

PH 4207, Mid Semester Exam (Total Marks 25)  
(22/02/19)

1. (a) Consider the unit vector on the Bloch sphere, given by  $\hat{n} = (\sin\theta \cos\phi, \sin\theta \sin\phi, \cos\theta)$ . Show explicitly using unitary rotation on the Bloch sphere, how can one go from the state  $|0\rangle$  in the Bloch sphere to  $\hat{n}$  in exactly one step. [4]

- (b) Suppose  $R_y(\theta)$  and  $R_z(\theta)$  are the rotation operators for a qubit about the y and z axes and suppose you are given a one qubit unitary operation  $W = R_z(\alpha)R_y(\theta)R_z(\alpha)$ . Show that W can be written as  $W = A\sigma_x B\sigma_x$ , where A and B are unitary operations given by  $A = R_z(\alpha)R_y(\frac{\theta}{2})$  and  $B = R_y(\frac{-\theta}{2})R_z(-\alpha)$ . What does the operator AB correspond to? [3]

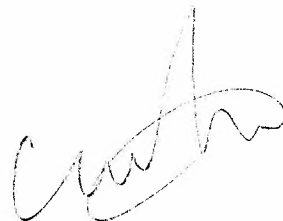
2. (a) Consider a quantum state of an electron consisting of a magnetic moment,  $\vec{\mu} = \gamma \hbar \vec{\sigma}$ , where,  $\vec{\sigma} = \hat{i} \sigma_x + \hat{j} \sigma_y + \hat{k} \sigma_z$ , evolving under a magnetic field. The Hamiltonian,  $H$  under a static magnetic field B applied along the x-axis is given by,

$$H = -\frac{1}{2} \hbar \omega \sigma_x,$$

where,  $\omega = \gamma B$  and  $\gamma$  is the gyromagnetic ratio. If initially the electron is in the state  $|0\rangle = |\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ , find out the minimum time necessary to flip the spin from  $|\uparrow\rangle$  to  $|\downarrow\rangle$ . Find also the time necessary to take the initial state,  $|\uparrow\rangle$  to an arbitrary state,  $\phi = \frac{|\uparrow\rangle + i|\downarrow\rangle}{\sqrt{2}}$ . Why this state cannot be distinguished from the state  $|\uparrow\rangle$ . [4]

- (b) Calculate the expectation value of  $\vec{\mu}$  in an arbitrary state after a time t since the evolution has started from it's initial state. Please give a physical interpretation of the result. [3]

3. Consider the singlet state,



$|\psi\rangle = \frac{1}{\sqrt{2}}[|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2]$ , where  $|\uparrow\rangle=|0\rangle=\begin{pmatrix} 1 \\ 0 \end{pmatrix}$  and  $|\downarrow\rangle=|1\rangle=\begin{pmatrix} 0 \\ 1 \end{pmatrix}$  are two orthogonal states.

- (a) Show that the quantum mechanical expectation value,  $E(n, m) = \langle\psi|(\vec{\sigma}^A \cdot \hat{n})(\vec{\sigma}^B \cdot \hat{m})|\psi\rangle$ , is given by  $E(n, m) = -\hat{n} \cdot \hat{m}$ , where,  $\hat{m}$  and  $\hat{n}$  are unit vectors and the superscript A and B refer to Hilbert spaces of Alice and Bob (as done in class). [3]
- (b) The CHSH-inequality is given by,  $|E(n, m) - E(n, m')| + |E(n', m') + E(n', m)| \leq 2$ . Find the angles where the inequality is maximally violated. Interpret the result. [2]

4. Consider the 4 particle state,

$$|W\rangle = \frac{1}{2\sqrt{2}}(|1100\rangle + \sqrt{2}|1010\rangle - |1001\rangle + |0011\rangle - \sqrt{2}|0101\rangle + |0110\rangle).$$

Perform the the Schmidt decomposition of  $|W\rangle$ , such that you partition the system into two sub-systems (13|24), i.e.,  $|W\rangle = \sum_i \lambda_i |e_i\rangle_{13} |e_i\rangle_{24}$ . Subsequently, evaluate the entanglement between these two sub-systems (13|24). How many e-bits of entanglement do you obtain? Is it a maximally entangled state? [6]