Indian Institute of Science Education and Research Kolkata
Department of Mathematics and Statistics
Sobolev Space: Theory and Applications (MA5214)

Spring Sem
May 7, 2019

IISER KOLKATA

Time: **10:00–12:30** *End-Semester Exam* Marks: **50**

1. State the following theorems

(i) Poincare's inequality.

(ii) Morrey's inequality.

(iii) Lax-Milgram Theorem

(iv) Fredholm alternative.

[10]

2. Let U be the unit open ball centered at origin. Show there exists a positive constant C, depending only on n, such that

$$\int_U u^2 dx \le C \int_U |Du|^2 \, dx$$

for all $u \in H_0^1(U)$. [8]

3. Show that

$$H^{3/4}(\mathbb{R}) \subset L^\infty(\mathbb{R}).$$

[8]

4. Let $I = (-1, 1)$. Consider the function $f : H_0^1(I) \to \mathbb{R}$ defined by

$$f(v) = \int_{-1}^{1} v(x) \sin x \, dx + v(0), \quad v \in H_0^1(I).$$

(i) Show that $f \in H^{-1}(I)$.

(ii) Find a function g in $L^2(I)$ such that

$$f(v) = \int_{-1}^{1} v(x) \sin x \, dx + \int_{-1}^{1} g(x) v'(x) dx$$

for all $v \in H_0^1(I)$.

(iii) Does the above g unique? [3+5+2]

5. (i) Let $f \in L^2(B(0,1))$. Give the definition of weak solution for the following problem

$$\begin{cases} -\Delta u + u = f & \text{in } B(0,1), \\ u = 0 & \text{on } \partial B(0,1). \end{cases}$$

(ii) Show that the above problem has a unique weak solution.

(iii) Consider the map $f \mapsto Tf$, where Tf denotes the unique solution of above problem corresponding to given $f \in L^2(B(0,1))$. Show that $T : L^2(B(0,1)) \to L^2(B(0,1))$ is continuous and compact. [4+4+6]

07/05/2019