MIDSEM EXAM: PH4208, SOFT CONDENSED MATTER PHYSICS, SPRING 2019

MARKS: 20, TIME: 1:30 HOURS, 21 FEBRUARY 2019. 03:30 PM

ANSWER A MINIMUM OF 20 MARKS FROM THE QUESTIONS. IF RUNNING SHORT IN TIME, ANSWER AS MUCH AS YOU CAN. WRITE EVERY STEP OF YOUR CALCULATION. GOOD LUCK!

1. The Lennard-Jones potential for molecular interaction is given by:

$$U(R) = 4\epsilon\left[\left(\frac{\sigma}{R}\right)^{12} - \left(\frac{\sigma}{R}\right)^{6}\right].$$

Sketch U/ϵ as a function of R/σ. Find the minimum and the zero of the curve. Explain briefly what this potential implies about the attraction and repulsion between molecules.

[2 marks]

2. Consider the classical ideal gas. What is the probability of finding a particle with speed v if the particles are non-relativistic (you need to normalize the probability to get the final answer)? Find the energy of the system from the velocity distribution.

[3 marks]

3. Take two liquid crystal phases of your choice and identify the a) invariances, b) order, c) rigidity, and d) new modes of long-wavelength fluctuations in those phases.

[3 marks]

4. Show that $dp = nd\mu + sdT$, where symbols have usual meaning and small letters mean volume densities. Further, find the isothermal compressibility as a function of number density and the chemical potential.

[4 marks]

5. How much heat must be added to a large system at 298K for the number of accessible states to increase by a factor of 10^6?

[2 marks]

6. For a system the number of accessible states $\Omega(N, V, E)$ varies as $\Omega = A\exp[\gamma(VE)^{1/2}]$, with A, γ as constants. At what value of the energy is the temperature zero?

[2 marks]

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7. Consider a lattice gas model of a system in which there are N sites, each of which can be empty or occupied by one particle, the energy cost being ϵ for each occupied particle. Further, each particle has a magnetic moment μ which in the presence of a magnetic field leads to a shift in energy into either μB or $-\mu B$. Calculate the average energy and the magnetization of the lattice gas at some temperature T.

[**4 marks**]

8. $\Sigma(N, V, E) = (\frac{V}{h^3})^N \frac{(2\pi m E)^{3N/2}}{N!(3N/2)!}$ for an ideal non-relativistic gas (symbols have usual meaning). Show that, $P = \frac{2}{3}\frac{E}{V}$ for this gas.

[**3 marks**]

9. Draw the pressure-volume phase diagram for a one-component classical fluid. Include short discussions on the phase behaviour above, at, and below the critical temperature and the coexistence curve. Identify one set of criteria to find the critical point.

[**3 marks**]

10. Show, in Bragg-Williams theory of Ising spins, $m = \pm[3(T_c - T)/T]^{1/2}$ for small values of m, where symbols have ususal meaning.

[**3 marks**]

Formulae you may find useful (but may not require to use all of them):

1. The first law of thermodynamics:

$$dE = TdS - PdV + \mu dN$$

2.

$$C_v = \left(\frac{\partial E}{\partial T}\right)_{N,V}, C_p = \left(\frac{\partial(E + PV)}{\partial T}\right)_{N,P}$$

3.

$$F = E - TS, \qquad G = F + PV,$$

where F, G are Helmholtz and Gibbs free energies respectively.

4.

$$\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -1$$

5.

$$\Gamma(\nu) = \int_0^\infty e^{-x}x^{\nu-1}dx; \qquad \nu > 0$$

$\Gamma(n) = (n-1)!,$ $\quad$ $\Gamma(n + 1/2) = (n - 1/2)(n - 3/2)...3/2.1/2.\sqrt{\pi},$ $\quad$ $\Gamma(1/2) = \sqrt{\pi}$

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6. Hyper-volume and hyper-surface area of n dimensional sphere of radius R:

$$V_n(R) = \frac{\pi^{n/2}}{(n/2)!}R^n, \qquad S_n(R) = \frac{2\pi^{n/2}}{\Gamma(n/2)}R^{n-1}$$

8. Stirling's formula:

$$\ln(N!) \simeq N \ln N - N, \qquad N \gg 1$$

9. 1 eV $= 1.6 \times 10^{-19}$ J, $h = 6.62 \times 10^{-34}\text{m}^2\text{.kg/s}$, $c = 3 \times 10^8\text{m/s}$, $m_e = 9.11 \times 10^{-31}\text{kg}$, $m_p = 1.67 \times 10^{-27}\text{kg}$, $k_B = 1.38 \times 10^{-23}\ \text{m}^2\ \text{kg}\ \text{s}^{-2}\ \text{K}^{-1}$.

17. Mean thermal wavelength, $\lambda = h/(2\pi m k_B T)^{1/2}$.

18. The grand potential in GCE is given by: $d\tilde{A} = -SdT - Nd\mu - PdV$.

19. $\ln(1+x) = x - x^2/2 + x^3/3 - x^4/4 + ..$

21. Single particle (non-relativistic classical gas) energy levels:

    1d: $\epsilon = (h^2/8mL^2)n_x^2$

    2d: $\epsilon = (h^2/8mL^2)(n_x^2 + n_y^2)$

    3d: $\epsilon = (h^2/8mL^2)(n_x^2 + n_y^2 + n_z^2)$

23. $\int_0^\infty e^{-\alpha y^2} y^\nu dy = \frac{1}{2\alpha^{(\nu+1)/2}}\Gamma\left(\frac{\nu+1}{2}\right), \qquad \nu > -1$

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