**Department of Mathematics and Statistics, IISER Kolkata**
**Mathematics-II (MA1201)**
**End semester exam, Total Marks: 50, Time: 2.5hrs.**

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**Question 1.** [6+4=10]

(a) State Fundamental Theorem of Calculus (First Form). Use it to calculate

$$\int_1^4 f dx, \text{ where } f(x) = [x], \ x \in [1,4]$$

and $[x]$ is the greatest integer not greater than $x$ .

(b) If $f : [0,\infty) \longmapsto \mathbb{R}$ is continuous on $[0,\infty)$ and

$$F(x) = \int_0^x (x-t)f(t)dt, \ x \geq 0.$$

Then show that

$$\frac{d^2F}{dx^2} = f(x) \ \ \forall \ x \geq 0.$$

**Question 2.** [6+4=10]

(i) Suppose $T : \mathbb{R}^2 \longmapsto \mathbb{R}^2$ is a bijective linear map. Let $\{v_1, v_2\}$ be a Basis of $\mathbb{R}^2$. Show that $\{T(v_1), T(v_2)\}$ is a Basis of $\mathbb{R}^2$.

(ii) Let $T : \mathbb{R}^2 \longmapsto \mathbb{R}^3$ be given by

$$T(x,y) = (x, x+y, y).$$

Write down the matrix of $T$ with respect to the basis $\{(1,0),(0,1)\}$ of $\mathbb{R}^2$ and Basis $\{(1,0,0),(0,0,1),(0,1,0)\}$ of $\mathbb{R}^3$.

**Question 3.** [5+5=10]

(i) Let $0 < r < 1$, then show that

$$\lim_{n \longmapsto \infty} nr^n = 0.$$

(ii) Test the convergence of the series

$$\frac{1+2}{2^3} + \frac{1+2+3}{3^3} + \frac{1+2+3+4}{4^3} + \ldots\ldots$$



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2/5/19

**Question 4.** [5+5=10]

Determine for each of the following sets, whether or not it is countable. Justify your answers.

(i) The Set $S$ of all functions $f : \{1, 2, 3, 4, 5\} \longmapsto \mathbb{N}$.

(ii) The set $I$ of all two element subset of $\mathbb{N}$.

**Question 5.** [10]

Justify True or False, If it is true prove it if not give counter example or proper reason.

(i) Let $\{a_n\}$ be a bounded sequence and $\{b_n\}$ converges to zero. Then $\{a_n b_n\}$ converge to zero.

(ii) Let $f : [0, 1] \longmapsto \mathbb{R}$ be Riemann integrable on $[0, 1]$ and $f(x) > 0 \ \forall \ x$ in $[0, 1]$. Then $\frac{1}{f}$ is Riemann integrable on $[0, 1]$.

(iii) The Power set of $\mathbb{N}$ i.e $\mathcal{P}(\mathbb{N})$ is countable.

(iv) If $F : \mathbb{R} \longmapsto \mathbb{R}$ is such that $F(0) = 0$ and $|F(x) - F(y)| = |x - y|$ then $F(x) = x$ or $F(x) = -x$.

**THE END.**