ENDSEM Quantum Mechanics -1 CH2102 30.11.2018

$h = 6.627\times10^{-34}$ **joule-sec** ; $m_e = 9.1\times10^{-31}$ **kg** ; **c=** 3×10^{8} **meter/sec.** ;
1 eV = 1.6 $\times$ **10^{-19} joule** ; $m_p = 1.67\times10^{-27}$ **kg** ; **Full Marks- 50; Time- 3 hrs.**

Answer the following questions:

1. The energy of the ground state of hydrogen atom is approximately -13.6 eV. Find the followings:

   (a) Approximate energy of n=3 state of Li^{++}.
   (b) Approximate energy difference (absolute) between n=4 and n=3 state for Li^{++}.
   (c) The state (quantum number) of Be^{3+} with approximate energy of -13.6 eV.

1+1+1

2. Consider the operator, $N = a^{\dagger}a$ where a and $a^{\dagger}$ are lowering and raising operators of the one-dimensional harmonic oscillator respectively i,e,

$$a^{\dagger}|\psi_n\rangle = \sqrt{n+1}\,|\psi_{n+1}\rangle,\quad n \text{ represents the quantum number of the state}$$

   (a) Show that $N|\psi_n\rangle = n|\psi_n\rangle$
   (b) Show that $[N,a^{\dagger}] = a^{\dagger}$ (Hint: Use $\left[a,a^{\dagger}\right]=1$)

2+2

3. For a particle confined in a one-dimensional box (0, L)

   (a) Evaluate $\langle x\rangle$ for any quantum number n.
   (b) Show that the probability of finding the particle in the region $\frac{L}{4}$ to $\frac{3L}{4}$ is $\frac{1}{2}$, if "n" is even and $\left(\frac{1}{2}+\frac{(-1)^k}{n\pi}\right)$ if "n" is odd, $(n = 2k+1, k = 0,1,2,........)$

1+4

4. Show that for operators $\hat{A}$, $\hat{B}$, $\hat{C}$

   (a) $[A,BC] = B[A,C]+[A,B]C$

   (b) $[AB,C] = A[B,C]+[A,C]B$

   (c) $[A,[B,C]]+[C,[A,B]]+[B,[C,A]]=0$

2+2+2

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5. Consider a particle with mass m in a square two-dimensional box. The potential energy function is

$V(x, y) = 0$ for $0 \le x \le a$ and $0 \le y \le a$

$V(x, y) = \infty$ for everywhere else

(a) Write down the Hamiltonian for the system.
(b) Write down the general normalized eigenfunctions for this system inside the box.
(c) Write down the energy eigenvalues for the system.
(d) Determine the degeneracies for the lowest 3 states. 1+1+1+3

6. The wavefunction of a particle in a 1-D box with infinite potential outside the box is $\psi = 0.1\phi_1 + 0.2\phi_2 + 0.3\phi_3$ where ϕ_1 , ϕ_2, ϕ_3 are normalized ground, first two excited states respectively. [Note ψ is not a stationary state.]
(a) Normalize the wavefunction ψ .
(b) What is the probability to observe the particle in the ground state?

1+1

7. The wavefunction of an electron which is confined on a line between x=1 and x=3 is $\psi(x) = 3x + 5$

(a) Normalize the wavefunction.
(b) Find the probability of the finding the electron between x=2 and x=3.

1+1

8. If $\hat{A}$ and $\hat{B}$ are two quantum operators for two observables and $f(\hat{A})$ is a polynomial of operator $\hat{A}$. Assume the relation $\left[\hat{A}, \hat{B}\right] = \hat{B}f(\hat{A})$

(a) Demonstrate that if ψ_a is an eigenfunction of the operator $\hat{A}$ with an eigenvalue 'a' then $\hat{B}\psi_a$ is also an eigenfunction of the operator $\hat{A}$.
(b) What is its corresponding eigenvalue? 2+2

9. Show that $[p_x, f(x)] = -i\hbar\frac{\partial f(x)}{\partial x}$ where $f(x)$ is a general polynomial of x .

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10. (a) Write down the Hamiltonian ($\hat{H}$) for hydrogen atom in spherical polar coordinate.

(Hint : $\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial}{\partial r}) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial}{\partial\theta}) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}$)

(b) Show that the square of the total orbital angular momentum operator $\hat{L}^2$ commutes with $\hat{H}$.
(c) Write down the eigenvalues of $\hat{L}_z$ and $\hat{L}^2$ operator when they operate on $Y_2^{-1}(\theta, \phi)$.
(d) Calculate the average distance of hydrogen atom in ''1s'' state. The normalized ''1s'' wavefunction of hydrogen atom is $\psi_{1s} = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$

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(Hint: $\int_0^\infty z^n e^{-z} dz = n!$)

(e) The force constant of $^{79}Br\,^{79}Br$ is 240 Nm^{-1}. Calculate the fundamental frequency and the zero-point energy of $^{79}Br\,^{79}Br$.

1+1+2+2+2

11. (a) Write down and draw the ground (n=0) and first (n=1) excited state eigenfunctions for 1D harmonic oscillator.

(b) Show that they are orthogonal to each other.

(c) The orientation of a rigid rotator is completely specified by two angles θ and ϕ. Write down the Schrodinger equation for a rigid rotator having reduced mass μ.

(d) Evaluate its eigenvalue in terms of moment of inertia $I = \mu r^2$.

(e) Assuming $\Psi_{n,l,m}(r,\theta,\phi) = R_{nl}(r) Y_l^m(\theta,\phi)$ are the eigenfunctions for hydrogen atom, write down all possible eigenfunctions for n=3 state mentioning their degeneracies.

2+1+1+2+2