

Endsem Exam (Many Body Physics: PH 4206) 3 May, 2019

1. The Hamiltonian for a 1D harmonic oscillator with mass  $m$  and frequency  $\omega_0$  is:

$$\hat{\mathcal{H}} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega_0^2\hat{x}^2.$$

Here, the momentum and position operators satisfy canonical commutation relations. The “greater” Green’s function corresponding to operators  $\hat{A}$  and  $\hat{B}$  is defined as:

$$G_{\hat{A}\hat{B}}^>(t, t') \equiv -i\langle \hat{A}(t)\hat{B}(t') \rangle.$$

Here,  $\langle \rangle$  implies average over quantum states at thermal equilibrium with temperature  $T$ . Find out  $G_{\hat{x}\hat{x}}^>(t)$  explicitly for the harmonic oscillator described above. By suitable fourier transform, derive the corresponding green’s function  $G_{\hat{x}\hat{x}}^>(\omega)$  in frequency domain. Please work out all intermediate steps (with explanations, as necessary) to arrive at your final results. You might find the use of ladder operators:

$$\hat{a} = \alpha_1\hat{x} + i\alpha_2\hat{p}, \quad \hat{a}^\dagger = \alpha_1\hat{x} - i\alpha_2\hat{p}$$

helpful for your purpose. Here,  $\alpha_1 = \sqrt{(m\omega_0/2\hbar)}$ , and  $1/\alpha_2 = \sqrt{2m\omega_0\hbar}$ .

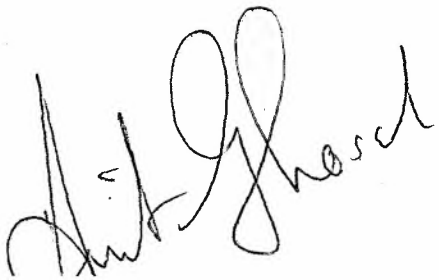
[points: 13]

2. It is given that the general formalism of *Equation of Motion* method yields:

$$\sum_{\nu''} (\delta_{\nu\nu''}(\omega + i\eta) - t_{\nu\nu''}) G^R(\nu'', \nu', \omega) = \delta_{\nu\nu'} + D^R(\nu, \nu', \omega)$$

$$\text{with : } D^R(\nu, \nu', \omega) = -i \int_{-\infty}^{\infty} dt e^{i(\omega+i\eta)(t-t')} \Theta(t) \langle \{ -[V_{\text{int}}, \hat{a}_\nu](t), \hat{a}_{\nu'}(t') \} \rangle.$$

Here, all notations carry standard meaning. Consider a single-impurity (with non-



interacting electrons on impurity sites, i.e.  $U = 0$ ) Anderson Model Hamiltonian:

$$\mathcal{H} = \sum_{k,\sigma} \varepsilon_k \hat{c}_{k\sigma}^\dagger \hat{c}_{k\sigma} + \sum_{\sigma} \varepsilon_d \hat{d}_{\sigma}^\dagger \hat{d}_{\sigma} + \sum_{k,\sigma} \left( t_k \hat{d}_{\sigma}^\dagger \hat{c}_{k\sigma} + \text{h.c.} \right)$$

where,  $\hat{d}_{\sigma}$  annihilates an electron with spin  $\sigma$  on the impurity site, all other notations carry the usual meaning. Obtain the equations of motion for the following Green's functions (in frequency domain) corresponding to:

- (a)  $G^R(d\sigma, t - t') = -\Theta(t - t') \langle \{ \hat{d}_{\sigma}(t), \hat{d}_{\sigma}^\dagger(t') \} \rangle,$
- (b)  $G^R(k\sigma, d\sigma, t - t') = -\Theta(t - t') \langle \{ \hat{c}_{k\sigma}(t), \hat{d}_{\sigma}^\dagger(t') \} \rangle,$
- (c)  $G^R(d\sigma, k\sigma, t - t') = -\Theta(t - t') \langle \{ \hat{d}_{\sigma}(t), \hat{c}_{k\sigma}^\dagger(t') \} \rangle,$
- (d)  $G^R(k\sigma, t - t') = -\Theta(t - t') \langle \{ \hat{c}_{k\sigma}(t), \hat{c}_{k\sigma}^\dagger(t') \} \rangle.$

[points: 8]

2. Consider a 1D harmonic oscillator (unit mass), coupled linearly to position  $\hat{x}$  with a time-dependent force field  $F(t)$ , described by the Hamiltonian:

$$\hat{\mathcal{H}} = \frac{\hat{p}^2}{2} + \frac{1}{2} \omega_0^2 \hat{x}^2 - \hat{x} F(t).$$

- (a) Using Heisenberg equations of motion:  $i\partial_t \hat{A} = [\hat{A}, \hat{\mathcal{H}}]$ , derive the equation of motion for the operator  $\hat{x}$ . Take  $\hbar = 1$  for simplicity.
- (b) "Retarded response function" is defined as:  $\chi^R(t) = i\Theta(t) \langle [\hat{x}(t), \hat{x}(0)] \rangle$  in linear response theory. Here, the expectation value is taken *in the absence of the perturbation*. Solving the equation of motion for  $\hat{x}$  with  $F(t) = 0$ , evaluate  $\chi^R(t)$  explicitly, Also obtain  $\chi^R(\omega)$  in the frequency domain.

[points: 4+5=9]