MA2103: Mathematics III
End Semester Examination
Autumn Semester 2018

Name:

Time : 90 min.

Roll Number:

Part - A

All questions are compulsory and demand short answer. [13 X 2]

1. Write down an explicit bijection between $\mathbb{R} \setminus \mathbb{N}$ and $\mathbb{R} \setminus \mathbb{Q}$.

2. Justify the following statement is true or flase: If S is a countable set then there exists an injective map $f : \mathbb{N} \to S$

3. Justify if the following statement is true or flase: For any $n \in \mathbb{N}$, there exist n uncountable sets which are not pairwise equipotent.

4. Justify the following statement is true or flase: If the sets A and B are uncountable then always $A \sim B$.

5. Justify the following statement is true or flase: $(0,1)^2 \sim \mathbb{R}^2$, when $\mathbb{R} \sim (0,1)$ is given.

6. Give an example (with justification) of a set, with a relation, which is a partial ordered set but not well ordered set.

7. With respect to the standard odering justify the following statements: $[0, 1]$ is a well orderd set.

8. With respect to the standard odering justify the following statements: $\mathbb{Z}$ is a well orderd set.

9. Justify the following statement is true or flase: For any $p \in \mathbb{N}$, if p devides a^n for some $a \in \mathbb{Z}$, then p devides a.

10. Prove or disprove that if $r_1, \ldots, r_k$ is a reduced residue system modulo m, which is odd, then $4r_1+b, \ldots, 4r_k+b$ is another reduced residue system modulo m for any integer b.

11. Prove or disprove that, $a|c$ and $b|c \implies ab|c$, where $a, b, c \in \mathbb{Z}$.

12. Prove or disprove that, for $p(\geq 2) \in \mathbb{Z}$, if $p|ab \implies p|a$ or $p|b$, where $a, b \in \mathbb{Z}$ then p is a prime number.

13. $3^{2018} \bmod 11 = ?$

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Part - B

All questions are compulsory and each carry equal marks. [6 X 4]

1. Prove that the set of all functions from $\mathbb{N}$ to $\{0, 1\}$ is equipotent with $\mathbb{R}$.
2. Prove that the number $2^{n+2} + 3^{2n+1}$ is divisible by 7.
3. Show that for all $a \in \mathbb{Z}$, a^8 is of the form $17k$ or $17k \pm 1$, $k \in \mathbb{Z}$.
4. (a) Find the solution, $(x, y) \in \mathbb{Z} \times \mathbb{Z}$, of the equation $41x + 47y = 2018$.
   (b) Let a, m be positive integers and relatively prime to each other. Then find the solution of the linear congruence $ax + 2018 \equiv 2019 \pmod{m}$.
5. Find the smallest even number $x \geq 4$ such that $3|(x+1), 4|(x+2), 5|(x+3)$ and $6|(x+4)$.
6. Show that $n^4 + 4^n$ is composite for all $n \in \mathbb{N}$, $n > 1$.

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