Indian Institute of Science Education and Research Kolkata
Department of Mathematics and Statistics
Ordinary differential Equations (MA4202)

IISER KOLKATA

Spring Sem
May 8, 2019

Time: 14:00–16:30 | *End-Semester Exam* | Marks: 50

1. Determine whether the following statements are TRUE or FALSE and justify your answer.

   (i) $\exp(A+B) = \exp(A)\exp(B)$ for every matrices $A, B \in \mathbb{R}^{n\times n}$.

   (ii) The following boundary value problem

   $$\begin{cases} y'' + \sin(x)y = 0, \\ y(0) = y(1) = 0, \end{cases}$$

   has two linear independent solutions.

   (iii) The origin is an unstable equilibrium point of the system

   $$\begin{cases} x_1' = 2x_1 + x_2, \\ x_2' = x_1 + 2x_2. \end{cases}$$

   (iv) Any non-trivial solution of

   $$y'' + qy' + ry = 0$$

   has atmost finitely many zeros in $[0, 1]$.

   (v) Any solution of

   $$y'' + (\sin x + 2)y = 0$$

   has infinite number of zeros in $\mathbb{R}$. [3× 5]

2. (a) State a sufficient condition for the asymptotic stability of a equilibrium point of

   $$\dot{x} = f(x).$$

   (b) Show that origin is a stable equilibrium point of the equation

   $$x'' + x' + q(x) = 0$$

   where q is a given continuous function satisfying $x\,q(x) > 0$ for all $x \neq 0$.

   [3+6]

Rajib [signature]
08/05/2019

3. Compute the exponential of the matrix

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}.$$

[7]

4. Find the Green's function G for the differential operator $Lu = \frac{d^2u}{dx^2} + u$, under the boundary condition

$$u(0) = 0 = u'(1).$$

Let f be a continuous function on $[0,1]$. Show that the function $u : [0,1] \to \mathbb{R}$, defined by

$$u(x) = \int_0^1 G(s,x)f(s)ds$$

satisfies

$$\begin{cases} u'' + u = f, \\ u(0) = 0 = u'(1). \end{cases}$$

[5+5]

5. State Sturm-Liouville theorem. Verify it for the following eigenvalue problem

$$\begin{cases} y'' + y + \lambda y = 0, \\ y(0) = 0 = y'(1). \end{cases}$$

[2+7]

End of exam