

MA1202 End Semester Examination

Total marks: 50

Time: 2 Hours 30 minutes

Answer Question number 1 and any two from the rest

$(30 + 10 \times 2 = 50)$

Q 1a) *Solve any five*

$[6 \times 5 = 30]$

a) For the matrix $A = \begin{pmatrix} 4 & -6 \\ 2 & -3 \end{pmatrix}$ find A^2 and $(I + A)^{10}$ where I is the 2×2 identity matrix.

b) Find the LU decomposition for the matrix $A = \begin{pmatrix} 1 & 6 & -5 \\ -2 & -5 & 11 \\ 1 & -15 & -10 \end{pmatrix}$ and use it to find $\det A^2$. Remember - we are asking for the LU decomposition, not the PLU decomposition!

c) Find the general and singular solutions of the equation

$$y + xy' + (y')^3 = 0$$

d) Solve the equation

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 0$$

subject to $y(1) = y(-1) = 0$.

e) Evaluate the integral $\oint_C \frac{2z-5}{(z+1)^3(z-1)^2(z^2+1)^2} dz$ where C is the circle of radius 5 centered at $1 + i$, traversed counterclockwise.

f) Your friend claims that she has found an analytic function of $z = x + iy$ whose real part is $\frac{y+1}{x^2+y^2+2x+2y+2}$. Could she be correct? Explain.

Dr. F. Khan
08/05/2019

Q 2) Find all eigenvalues and eigenvectors of the matrix

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

Q 3) Use the factorization $\left(\frac{d}{dx} + i\right)\left(\frac{d}{dx} - i\right) = \frac{d^2}{dx^2} + 1$ to solve the differential equation

$$\frac{d^2 y}{dx^2} + y = \cos x$$

subject to $y(0) = y'(0) = 0$.

Q 4) Evaluate the improper integral

$$\int_{-\infty}^{\infty} \frac{dx}{x^4 + 15x^2 + 20}$$

using the method of residues.

Q 5) Convert

$$\int_0^\pi \frac{d\theta}{1 + \sin^2 \theta}$$

to a complex integral over a closed contour and evaluate it by the method of residues.

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