Evolutionary Dynamics (ID4201) Final Exam (May 1, 2019)

Answer all questions. Total Time=2.5 hours. Total Marks=50. The marks are specified in brackets.

1. Consider two stochastic strategies with last round memory defined by S1(p1,p2,p3,p4) and S2(q1,q2,q3,q4) where p1,p2,p3,p4 are S1's probabilities of cooperating when the state of the game in the last round was CC, CD, DC, DD respectively.

(a) Write down the matrix giving the transition probabilities between the 4 possible Markov states. (2)

(b) If μ is the mutation probability of making a mistake (behavioural noise); obtain the transition probabilities in going from (i) *CC to CC* (ii) *DC to DD* in the presence of such behavioural noise. Justify each term in your expressions for the transition probabilities (4)

2.a) The following reactions lead to Rock-Paper-Scissors dynamics. Write down the set of dynamical equations for the frequencies ($x_A; x_B; x_C$) of A, B, C respectively. ϕ – empty site (3)

$$\begin{array}{lll} AB \xrightarrow{p} A\emptyset\,, & BC \xrightarrow{p} B\emptyset\,, & CA \xrightarrow{p} C\emptyset, \\ AB \xrightarrow{z} AA\,, & BC \xrightarrow{z} BB\,, & CA \xrightarrow{z} CC, \\ A\emptyset \xrightarrow{q} AA\,, & B\emptyset \xrightarrow{q} BB\,. & C\emptyset \xrightarrow{q} CC, \end{array}$$

b) If in addition to the above reactions, A, B, C can mutate to each other with a rate μ i.e. (3)

$$A \xrightarrow{\mu} \begin{cases} B \\ C \end{cases}, \quad B \xrightarrow{\mu} \begin{cases} A \\ C \end{cases}, \quad C \xrightarrow{\mu} \begin{cases} A \\ B \end{cases}$$

Write down the new set of dynamical equations for the frequencies $x_A; x_B; x_C$

3. a) One mechanism for ensuring that altruistic behaviour spreads in the population involves increasing the probability Pr(A|A) of altruists interacting with other altruists and reducing the chances Pr(A|N) of altruists interacting with non-altruists.

(i) Using a typical PD game with two parameters b: benefit of cooperation and c: cost of cooperation; write down the fitness of altruists and non-altruists in terms of these probabilities and obtain the condition for spread of altruism in the population. (1)

(ii) Preferential interaction between altruists can be facilitated by considering interactions between individuals related by common descent. If the probability that two individuals are related by common descent is r and p is the frequency of the altruism allele in the population, write the probabilities Pr(A|A) etc. in terms of r and p and obtain the Hamilton's rule. (2)

b) Consider a model of dispersal of asexual organisms where N sites each occupied by an asexual adult which produces ***k*** offspring. Out of these a fraction ***v*** emigrate (disperse) from its birth-site to another site and out of these a fraction ***p*** survives. Emigrants from a given site that survive migration are equally likely to reach and compete for any one of the (N-1) remaining sites. Suppose a mutant arises at one site that disperses ***(v+δ)*** fraction of the ***k*** offsprings it produces.

(i) What is the probability Pr(M|WT) that a mutant takes over a site originally occupied by a WT in the next generation. Justify your answer by explaining the meaning of each term in the expression for Pr(M|WT). (2)

(ii) What is the probability Pr(M|M) that a mutant offspring replaces its parent at the home site where it was born in the next generation. Justify your answer by explaining the meaning of each term in the expression for Pr(M|WT). (2)

(iii) In the limit $N \to \infty$; obtain an expression for the total # of successful mutants at ALL sites (2)

4. (a) Imagine a structured population described by a burst graph where a node at the periphery can only be replaced by the central node but the central node cannot be replaced by any peripheral node. What is the probability of fixation of a mutant that arises in such a structured population? Justify your answer! (2)

(b) In a cycle graph, every member can only be replaced in the next generation by the member *preceding* it or *following* it. If the #B members are ***m*** and #A=***N-m***, what is the probability that the #B individuals (i) decrease by 1 and (ii) increase by 1? Justify your answer in each case. Assume fitness of B=r, fitness of A=1. (2+2)

(c) Inactivating a tumour suppressor gene requires two mutations, one in each allele. The first mutation that inactivates one of the alleles occurs with the rate u1 and the second mutation that inactivates the remaining allele occurs with a rate u2. Type 0 cells have two unmutated alleles and Type 1 cells have only one mutated allele. If there are ***i*** cells of Type 1 and ***(N-i)*** cells of type 0, write down the probabilities for number of Type 1 cells to (i) decrease by 1 (ii) increase by 1 (iii) remain unchanged. Briefly justify your answers. (2+2+2)

5. (a) (i) Consider a PD game on a regular network, where each node has k-connections. If $q_{A|B}$ represents the probability that a B-player has a A-player as a neighbour etc.

Write down the payoff p_i to the ith-player (i=A or B); explain each term in the expressions for the two payoffs. (2)

(ii) If a B-player is selected for death in the network, obtain an expression for the probability that the B-player is replaced by one of it's A-neighbours. Explain the significance of each term in the expression. (4)

(b) Consider a dynamical network formed due to connections between nodes that play either of two strategies A or B. X(t),Y(t),Z(t) are the number of AA, AB, BB links respectively. α_A & β_B are the likelihood of A and B players to form new links and the death rate of AA, AB, BB links are β_{AA} , β_{AB} , β_{BB} respectively.

(i) Obtain a set of 3 dynamical equations for X, Y, Z. Explain the terms that appear in the dynamical equation for X(t) (3)

(ii)Solve the equations to determine the *fraction* of active AA, AB, BB links at equilibrium (2)

(iii)Assuming the fitness of A and B arises only due to interactions (i.e. intrinsic fitness = 0); write down the fitness of A and B players in such a dynamical network. Explain each term in the expressions. (2)

(iv) The invasion probability of A : $\rho_A > 1/N$ implies a+2b>c+2d in the weak selection limit; where a,b,c,d are the elements of the payoff matrix of the game between strategies A and B. In the limit where the active linking time-scale is much smaller than the evolutionary updating time-scale; obtain the condition for the invasion of cooperators in terms of the birth and death rate of network links if strategy A is a cooperator and strategy B is a defector. USe E(C,C)=b-c; E(C,D)= -c; E(D,C)=b, E(D,D)=0. (2)

(v) Assuming that the death rates of links are fixed, prove that there is an optimal value of α_C for which cooperators are most likely to invade defectors in such a dynamical network and obtain that optimal value of α_C (2)