IISER Kolkata **End-semestral Examination**

MA 3201: Topology

Date: April 30, 2019 **Duration: 2 hours 30 minutes**

Maximum marks 50

1. Prove or disprove by a counterexample (there is no credit without a proof or a counterexample)
   (i) product topology on the cartesian product of any collection of metric spaces is a metrizable space.
   (ii) the cartesian product of any collection of Hausdorff topological spaces is Hausdorff.
   (iii) if Y is a compact subspace of a topological space X then Y is closed in X.
   (iv) consider a bijection $f : \mathbb{Z} \to \mathbb{Q}$ as a function from $\mathbb{Z}$ to $\mathbb{R}$. Then f is necessarily the restriction of some continuous function $g : \mathbb{R} \to \mathbb{R}$.
   (v) there exists a bijection $f : \mathbb{Q} \to \mathbb{Z}$ (viewed as a function from $\mathbb{Q}$ to $\mathbb{R}$) such that f is the restriction of some continuous function $g : \mathbb{R} \to \mathbb{R}$.
   (vi) any continuous image of a locally compact space is locally compact.
   (vii) any locally compact Hausdorff space is completely regular.
   (viii) any normed linear space is second countable.
   (ix) there exists a non-constant continous function from $\mathbb{R}$ to $\mathbb{Z}$.
   (x) suppose that X is a T_1-space and A is any closed subset of X such that any continuous function $f : A \to \mathbb{R}$ has a continuous extension to X, then X is normal.

   4+ 3+2+5+4+3+3+5+4+5
2. Suppose that $f : (0,1) \to [0,1]$ is a one-to-one continuous function. Prove that f cannot be surjective. 5

Indicate which of the following statements are TRUE or FALSE. Each question carries two marks. If a statement is *false* then provide a counterexample (with a proof of why it is indeed a counterexample), without counterexample no marks will be given for FALSE statements. For wrong indication of TRUE or FALSE you will get −1 (negative one). [7 × 2]

3. If a topological space contains uncountably many open sets then it cannot be separable.
4. If $A = \mathbb{N}$ and $B = \{n + \frac{1}{2n} \mid n \in \mathbb{N}\}$, then there exists disjoint open subsets U_A and U_B of $\mathbb{R}$ such that $A \subseteq U_A$ and $B \subseteq U_B$.
5. A subspace of a metric space is compact $\Longleftrightarrow$ it is closed and bounded.
6. Suppose that X is a normal space, then X is metrizable $\Longleftrightarrow$ X is second countable.
7. A bounded continuous function on $(0,1]$ has a continuous extension to $[0,1]$.
8. *Let $g : X \to Y$ be a mapping. A subset $E \subseteq X$ is said to* **determine** *g, if $h : X \to Y$ is another function such that $g_{|E} = h_{|E}$, then $g = h$ on X.*

   Suppose that $f : X \to Y$ is a continuous mapping of a metric space into another, X is compact. Then there is a countable subset of X which determines f.

9. Let X and Y be topological spaces, A a subspace of X. Then a continuous mapping of A into Y has at most one continuous extension.