Indian Institute of Science Education Research Kolkata

Mid-semester Examination:

CH4205 (Statistical Thermodynamics): 20/02/2019

Time: 10:00 am to 11:30 am **Full Marks: 20**

1) Consider two, two-state paramagnetic systems. Each has total N ($N\downarrow + N\uparrow$) = 25 spins. The two systems are in weak thermal contact with each other. System A starts with $N\uparrow = 10$ and system B starts with $N\uparrow = 15$. Calculate entropy of system A, system B and the combined system in their initial states. **2 marks**

2) A rectangular box contains 100 molecules of a gas. What is the probability that all the molecules are situated in the leftmost 99% of the box? **2 marks**

3) Show that when two systems are in thermal equilibrium, then temperature of both the systems are equal. **3 marks**

4) Explain phase space, μ-space and τ-space. **3 marks**

5) **Classical harmonic oscillators in the microcanonical ensemble:**
A system consists of N non-interacting harmonic oscillators with coordinates q_i and momenta p_i. The Hamiltonian reads

$$H = \sum_{i=1}^{N} \left(\frac{1}{2} p_i^2 + \frac{1}{2} q_i^2 \right)$$

i.e., the masses and frequencies of the oscillators are set to unity.

a) Calculate energy E and the heat capacity C_v as functions of T and N. **3 marks**

6) **Entropy of N spin-1 atoms:**
Consider a lattice with N (distinguishable) spin-1 atoms. Each atom can be in one of the three spin states $S_z = -1, 0, +1$. Let n_-, n_0, n_+ denote the respective numbers of atoms in each of those spin states ($N = n_- + n_0 + n_+$)

a) Calculate the entropy as a function of n_-, n_0 and n_+. Use the Stirling formula to approximate the factorials.

b) Which macrostate, i.e., which n_-, n_0, n_+ maximizes the entropy? **(2 + 2) marks**

7) Consider three spin ½ particle in a magnetic field H along z axis. Corresponding to the quantum number m_i of each spin, the particles have an energy $\pm\mu H$ (energy of a magnetic moment μ in a field H is $-\mu H$). Calculate average moment of the first spin. **3 marks**