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**MA 3201: Topology**

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Date: February 18, 2019

Duration: 1 hour 30 minutes

Maximum marks 20

1. Let  $F$  be a closed subset of a metric space  $X$ . Does there exist a continuous function  $g : X \rightarrow \mathbb{R}$  such that  $F = g^{-1}(\{0\})$ ? Prove your claim or give an example of such a function. 5
2. Let  $(X, d)$  be a metric space and  $E$  be a subset of  $X$ . For a point  $x \in X$ , define *the distance from  $x$  to  $E$*  by
$$\text{dist}(x, E) = \inf\{d(x, y) \mid y \in E\}.$$
Prove that the function  $f : X \rightarrow \mathbb{R}$ , given by  $f(x) = \text{dist}(x, E)$  is uniformly continuous on  $X$ . 5
3. Let  $X$  be any topological space. Does there exist a topological space  $Y$  containing more than one point such that any function  $f : X \rightarrow Y$  is continuous? Prove your claim or give an example of such a topological space. 3
4. Is it true that any one-to-one continuous mapping of a topological space onto another is a homeomorphism? Prove your claim or give a counterexample. 4
5. Is it true that any compact topological space is necessarily first countable? Prove your claim or give a counterexample. 6

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