Indian Institute of Science Education and Research Kolkata

Department of Mathematics and Statistics

Subject: Analysis II (MA2201)

Mid-Sem. Exam., ~~Class Test~~ (20.02.2019) Full marks: 20 Max. time: 60 minutes

****

Instructions: In order to get full credits, all the intermediate steps must be provided (including statement of theorem(s), if any) and the answers must be given in its most simplified forms. The notations have their usual meaning. The number inside the square brackets at the right of each question denotes the marks corresponding to this question.

****

1. Let a function $f : \mathbb{R} \to \mathbb{R}$ be given by
$$f(x) = \begin{cases} e^{-x}, & \text{if } x < 1 \\ ax + b, & \text{if } x \geqslant 1 \end{cases}$$
Can you find the values of a and b such that f is differentiable? [2]

2. Consider the function $f : \mathbb{R} \to \mathbb{R}$ given by
$$f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$
Verify whether $f'(x)$ exists. If yes, verify whether f' is continuous. [4]

3. If f' exists in $\left[0, \frac{\pi}{2}\right]$, verify whether the equation
$$4\cos^2 x \left[f\left(\frac{\pi}{2}\right) - f(0)\right] = \pi f'(x)$$
has any root in $\left(0, \frac{\pi}{2}\right)$. [4]
Hint: Use Cauchy's mean value theorem.

4. If it is known that
$$a \leqslant \int_{\pi/6}^{\pi/2} \frac{x}{\sin x}\, dx \leqslant 2a,$$
on using mean value theorem, find the value of a. [3]

5. Let $f : [0, 2] \to \mathbb{R}$ be defined as
$$f(x) = \lim_{n\to\infty} \frac{x^n + 2}{x^n + 3},\ 0 \leqslant x \leqslant 2.$$
Verify whether f is Riemann integrable over $[0, 2]$. If yes, evaluate $\int_0^2 f(x)dx$. [2]

6. Let $a_1, a_2, \ldots, a_n \in \mathbb{R}$ for any fixed $n \in \mathbb{N}$. Evaluate
$$\lim_{x\to\infty} \left\{x - \sqrt[n]{(x - a_1)(x - a_2)\ldots(x - a_n)}\right\},$$
if exists. [5]

_____________*********_____________

20/2/2019