May 2, 2019

Semestral Examination

MA2202

Instructor: Dr. Soumya Bhattacharya

Exercise 1. (20 points)
Let $X \sim$ Bernoulli(p), $Y \sim$ Bernoulli(q) be independent and let $Z := X + Y - XY$.

(a) Find the probability distribution of Z.
(b) Find the conditional probability distribution of $Y|Z = 1$.

Exercise 2. (30 points)
Let X and Y be iid Poisson(λ) random variables. Let $Z := \min\{X, Y\}$ and $W := \max\{X, Y\}$.

(a) Determine the correlation coefficient of $Z + W$ and $X - Y$.
(b) Write down the joint pmf of Z and W.
(c) Compute $E(Z|W)$.

Exercise 3. (10 points)
Let X be a random variable and let Y be a positive random variable such that $X|Y \sim$ Normal$\left(0, \frac{1}{Y^2}\right)$. Show that $\mathrm{Var}(X) \geq 1/E(Y^2)$.

Exercise 4. (20 points)
A gambler plays a game in which on each play he wins Rs. 1/- with probability p and loses Rs. 1/- with probability $q := 1 - p$. However, once he loses all his money, he can not gamble any more. The gambler starts with Rs. R/-, where R is a positive integer. To evade the possibility of losing all his money, he chooses an integer $M > R$ and decides to quit gambling as soon as he has Rs. M/-. Find the probability that he leaves the game with Rs. M/- in both of the following cases:

(a) $p \neq q$.
(b) $p = q = 1/2$.

Soumya Bhattacharya

Maximum score: 50 points. Time: 2.5 hours