

Answer all questions

27-11-2018, Marks = 100

Q.1

For the Maxwell Lagrangian density $\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$.

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

(a) Identify the true degrees of freedom in terms of A_μ [5]

(b) Identify the constraints and interpret, [5]

(c) Find out the energy and momentum expressions after quantization: [5]

$$[a_\mu^i, a_\nu^{\dagger j}] = \delta_{\mu\nu} \delta_{ij}, \text{ where } i, j \text{ represent polarization}$$

Q.2

Find the coherent state $\langle \alpha | \alpha \rangle = \langle \alpha | \alpha \rangle$ for the single mode case and find the photon statistics with its physical interpretation. [10]

Write the Wigner function [15] for the photon added coherent state.

Hint: you may use $a = \frac{1}{\sqrt{2}} (\hat{p} + ix)$ for explicitness. $= \frac{1}{\sqrt{2}} (\frac{p}{\hbar} + ix)$

Q.3

For the non-linear Schrödinger equation, (2)

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + g|\psi|^2 \psi - m\psi$$

find the Lagrangian density.

For the grey soliton

$$\psi = \sqrt{\rho_0} \left[i \sin \theta + \cos \theta \tanh \left(\frac{\rho_0 x}{\ell} \right) \right]$$

find ℓ and the soliton

energy, $E = \int dx \mathcal{H}$, where

\mathcal{H} is the Hamiltonian density

Marks: $12\frac{1}{2} + 12\frac{1}{2} = 25$

Q.4

Define the Stokes vector means of Jones matrix and write down the physical requirements of a Mueller matrix. [15]

Use a 'classically' entangled electric field to illustrate the above point. [10]