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Mid-semestral Examination, Spring 2019

ANALYSIS IV (MA 3204)

Date: February 23, 2019 Maximum Marks: 20 Time: 1000 – 1130

Note: You need to write precise statement of the theorem that you are using, provided that you are not asked to prove the theorem itself.

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(1) (a) Let $E \subseteq \mathbb{R}^d$. Show that the following are equivalent:

(i) E is measurable,

(ii) for each $\varepsilon > 0$, there exists an open set $U \supseteq E$ such that $m_*(U \setminus E) < \varepsilon$,

(iii) there exists a G_δ-set $G \supseteq E$ such that $m_*(G \setminus E) = 0$.

(b) Show that if E is measurable, then $E + x$ is measurable for all $x \in \mathbb{R}^d$ and $m(E) = m(E + x)$. [8 + 4 = 12]

(2) (a) Let $\{f_n\}_{n\in\mathbb{N}}$ be a sequence of measurable functions defined on a measurable set in $\mathbb{R}^d$. Show that $\sup\{f_n : n \in \mathbb{N}\}$ is measurable.

(b) Show that $\sup\{f_\alpha : \alpha \in A\}$ is not necessarily measurable even if f_α is. [3 + 3 = 6]

(3) Give an example of a function such that $|f|$ is measurable but f is not. [3]

(4) Prove or disprove: Measurable function of a continuous function is measurable. [4]

(5) Let $f : [0, 1] \to \mathbb{R}$ defined by $f(0) = 0, f(x) = x \sin \frac{1}{x}$ for $x > 0$. Find the measure of the set $\{x \in [0, 1] : f(x) \geq 0\}$. [5]
