December 6, 2018 10:00 a.m. - 1:00 p.m.

Semestral Examination
MA4103
Instructor: Dr. Soumya Bhattacharya

In the following exercises, A denotes a commutative ring with identity and $R := A[x]$.

Exercise 1. (10 points)
Show that $\operatorname{Jac} R = \operatorname{rad} 0_R$.

Exercise 2. (10 points)
Determine all the units in the ring $(\mathbb{Z}/36\mathbb{Z})[x, y]$.

Exercise 3. (10 points)
Let $0 \to M \to N \to R \to 0$ be an exact sequence of A-modules. Prove that the sequence splits.

Exercise 4. (10 points)
Show that for each ideal $I \subseteq A$, the A-modules $I \otimes A/I$ and I/I^2 are naturally isomorphic.

Exercise 5. (20 points)
a) Determine $\dim (\mathbb{Z}/20\mathbb{Z})[x, y]$.
b) Show that $\mathbb{Q}[x, y, z, t]/\langle t^2 \rangle$ has no prime ideal of height 9.

Exercise 6. (10 points)
Find all the maximal ideals in the ring of complex valued continuous functions on the closed complex unit disc.

Soumya Bhattacharya
6/12/2018

Maximum score: 50 points.