Indian Institute of Science Education and Research Kolkata
Department of Mathematics and Statistics
Ordinary differential Equations (MA4202)

IISER KOLKATA

Spring Sem
February 18, 2019

Time: **15:30–17:00** *Mid-Semester Exam* Marks: **20**

1. Let $t \mapsto (x_1(t), x_2(t))$ be a solution of

$$\begin{cases} \dot{x_1} = x_2^2 + \sin(tx_1), \\ \dot{x_2} = \sin x_1, \\ (x_1(0), x_2(0)) = (0, 0). \end{cases}$$

Show that $x_1(t) = 0 = x_2(t)$ for all t. [4]

2. Let $c, r > 0$ and $f : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ be continuous function. Define

$$M := \sup\Big\{ |f(t, x)| : t \in [-c, c], x \in [-r, r] \Big\}$$

and

$$h = \min\left\{c, \frac{r}{M}\right\}.$$

Let us define

$$\mathcal{S} = \Big\{ X \in C^1\Big((-h, h); [-r, r]\Big) \cap C\Big([-h, h]; [-r, r]\Big) : X'(t) = f(t, X(t)) \text{ for all } t \in (-h, h), \text{ and } X(0) = 0 \Big\}.$$

Let $p \in [-h, h]$. Show that the set

$$S_p = \Big\{ X(p) : X \in \mathcal{S} \Big\}$$

is non-empty closed and bounded set. [8]

3. Determine the maximal interval of existence for the following initial value problem:

(i)
$$\begin{cases} \dot{x} = x\cos^2 x + \sin t \cos x + 1, \\ x(0) = 0. \end{cases}$$

(ii)
$$\begin{cases} \dot{x} = -y(x^2 + y^2), \\ \dot{y} = x(x^2 + y^2), \\ x(0) = 1 = y(0). \end{cases}$$

[2×4=8]

End of exam

Rajib Dutta
18/2/19