

MA2103: Mathematics III  
Mid Semester Examination  
Autumn Semester 2018

Time : 1h.

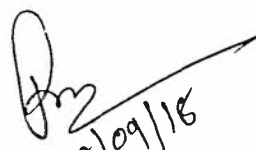
Notations used:

- (a)  $A \sim B$  :  $A$  and  $B$  are equipotent, i.e. there exists a bijective map from  $A$  to  $B$ .
- (b)  $A^B$  : set of all functions from  $B$  to  $A$ .
- (c)  $A \setminus B$  :  $\{x \in A : x \notin B\}$ .

All questions are compulsory and each carry equal marks.

1. Find an explicit bijection between  $[0, 1]$  and  $[0, 1)$ .
2. Let  $A, B, X$  be non-empty sets such that  $A \sim B$ . Then prove or disprove that  $X^A \sim X^B$ .
3. Let  $A, B, X$  be non-empty sets such that  $A^X \sim B^X$ . Then prove or disprove that  $A \sim B$ .
4. Show that  $(0, 1)^{\mathbb{N}} \sim ((0, 1) \setminus \mathbb{Q})^{\mathbb{N}}$ . [If you use the fact  $(0, 1) \sim (0, 1) \setminus \mathbb{Q}$ , then you have to prove it.]
5. Prove that  $\mathbb{N}^{\mathbb{N}} \sim (0, 1) \setminus \mathbb{Q}$ .

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18/09/18

Shirshendu  
Chowdhury  
18/09/18