Indian Institute of Science Education and Research Kolkata
Department of Physical Sciences

PH4205 :: General Theory of Relativity and Cosmology
End-semester Examination (Spring 2019)
Marks: 50, Duration: 2.5 hour

1. Show that 4-velocity of a particle $u^\alpha = \frac{dx^\alpha}{d\tau}$ is a contra-vector under general coordinate transformation where τ is proper time. Further, show that its squared magnitude is $-c^2$ and the quantity $g_{\alpha\beta}u^\alpha$ transforms as a tensor of rank $(0,1)$. [3+3+4]

2. By computing all Christoffel symbols and expressing equation for *great circles* in (θ, ϕ) coordinates, show that *geodesics* on a two-sphere with the metric $ds^2 = 2d\theta^2 + (1 - \cos 2\theta)d\phi^2$ are *great circles.* [4+3+5]

3. Write down the Killing equation for the metric $ds^2 = -c^2dt^2 + dy^2$. Find the general solution of the corresponding Killing equation. Show that there are 3 independent *Killing vectors* and write them down explicitly. [1+3+4]

4. Compute the stress-energy tensor for a scalar field ϕ described by the action $S_\phi = \int d^4x\sqrt{-g}[-\frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi]$. By treating the stress-energy tensor as a perfect fluid, compute the corresponding equation of state. In a fluid medium the squared speed of sound is $c_s^2 = (\partial P/\partial\rho)$ in natural units. How does the speed of sound in the given perfect fluid medium compare with the speed of light? Given $\delta g = gg^{\mu\nu}\delta g_{\mu\nu}$. [4+3+3]

5. It's known that the stress-energy tensor for Maxwell fields is *trace-free.* Show that the corresponding equation of state is $w = \frac{1}{3}$ when treated it as a perfect fluid. Solve Friedman equation for a flat FRW universe containing such a fluid. Plot corresponding scale factor $a(t)$ and energy density $\rho(t)$ and $\rho(a)$. [3+4+3]

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