MA1101

Room no → 107

SHORT QUESTIONS: TIME 45 MINUTES

**Name:** **Roll Number:**

**Choose the correct option and tick. Each question carries 1 mark. More than one tick will give you 0!**

1. What is the minimum number of steps required to solve the tower of Hanoi problem with $k$ discs?
   (a) $2k - 1$.
   (b) $2^k - 1$.
   (c) $2^k + 1$.
   (d) None of the above.

2. Let $f(x) = x^3 + 6x^2 + 6$ and consider the interval $[-6, 0]$. Suppose $c$ is the number which satisfies the mean value theorem for $f$ on the mentioned interval. Which value of $c$ will satisfy the theorem?
   (a) $-5$.
   (b) $-4$.
   (c) $-3$.
   (d) Can not be determined.

3. $f(x) = x^7 + x^5 + x^3 + 1$. How many real solutions does $f(x)$ have?
   (a) 7.
   (b) 5.
   (c) 1.
   (d) None.

4. If $f : D \to \mathbb{R}$ is not continuous at a point $c \in D$, then which statement among the following is true?
   (a) $\exists\, \epsilon > 0$ such that $\forall\, \delta > 0\ \exists\, x \in D$ with $|x-a| < \delta$ and $|f(x) - f(a)| \geq \epsilon$.
   (b) $\exists\, \epsilon > 0$ such that $\forall\, \delta > 0\ \forall\, x \in D$ with $|x-a| < \delta$ and $|f(x) - f(a)| \geq \epsilon$.
   (c) $\exists\, \epsilon > 0$ such that for some $\delta > 0\ \exists\, x \in D$ with $|x - a| < \delta$ and $|f(x) - f(a)| \geq \epsilon$.
   (d) $\forall\, \epsilon > 0$ such that for some $\delta > 0\ \forall\, x \in D$ with $|x - a| \geq \delta$ and $|f(x) - f(a)| \geq \epsilon$.

5. If the roots of $x^2 - 5x + a = 0$ are real (with $a \neq 0$) then the value of $a$ is
   (a) 5.
   (b) $25/4$.
   (c) $-25/4$.
   (d) $-5$.

6. If $\alpha, \beta, \gamma$ are the three nonzero roots of $x^3 + px^2 + qx + r = 0$, then the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$ is

   (a) $-q/r$.
   (b) $-p/r$.
   (c) $q/r$.
   (d) $-q/p$.

7. There are 12 chairs in a row. Then how many people should be seated to guarantee that at least 3 persons are seating consecutive to each other?

   (a) 8.
   (b) 11.
   (c) 9.
   (d) None of the above.

8. Given that the product of two roots of the equation $x^4 + 2x^3 - 25x^2 + 26x + 120 = 0$ is 8. The roots are

   (a) $4, 2, 8, 8/3$.
   (b) $4, 2, -3, -5$.
   (c) $-4, -2, -3, -5$.
   (d) None of the above.

9. Find how many positive and negative real roots of the equation $x^3 - 6x^2 + 7x + 4 = 0$ does have.

   (a) two positive and one negative.
   (b) one positive and two negative.
   (c) all three are negative.
   (d) None of the above.

10. Let $f : \mathbb{R} \to \mathbb{R}$ be defined as

$$f(x) = \begin{cases} 0 & \text{if } x = 0, \\ x \sin(1/x) & \text{otherwise.} \end{cases}$$

   Then

   (a) $f$ is differentiable at 0.
   (b) $f$ is not differentiable at 0.
   (c) $f$ is not continuous at 0.
   (d) Nothing can be said about $f$ at 0.

Satyaki Mazumder

Room No ⊃ 107

# MA1101: End Sem Exam

Full Marks: 50

26.11.2018

**1. Attempt every question. Each question carries 3 marks.**

1A Using A.M.$\geq$ G.M. prove that, for $n > 1$,

$$n(n+1)^2 > 4(n!)^{3/n}.$$

1B Suppose the equation $ax^3 + 3bx^2 + 3cx + d = 0$, with real coefficients, has two equal roots. Prove that

$$(bc - ad)^2 = 4(b^2 - ac)(c^2 - bd).$$

1C Give an example of a function $f : [a, b] \to \mathbb{R}$ such that $f$ is continuous on $[a, b]$, differentiable in $(a, b)$ but NOT differentiable on $[a, b]$, where $a < b \in \mathbb{R}$.

1D Let $c \in \mathbb{R}$ and $f : \mathbb{R} \to \mathbb{R}$ be such that $\lim_{x \to c}(f(x))^2 = L$. If $L \neq 0$ then show, by providing an example, that $f$ may not have limit at $x = c$.

1E Let $a \in \mathbb{R}$ be such that for every $\epsilon > 0$, $0 \leq a < \epsilon$. Prove that $a = 0$.

**2. Attempt every question.**

2A If $x, y, z$ are positive real numbers and $x + y + z = 1$, then prove that

$$8xyz \leq (1-x)(1-y)(1-z) \leq \frac{8}{27}.$$

[5]

2B How many real roots the equation

$$x^5 - 5x + 2 = 0$$

have? Determine how many of them are positive and how many of them are negative using Sturm's method. [5]

2C Let $f : [a, b] \to \mathbb{R}$ be continuous on $[a, b]$ and differentiable in $(a, b)$ with $a < b \in \mathbb{R}$. If $\lim_{x \to a} f'(x) = A$, where $A \in \mathbb{R}$, then show that

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a} = A.$$

[5]

2D Let $f : \mathbb{R} \to \mathbb{R}$ be such that

$$f(x) = \begin{cases} x & \text{if } x \in \mathbb{Q}, \\ 0 & \text{if } x \in \mathbb{R} \setminus \mathbb{Q}. \end{cases}$$

(a) Show that $f$ is not continuous at $a$, where $a \in \mathbb{R} \setminus \mathbb{Q}$. [2]

(b) Is $f$ continuous at 0? Give a complete justification to your answer. (Without proper justification no marks will be given). [3]

2E Given $n$ integers $a_1, a_2, \ldots a_n$, not necessarily distinct, show that there exist integers $k$ and $\ell$ with $0 \leq k < \ell \leq n$ such that the sum $a_{k+1} + a_{k+2} + \ldots + a_\ell$ is multiple of $n$. [5]

Satyaki Mazumder