Mid semester exam: MA1101

IISER Kolkata

17 September 2018 | Full Marks: 20 | TIME: 1 HOUR

1. Let A, B and C be subsets of a universal set U such that

$$A \cup B = A \cup C \text{ and } A \cap B = A \cap C.$$

Prove that $B = C$. [4]

2. Let $A = \{2, 3, 5, 6, 7\}$ and $\rho = \{(2,2), (2,3), (2,5), (3,3), (5,5), (6,6)\}$. Find ρ^{-1}. Compute $\rho^{-1} \circ \rho$ and $\rho \circ \rho^{-1}$. [1+2+1]

3. Let A be a finite non-empty set with n elements and $f : A \to A$ be surjective. Will f be injective? Justify your answer. [1+3]

4. Prove that, using the definitions of integers and multiplication and addition on $\mathbb{Z}$,

   (a) $-2 \times 1 = -2$. [2]

   (b) For $a, b, c \in \mathbb{Z}$, $a \times (b + c) = a \times b + a \times c$. [2]

5. Let $m, n, p \in \mathbb{N}$ such that $m + p = n + p$. Then show that $m = n$. [4]
   (Hint: use induction on p).

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