{-27}$ kg Full Marks=20

Answer the following questions:

1. If a non -stationary state ψ is expanded in eigenfunctions of Hamiltonian operator as $\psi = \frac{1}{\sqrt{3}}\phi_1 + \frac{2}{\sqrt{3}}\phi_2$; where ϕ_1 and ϕ_2 are the normalized eigenfunctions of Hamiltonian operator $\hat{H}$ with corresponding eigenvalues 1 and 2 units of energy. What is the value of the average energy $\int \psi^* \hat{H}\psi d\tau$ of the system? 4

2. Show which are necessarily Hermitian operators or non-Hermitian operators, where $\hat{A}$ is an arbitrary operator:

(i) $\hat{A}+\hat{A}^\dagger$ (ii) $\hat{A}\hat{A}^\dagger$ (iii) $\hat{A}-\hat{A}^\dagger$ (iv) $\exp(\hat{A}-\hat{A}^\dagger)$ (v) $\exp(\hat{A}+\hat{A}^\dagger)$ 0.5x5=2.5

3. Let $\psi = \sum_i c_i\phi_i$ and is normalized to unity and $H\phi_i = \varepsilon_i\phi_i$ show that

$$\sum_i |c_i|^2 \varepsilon_i = \int \psi^* \hat{H}\psi d\tau$$ 3

4. For a stationary state of energy, derive the time part of the wavefunction and show that the average value of any time-independent operator in this state does not change with time. 2+2

5. (a) Write down the Hamiltonian operator in three dimensional space (x,y,z) for a particle having potential $V(x,y,z)$.

(b) If an electron is subjected to a potential difference of V volts show that the corresponding de Broglie wavelength is $\lambda = \frac{12.26}{\sqrt{V}}$ Å 0.5+1

(c) An electron remains in excited state for 10^{-11} sec. What is the minimum uncertainty in the energy of an excited state. Find also the uncertainty in the frequency of light emitted at 10^{-11} sec. 1.5X2

(d) Calculate the normalization constant "A" of the following wavefunction

$\psi = A\sin\left(\frac{5\pi x}{L}\right)$ within $x=0$ to $x=L$. 2