MA1202 Mid Semester Examination

Total marks: 20　　　　　　　　　　　　　　　　　　　　　Time: 1 Hour

Group A

Answer any two questions

Q 1a) We have $B = P^{-1}AP$ where A, B and P are $n \times n$ matrices. The eigenvectors of B, corresponding to the the eigenvalues $\lambda_1 = 3$, $\lambda_2 = -1$ and $\lambda_3 = 6$ are x_1, x_2 and x_3, respectively, where

$$x_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad x_2 = \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix}, \quad x_3 = \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix}, \quad \text{and} \quad P = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{pmatrix}$$

Determine the eigenvalues and eigenvectors of A.

b) Determine $B^{50}x_2$.　　　　　　　　　　　　　　　　　　　　　　　　$(3+1)$

Q 2) a) Prove that the determinant of an orthogonal matrix is either +1 or -1. Remember, an orthogonal matrix is one for which the transpose is equal to its inverse.

b) Show that the product of a matrix and its transpose is a symmetric matrix. Does this require the original matrix to be square?　　　　　　　　　　　　　　　　$(2+2)$

Q 3) a) Diagonalize the matrix

$$A = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Remember, diagonalizing a matrix A means finding an invertible matrix P and a diagonal matrix D such that $P^{-1}AP = D$.

b) By using the diagonalization or otherwise determine A^{1000}.　　　　　　　　$(3+1)$

Group B

Answer any one

Q 4)a) Determine the LU decomposition of the matrix A

$$A = \begin{pmatrix} 3 & 1 & -2 \\ -6 & -1 & 4 \\ 9 & 5 & -1 \end{pmatrix}$$

b) What is the determinant of A?

c) Using this decomposition, solve for x_1, x_2 and x_3 where $Ax_i = b_i$ with the column vectors b_i given by

$$b_1 = \begin{pmatrix} 15 \\ 0 \\ 0 \end{pmatrix}, \quad b_2 = \begin{pmatrix} 15 \\ 15 \\ 0 \end{pmatrix}, \quad b_3 = \begin{pmatrix} 15 \\ 15 \\ 15 \end{pmatrix}.$$

$$\left(\frac{4+2+6}{(4\times3)} \right)$$

Q 5a) Use Gauss-Jordan elimination to solve the set of equations

$$y - 3z + 4t = 1$$
$$2x - 2y + z = -1$$
$$2x - y - 2z + 4t = 0$$
$$-6x + 4y + 3z - 8t = 1$$

b) Find the determinant of the matrix

$$X = \begin{pmatrix} \frac{1}{4} & \frac{1}{2} & 1 \\ 1 & 1 & 1 \\ 4 & 2 & 1 \end{pmatrix}$$

by Gauss elimination.

c) Are the three column vectors

$$u = \begin{pmatrix} 1 \\ 4 \\ 16 \end{pmatrix}, \quad v = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}, \quad w = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

linearly dependent or independent? Explain. (4×3)

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