INDIAN INSTITUTE OF SCIENCE EDUCATION AND RESEARCH KOLKATA

MA 4101: Complex Analysis **End-semester Exam**
Duration: 3 hours **Date : November 29, 2018**

Total Marks: 50

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1. Indicate True or False with proper explanation (no marks will be given if one indicates True of false without explanation)

$f \in \mathcal{O}(\Omega)$ &

i) *Let Ω be a domain in $\mathbb{C}$ such that $\overline{\mathbb{D}} \subset \Omega$. If, $2|f(z)| > 3$ ~~for~~ $|z| = 1$, and $f(0) = 1$ then f has a zero in $\mathbb{D}$.*

ii) *Let $f \in \mathcal{O}(\mathbb{C})$ and*

$$|f(z)| \leq M + N|z|^n \quad \forall z \in \mathbb{C} \setminus D(0; R),$$

for some constants $M, N > 0$ and $R > n$. Then f is a polynomial of degree at most n.

iii) *There exists no nonconstant entire function f such that $f(\mathbb{C}) \subset \mathbb{C} \setminus \{z \in \mathbb{C} : \Im\mathfrak{m} z = 0, \Re\mathfrak{e} z \leq 0\}$.*

iv) *There exists a unique holomorphic map $f : \mathbb{D} \to \mathbb{D}$ such that $f(1/2) = 0$ and $f(0) = 1/2$.*

v) *There exists a Möbius transformation which maps the domain*
$\left\{z = x + iy \in \mathbb{C} : x^2 + \dfrac{y^2}{4} < 1\right\}$ *onto* $\{z = x + iy \in \mathbb{C} : y > 0\}$.

vi) *There is no holomorphic function f on the unit disc $\mathbb{D}$ such that*

$$\text{nonconstant} \left(f\left(\frac{1}{n}\right)\right)^3 = \frac{1}{n} \quad \forall n \in \mathbb{N}.$$

vii) *There exists a nonconstant holomorphic function f on $\mathbb{C} \setminus \{0\}$ whose range lies in the unit disc.*

viii) *Let $D(a; R)$ be the open disc centred at a and radius r. Suppose $b \notin \overline{D(a, R)}$. Then there exists a sequence of polynomials $\{p_n\}$ in z which converges to $f(z) := \dfrac{1}{z - b}$ uniformly on $\overline{D(a; R)}$.*

($8 \times 5 = 40$ marks)

2. *Suppose f is holomorphic function on $D(0; R)$ such that f has a zero of order m at $z = a$, $|a| < R$. Then show that there exist $\varepsilon > 0$ and $\delta > 0$ such that for $0 < |w| < \varepsilon$ the function $f(z) = w$ has exactly m distinct zeros in $B(a; \delta)$.*

(10 marks)

Note: You can use well-known theorems taught in class, but you need to write precise statement of the theorem that you are using.

29/11/2018