Gaurav
2-5-19

Indian Institute of Science Education and Research, Kolkata
End Semester Examination, Spring 2019
Mineral Physics (ID 4213)

Time : 2 hours 30 mins Maximum Marks : 50

1. Assuming the linear variation of bulk modulus with pressure, $K = K_0 + K_0'P$, obtain the expression for pressure as a function of density. (2 Marks)
2. For a compression under hydrostatic pressure, the Hencky strain is defined as
$$\epsilon_H = \frac{1}{3} ln\left(\frac{V}{V_0}\right) = \frac{1}{3} ln\left(\frac{\rho_0}{\rho}\right).$$
Expanding the free energy as a function of Hencky strain, obtain the second order equation of state (Pressure vs volume relation). Using the obtained equation of state, determine the pressure variation of bulk modulus. (7 marks)
3. The strain energy density of the system is given as
$$\phi(\epsilon) = \frac{1}{2}\sum_{ijkl} C_{ijkl}\epsilon_{ij}\epsilon_{kl}.$$
Obtain the expression for generalized Hooke's law. (3 Marks)

4. Identify the possible reflection and rotation symmetry operations for orthorhombic crystal system ($a \neq b \neq c, \alpha = \beta = \gamma = 90°$). Using the matrix representation for reflection with respect to *xy* plane, show that that stress tensor transforms as
$$\sigma'_{11} = \sigma_{11}$$
$$\sigma'_{22} = \sigma_{22}$$
$$\sigma'_{33} = \sigma_{33}$$
$$\sigma'_{12} = \sigma_{12}$$
$$\sigma'_{23} = -\sigma_{23}$$
$$\sigma'_{13} = -\sigma_{13}.$$
Using the invariance of the elastic tensor under reflection symmetry with respect to *xy* plane, show that C_{1111} and C_{1122} are nonzero, whereas C_{1123} and C_{2312} are zero.
(You may need the following information: Components of second rank tensor such as stress and strain tensors transform as $A'_{ij} = \sum_{kl} Q_{ik}Q_{jl}A_{kl}$ whereas components of forth rank tensor transform as $C'_{ijkl} = \sum_{pqrs} Q_{ip}Q_{jq}Q_{kr}Q_{ls}C_{pqrs}$).

(10 Marks)

5. The elastic tensor for isotropic materials is given as
$$C_{ijkl} = \lambda\delta_{ij}\delta_{kl} + \mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$$
Express the 6×6 elastic constant matrix in terms of λ and μ. (Voigt Notation: $11 \to 1, 22 \to 2, 33 \to 3, 23 \to 4, 13 \to 5, 12 \to 6$) (3 Marks)

6. Consider the nucleation of a small spherical particle of phase β within a matrix of the parent phase α. What is the main driving force behind the nucleation. What are the factors that oppose

nucleation. Hence write an expression for the overall change in free energy that takes place when a spherical nucleus of radius 'r' forms. Plot the overall change in free energy as a function of radius 'r'. Explain the significance of any extremum which appears in your graph. Plot and explain what would be the effect of temperature on the nature of this graph. Obtain an expression for the critical radius at which this extremum appears. (9 Marks)

7. Consider that the shape of the nucleating phase (which is progressively growing in size) is such that the ratio of the radius r, to its thickness t is constant. Show how the surface energy and strain energy terms evolve as a function of the plate thickness, t. Also comment on the coherency of the interface as the nucleus grows. (5 Marks)

8. Show how you can determine the activation energy for nucleation and the energy barrier for nucleation from a TTT phase diagram. (5 Marks)

9. Show that for any system at equilibrium, the fraction 'f' of the total number of particles having a thermal energy not less than H_a is given by $f = \exp(-H_a/RT)$, where H_a is the activation energy per mole. (3 Marks)

10. Let the activation energy for cationic diffusion be 300 kJ/mole. Find out by how many times the rate of diffusion increases as you increase the temperature from 300 K to 1500 K. (3 Marks)