

Indian Institute of Science Education and Research Kolkata

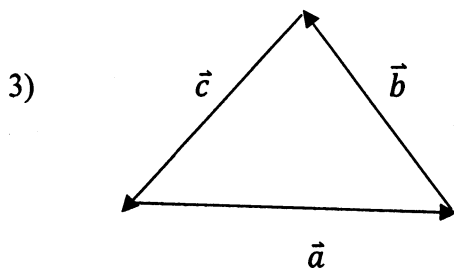
End-Semester Examination @ ID4108 (07/12/2018)

Time: 3 Hours

Full Marks: 50

1) Show that points A(1, 2, 7), B(2, 6, 3) and C(3, 10, -1) are collinear. **2 marks**

2) A vector $\vec{r} = \hat{i}x + \hat{j}y + \hat{k}z$ makes angle α , β and γ with \hat{i} , \hat{j} and \hat{k} direction respectively. Find out a relation between α , β and γ . **2 marks**



In the above triangle angle between \vec{c} and \vec{b} is A , angle between \vec{c} and \vec{a} is B , angle between \vec{a} and \vec{b} is C . Show that

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \quad \mathbf{2 \text{ marks}}$$

3) When a set of vectors $\vec{A}_1, \vec{A}_2, \vec{A}_3, \dots, \vec{A}_n$ are said to be linearly independent? **2 marks**

4) Show how to convert a point P at a location in (x, y, z) in the cartesian coordinate to the spherical polar coordinate. **2 marks**

5) Suppose you are walking from RC to your hostel with a velocity 5 km/hour (which is constant). Show that the dot product between the velocity and acceleration is zero. **2 marks**

6) Suppose force is given by $\vec{F}(x, y)\hat{i} = 3xy^2\hat{i} + (3x^2y + 4y^2)\hat{j}$. Explain whether work done by this force will be path dependent? **2 marks**

7) Solve the set of three linear equation with three variables by Cramer's rule:

$$x + 2y + 3z = -5$$

$$3x + y - 3z = 4$$

$$-3x + 4y + 7z = -7$$

2 marks

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8) Find the inverse of the following matrix

$$\begin{bmatrix} 0 & -3 & -2 \\ 1 & -4 & 2 \\ -3 & 4 & 1 \end{bmatrix}$$

4 marks

9) Write the wave function for Carbon atom in terms of Slater determinant. 3 marks

10) Solve the following 1st order linear differential equation: 3 marks

$$\frac{dy}{dx} + \frac{2}{x}y = 10x$$

11) Solve the following 2nd order linear differential equation: 3 marks

$$4\frac{d^2y}{dx^2} + 12\frac{dy}{dx} + 9y = 0$$

12) Suppose the non-homogeneous second order linear ordinary differential equation is given by $\frac{d^2y}{dx^2} + A(x)\frac{dy}{dx} + B(x)y = C(x)$. $y_1(x)$ and $y_2(x)$ are two linear independent solutions for the homogeneous equations. Find out the general solution for the inhomogeneous equation in terms of $y_1(x)$, $y_2(x)$ and their derivatives. 5 marks

12) Let $f(x)$ be a periodic function of period 2π such that

$$f(x) = \pi^2 - x^2 \text{ for } -\pi \leq x \leq \pi$$

Write down the Fourier series of $f(x)$.

5 marks

13) Show that, if a function $g(x) = \frac{d^2}{dx^2}f(x)$, then Fourier transformation of $g(x)$ can be written as below:

$$\tilde{g}(k) = -k^2 \tilde{f}(k)$$

2 marks

14) Solve the differential following second order differential equation

$$(x+2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$$

Boundary conditions are: $y(0) = 1$ and $y'(0) = -1$

5 marks

15) Find the Fourier transformation of the function

$$f(x) = xe^{-x^2}$$

4 marks