Evolutionary Dynamics (ID4201) Mid-Sem exam (Feb.22, 2019)

Answer all questions. Total Time=1 hour. Total Marks=20.The mark for each question is specified in brackets.

1. a) For a population consisting of two types of strategies A and B whose frequency dependent fitness's are f_A and f_B respectively, write down the quasi-species equation (assuming no mutation) showing the rate of change of frequency of type A. If a>c and d>b, obtain the possible equilibrium frequencies for A. a,b,c,d correspond to the matrix elements E(A,A), E(A,B), E(B,A), E(B,B) respectively of the payoff matrix. (2)

b) Prove the equilibrium solution obtained in ~~b~~ a) is an *unstable* equilibrium. (2)

c) The frequencies of two subpopulations that evolve under selection but not mutation is given by the non-linear equations

$dx/dt = ax^c - \phi x$; $dy/dt = by^c - \phi y$;

where $\phi = ax^c + by^c$ is the average fitness and a,b are the growth rates of type A and B respectively; 0<c<1 Obtain an expression for the equilibrium frequency of type A when both sub-populations *coexist* at equilibrium and prove that it is a stable equilibrium state (2)

d) If A and B are two strategies making up a population of size N, ρ_A and ρ_B are the respective invasion probabilities, derive the condition for A to be risk dominant given $\frac{\rho_A}{\rho_B} = \prod_{i=1}^{N-1} \frac{f_i}{g_i}$ (3)

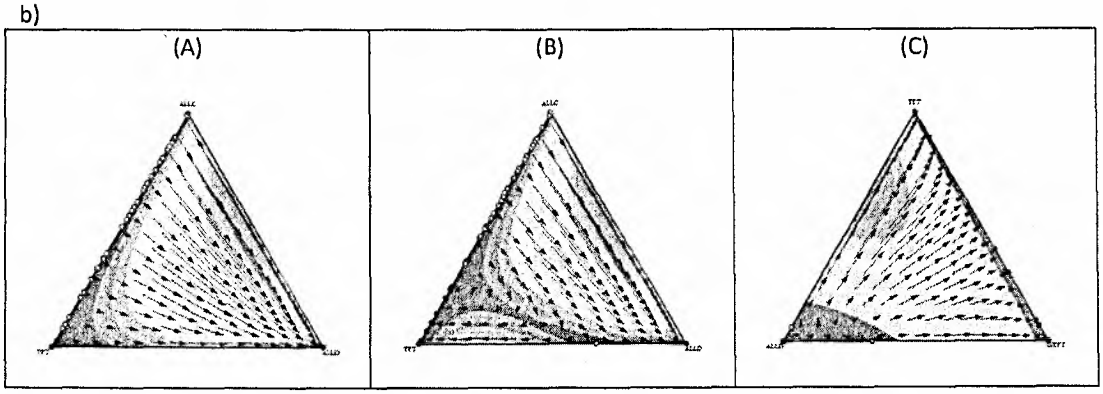
e) Obtain the above condition in the limit N>>1 and derive the resulting constraint such a condition imposes on the value of the mixed-state equilibrium frequency of A (1)

2. a) Consider a game between TFT, TF2T and ALLD where the expected number of rounds is $\bar{m}$. TF2T is a more cooperative strategy than TFT that cooperates with a defector twice before starting to defect.

(i) Write down the payoff matrix for such a game and derive the condition for which TFT is an ESS that can resist invasion by TF2T (3)

(ii) Derive the condition on the frequency for which a small fraction of ALLD players can invade a mixed population of TFT and TF2T players. (2)

b)



The figures show the phase diagram of a game between ALLC, TFT, ALLD (A&B) with different number of rounds and (C) a game between TFT, ALLD, GTFT without noise where $\bar{m} \rightarrow \infty$

(i)Explain using a quantitative argument, which of the first two figures have a larger value of $\bar{m}$, the average number of rounds, and why? (2)

(ii) If the initial state in fig (C) has a finite fraction of all 3 strategies with sufficiently large fraction of TFT and GTFT to prevent invasion by ALLD, explain why the arrows indicate that the population move towards the TFT, GTFT edge of the simplex and not to either the GTFT only or TFT only final state? (1)

c) In a game of WSLS vs ALLD, where $\bar{m}$ is the average number of rounds, obtain the condition on the parameters of the payoff matrix for WSLS to be stable against invasion by ALLD. (2)