Department of Mathematics and Statistics, IISER Kolkata
Analysis-III (MA3101)
Mid Semester Examination, Total Marks: 20, Time: 1 hr.

Instructor: Shirshendu Chowdhury — Date: 24th September , 2018.

Question 1. [5]

Let $f : \mathbb{R}^2 \longmapsto \mathbb{R}$ be defined by $f(0,0) = 0$ and

$$f(x,y) = xy\frac{x^2 - y^2}{x^2 + y^2} \text{ if } (x,y) \neq (0,0).$$

(i) For which vectors $u \neq 0$ does $f'(0,u)$ exist? Evaluate it when it exists.

(ii) Do $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ exists at 0?

(iii) Is f differentiable at 0?

(iv) Is f continuous at 0?

Question 2. [6]

Let $f : \mathbb{R}^2 \longmapsto \mathbb{R}^2$ be

$$(u,v) = f(x,y) = (x^2 + y^2, 2xy)$$

. Where is f invertible? State inverse function theorem and using that find

$$\frac{\partial x}{\partial u}, \frac{\partial x}{\partial v}, \frac{\partial y}{\partial u}, \frac{\partial y}{\partial v}.$$

Question 3. [9]

Justify True or False. If it is true prove it, if not give counter example or justify with proper reason.
(a) Let $f : \mathbb{R}^n \longmapsto \mathbb{R}^n$ be differentiable. Suppose that f is invertible on a neighbourhood of a point a and that $det[Df(a)] = 0$. Then f^{-1} is not differentiable at $f(a)$.

(b) Let S be an open set in $\mathbb{R}^n$ and $f : S \longmapsto \mathbb{R}^n$ be in $C^1(S)$. Let $det[Df(x)] \neq 0 \ \forall\, x \in S$. Then $f(W)$ is open set in $\mathbb{R}^n$ if $W \subset S$ is open.

(c) If $f : \mathbb{R}^2 \longmapsto \mathbb{R}$ is directionally differentiable on $\mathbb{R}^2$ and $g : \mathbb{R}^2 \longmapsto \mathbb{R}^2$ is directionally differentiable on $\mathbb{R}^2$. Then the composite function $f \circ g$ is directionally differentiable at $(0,0)$ in every directions.

THE END.

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24/09/18