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Final Exam MA 3103
Swarnendu Datta

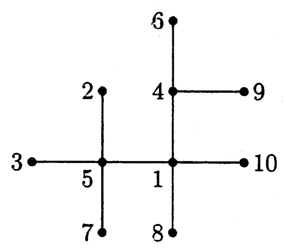
Final Exam, MA 3103

November 26, 2018, 10 — 1pm

No calculators, books, notes etc, are allowed. To get full credit, you must give justifications to all your statements.

1. (5 + 5 + 5 pts) For parts (a) and (b), please double check your answer, as there will be no partial marking.

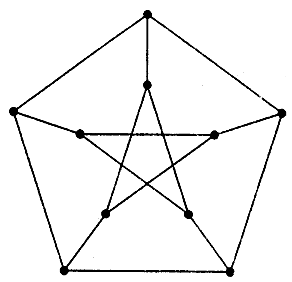
   a) Compute the Prufer code of the following tree:



   b) Construct the labelled tree whose Prufer code is 99032362.

   c) Suppose the Prufer code of a tree consists of only one number repeated several times. Find the structure of the tree.

2. (10 pts) Find the minimal number of edges which needs to be removed from the following graph in order to have an Eulerian cycle.

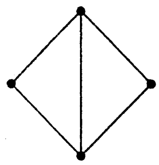


   In order the get full points, you must justify why deleting fewer edges won't work and also find an Eulerian cycle in your graph.

3. (5 + 10 pts)

   a) State the Inclusion-Exclusion principle.

   b) Find the number of ways of coloring the following graph with n colors such that the endpoints of each edge receives different colors (the answer should be a polynomial of n).



Hint: For each edge, consider the set of those colorings such that the endpoints of the chosen edge have distinct colors.

4. (6 + 4 + 6 pts) Let G be a planar graph with v vertices and e edges.

   a) Show that $e \leq 3v - 6$.

   b) Now suppose that no face of G is a triangle. Then prove that

$$e \leq 2v - 4$$

   c) Let $K_{r,s}$ denote the complete bipartite graph with $r + s$ vertices (partitioned into two sets of size r and s). Show that $K_{3,3}$ is not planar. Find all pairs (r, s) for which $K_{r,s}$ is planar.

5. (10 pts) Compute the maximum number of edges a graph with 27 vertices can have provided that the graph is triangle free. You need to give a self contained argument.