Endsem Exam (Many Body Physics: PH 4206) 3 May, 2019

1. The Hamiltonian for a 1D harmonic oscillator with mass m and frequency ω_0 is:

$$\hat{\mathcal{H}} = \frac{\hat{p}^2}{2m} + \frac{1}{2} m\omega_0^2 \hat{x}^2.$$

Here, the momentum and position operators satisfy canonical commutation relations. The "greater" Green's function corresponding to operators $\hat{A}$ and $\hat{B}$ is defined as:

$$G^{>}_{\hat{A}\hat{B}}(t,t') \equiv -i\langle \hat{A}(t)\hat{B}(t')\rangle.$$

Here, $\langle\rangle$ implies average over quantum states at thermal equilibrium with temperature T. Find out $G^{>}_{\hat{x}\hat{x}}(t)$ explicitly for the harmonic oscillator described above. By suitable fourier transform, derive the corresponding green's function $G^{>}_{\hat{x}\hat{x}}(\omega)$ in frequency domain. Please work out all intermediate steps (with explanations, as necessary) to arrive at your final results. You might find the use of ladder operators:

$$\hat{a} = \alpha_1 \hat{x} + i\alpha_2 \hat{p}\,, \quad \hat{a}^\dagger = \alpha_1 \hat{x} - i\alpha_2 \hat{p}$$

helpful for your purpose. Here, $\alpha_1 = \sqrt{(m\omega_0/2\hbar)}$, and $1/\alpha_2 = \sqrt{2m\omega_0\hbar}$.

[points: 13]

2. It is given that the general formalism of *Equation of Motion* method yields:

$$\sum_{\nu''} \left(\delta_{\nu\nu''}(\omega + i\eta) - t_{\nu\nu''}\right) G^R(\nu'',\nu',\omega) = \delta_{\nu\nu'} + D^R(\nu,\nu',\omega)$$

$$\text{with} : D^R(\nu,\nu',\omega) = -i\int_{-\infty}^{\infty} dt e^{i(\omega+i\eta)(t-t')}\Theta(t)\left\langle\{-[V_{\text{int}},\hat{a}_\nu](t),\hat{a}_{\nu'(t')}\}\right\rangle.$$

Here, all notations carry standard meaning. Consider a single-impurity (with noninteracting electrons on impurity sites, i.e. $U = 0$) Anderson Model Hamiltonian:

$$\mathcal{H} = \sum_{k,\sigma} \varepsilon_k \hat{c}^\dagger_{k\sigma}\hat{c}_{k\sigma} + \sum_{\sigma} \varepsilon_d \hat{d}^\dagger_\sigma \hat{d}_\sigma + \sum_{k,\sigma} \left(t_k \hat{d}^\dagger_\sigma \hat{c}_{k\sigma} + \text{h.c.} \right)$$

where, $\hat{d}_\sigma$ annihilates an electron with spin σ on the impurity site, all other notations carry the usual meaning. Obtain the equations of motion for the following Green's functions (in frequency domain) corresponding to:

(a) $G^R(d\sigma, t-t') = -\Theta(t-t')\left\langle \{\hat{d}_\sigma(t), \hat{d}^\dagger_\sigma(t')\} \right\rangle$,

(b) $G^R(k\sigma, d\sigma, t-t') = -\Theta(t-t')\left\langle \{\hat{c}_{k\sigma}(t), \hat{d}^\dagger_\sigma(t')\} \right\rangle$,

(c) $G^R(d\sigma, k\sigma, t-t') = -\Theta(t-t')\left\langle \{\hat{d}_\sigma(t), \hat{c}^\dagger_{k\sigma}(t')\} \right\rangle$,

(d) $G^R(k\sigma, t-t') = -\Theta(t-t')\left\langle \{\hat{c}_{k\sigma}(t), \hat{c}^\dagger_{k\sigma}(t')\} \right\rangle$.

[points: 8]

2. Consider a 1D harmonic oscillator (unit mass), coupled linearly to position $\hat{x}$ with a time-dependent force field $F(t)$, described by the Hamiltonian:

$$\hat{\mathcal{H}} = \frac{\hat{p}^2}{2} + \frac{1}{2}\omega_0^2\hat{x}^2 - \hat{x}F(t).$$

(a) Using Heisenberg equations of motion: $i\partial_t\hat{A} = [\hat{A}, \hat{\mathcal{H}}]$, derive the equation of motion for the operator $\hat{x}$. Take $\hbar = 1$ for simplicity.

(b) "Retarded response function" is defined as: $\chi^R(t) = i\Theta(t)\langle[\hat{x}(t), \hat{x}(0)]\rangle$ in linear response theory. Here, the expectation value is taken *in the absence of the perturbation*. Solving the equation of motion for $\hat{x}$ with $F(t) = 0$, evaluate $\chi^R(t)$ explicitly, Also obtain $\chi^R(\omega)$ in the frequency domain.

[points: 4+5=9]