Mid Semestral Examination
MA2202
Instructor: Dr. Soumya Bhattacharya

Exercise 1. (10 points)

Let $(\Omega, \mathcal{E}, P)$ be a probability space. Let $A_1, A_2, \ldots, A_n \in \mathcal{E}$ and let $A := A_1 \cup A_2 \cup \cdots \cup A_n$. Show that

$$P(A) \leq \sum_i P(A_i) - \sum_{i<j} P(A_i \cap A_j) + \sum_{i<j<k} P(A_i \cap A_j \cap A_k),$$

where the sums are over $i, j, k \in \{1, 2, \ldots, n\}$.

Exercise 2. (10 points)

A fair coin is tossed five times. Find the conditional probability of obtaining five heads given that there are at least four heads.

Exercise 3. (10 points)

Let X and Y be mutually independent random variables each having a geometric distribution with parameter p. Find the distribution of $Z := \min\{X, Y\}$.

Exercise 4. (10 points)

Let X be a nonnegative integer-valued random variable with finite expectation. Show that

$$E(X) = \sum_{l=1}^{\infty} P(X \geq l).$$

Exercise 5. (10 points)

Suppose, the average time between you receive two phone calls is ninety minutes. Assuming that you came to the examination venue in time, find the probability of there being three missed calls on your phone after this one-hour examination is over.

Soumya Bhattacharya

Maximum score: 20 points   Duration: 1 hour