IISER Kolkata
End-Semester Examination: Third Year Semester II
PH3201 (Basic Statistical Mechanic)
Time: 2 hours 30 minutes; Full Marks 50
Answer all questions

1. a) A set of N localized particles, each of which can exist in levels of energy $0, \epsilon, 2\epsilon, 3\epsilon$ with degeneracies $1, 3, 3, 1$ respectively at a temperature T. Show that the Helmholtz free energy F of the set of particles is

$$F = -3Nk_BT\ln(1 + \exp(-\tfrac{\epsilon}{k_BT})).$$

b) A 3-dimensional isotropic harmonic oscillator has energy levels

$$\epsilon_{n_1,n_2,n_3} = \hbar\omega(n_1 + n_2 + n_3 + \tfrac{3}{2}),$$

where each n_i can take values $0,1,2,3......$etc. Find the degeneracies of the levels $\frac{7\hbar\omega}{2}$ and $\frac{9\hbar\omega}{2}$. If the system is in equilibrium at a temperature T, find the condition so that the level $\frac{7\hbar\omega}{2}$ is less populated than the other.

c) Consider an LC circuit where the emf is switched off. The circuit is in equilibrium with a temperature T. Find the root mean square value of the current as a function of the temperature.

2. a) H is the Hamiltonian of a system and ρ is the corresponding density matrix. Show that

$$i\hbar\tfrac{\partial\rho}{\partial t} = -[\rho, H].$$

b) A uniform magnetic field is applied in the z-direction to a distribution of electrons at temperature T. Find out the density matrix. Using this, arrive at the ensemble average of σ_z, where σ_i's are the Pauli matrices. Also find ot the susceptibility of the system.

3. a) Consider a system of non-interacting electrons. Show that the probability of finding an electron with an energy Δ above the chemical potential is equal to the probability of finding no electron in a state with energy Δ below the chemical potential.

b) Estimate the root mean square fluctuation in the number of photons of mode frequancy ω in a cavity. Compare this with $<n>$.

c) Using the Bose-Einstein distribution, argue that the chemical potential of a boson has to be negative.

4. Thermodynamic potential Φ of a sytem is given in terms of the order parameter ϕ as

$$\Phi = \Phi_0 + \tfrac{a}{2}\phi^2 + \tfrac{b}{4}\phi^4.$$

a) Plot the potential qualitatively against ϕ for (i) $a > 0$ and (ii) $a < 0$.
b) Find the equilibrium of the system in terms of ϕ for (i) $a > 0$ and $a < 0$.
c) Assume that $a = \alpha(T - T_c)$ where α is a constant and the subscript c stands for a criticality. Argue that for $b > 0$ there is a second order phase transtion as the temperature is increased.
d) Show that there is no latent heat in the transition, but there is a discontinuity in the specific heat.