3/12/18

INDIAN INSTITUTE OF SCIENCE EDUCATION AND RESEARCH KOLKATA

PH4101 **ENDSEM** **Date: 03-12-2018**

Total Marks: 50 **Time: 180 mins**

ANSWER ANY TEN (10) QUESTIONS [Some values: Sin30°=0.5, Cos30°=√3/2]

1. A beam of X-rays is incident on a BCC crystal. If the difference between the incident and scattered wave vectors is $\vec{K} = h\hat{x} + k\hat{y} + l\hat{z}$, where $\hat{x}, \hat{y}, and\ \hat{z}$ are the unit vectors of the associated cubic lattice. What is the necessary condition for the scattered beam to give a Laue maximum? Justify your answer. **[Marks = 5]**

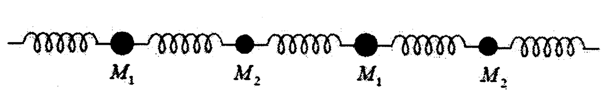
2. The second order maximum in the diffraction of X-rays of 0.20 *nm* wavelength from a simple cubic unit cell is found to occur at an angle of thirty degrees to the (1 0 0) crystal plane.(a) Calculate the distance between the lattice planes. (b) If the X-rays are reflected from the (1 1 1) plane of the unit cell, calculate the corresponding Bragg angle. **[Marks = 2x2.5]**

3. (a) Give a physical explanation of Debye frequency in solids. (b) Suppose the frequency of phonons (ω) in a one-dimensional chain of atoms is proportional to the wave vector. If n is the number density of atoms and c is the speed of the phonons, then calculate the Debye frequency, ω_D. **[Marks = 2+3]**

4. In a band structure calculation, the dispersion relation for electrons is found to be: $E(k) = \beta(\cos k_x a + \cos k_y a + \cos k_z a)$, where β is a constant and a is the lattice constant. Calculate the effective mass (a) at the boundary of the first Brillouin zone and (b) near bottom of the band. **[Marks = 2.5+2.5]**

5. Calculate the radius of the Fermi sphere of free electrons in a monovalent element with an BCC structure, in which the volume of the unit cell is a^3. Using the concept of reciprocal lattice vector explain why it should be a metal. **[Marks = 2+3]**

6. The phonon dispersion for the following one-dimensional diatomic lattice with masses M_1 and M_2 (as shown in the figure)



Is given by,

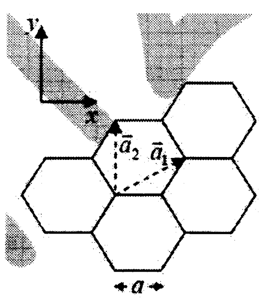
$$\omega^2(q) = K\left(\frac{1}{M_1} + \frac{1}{M_2}\right)\left[1 \pm \sqrt{1 - \frac{4M_1M_2}{(M_1+M_2)^2}\sin^2\left(\frac{qa}{2}\right)}\right],$$

where a is the lattice constant and $\boldsymbol{K}$ is the spring constant. Calculate the velocity of sound. **[Marks = 5]**

7. If the energy dispersion of a two-dimensional electron system is $E = uk$, where u is a constant and k is the momentum. (a) Show that the density of states $D(E)$ is proportional to E. (b) Show that at T = 0K, the Fermi energy E_F is proportional to electron number density n as, $E_F \propto \sqrt{n}$. **[Marks = 2+3]**

8. Consider an electron in b.c.c. lattice with unit cell lattice constant a. Prove that a single particle wave function that satisfies the Bloch theorem will have the form $f(\vec{r})e^{\vec{k}.\vec{r}}$, where $f(\vec{r}) = 1 + \cos\left[\frac{2\pi}{a}(x+y)\right] + \cos\left[\frac{2\pi}{a}(y+z)\right] + \cos\left[\frac{2\pi}{a}(z+x)\right]$. **[Marks = 5]**
9. The band energy of an electron in a crystal for a particular k -direction has the form $E(k) = A - B\cos(2ka)$, where A and B are positive constants and $0 < ka < \pi$. (a) Calculate the group velocity at Brillouin zone boundary. (b) Show that in range $\frac{\pi}{4} < ka < \frac{3\pi}{4}$ electron has a hole-like behaviour. **[Marks = 2+3]**

10. Consider a hexagonal lattice with basis vectors as shown in the figure below. Calculate the Reciprocal lattice vectors. [Assume $\vec{a_3} = a\hat{z}$]. **[Marks = 5]**



11. An intrinsic semiconductor with a band gap E_g is exposed to photons of energy $h\nu$. If $h\nu > E_g$, electron hole pairs are created through direct excitation of electrons from the valence band. If m_e is the mass of electrons and m_h is the mass of holes, then determine the wave vector $\vec{k}$ and the energy of the electron and holes. If necessary use schematic illustration of bands and direct transitions for a direct band gap semiconductor in support of your calculations. **[Marks = 2.5+2.5]**

12. Show that the paramagnetic susceptibility of free conduction electrons in a metal is independent of temperature. If necessary use schematic illustrations to explain your calculations. **[Marks = 5]**

13. Give a qualitative justification for the linear temperature dependence of electronic specific heat of metal using necessary schematic illustrations. **[Marks = 5]**