MA 5215: MID SEMESTER EXAM

**IISER Kolkata, Spring Semester, 2018-19.**
Instructor: **Imran H. Biswas**
Date: **21.02.2019** Time: 15:30-17:00 Full marks: 20

(Q.1) 4+4+3

**a.)** Let $X$ be a random variable that follows standard normal distribution (i.e. normal with $\mu = 0$ and $\sigma = 1$) and $Y = X^2$. Find out the probability density function of $Y$.

**b.)** Let $X$ be a real valued random variable such that $X \geq 0$. (Do not assume integrability of $\frac{1}{X}$) Show that

$$\lim_{y \to \infty} yE(X^{-1} I_{X>y}) = 0.$$

**c.)** Let $X$ be random variable such that $X \geq 0$ and $0 < E(X^2) < \infty$. Show that

$$P(X > 0) \geq \frac{(E(X))^2}{E(X^2)}.$$

(Q.2) 5+4

**a.)** Let $X_1, X_2, ....$ be an i.i.d sequence of random variables with $P(X_i > x) = e^{-x}$ for $x \geq 0$. Show that $\limsup_{n \to \infty} \frac{X_n}{\log n} = 1$ a.s.

**b.)** Let $(\Omega, \mathcal{F}, P)$ be a probability space and $\mathcal{G}$ be a sub $\sigma$-algebra of $\mathcal{F}$. Let $X$ be an integrable random variable and $Z = E[X|\mathcal{G}]$. Show that if $E[X^2] = E[Z^2] < \infty$, then $X = Z$ almost surely.

**Good luck !!!**

Imran Habib Biswas