MA3203: Algebra II (Spring 2019)
Mid Semester, Duration: 90 minutes

Answer all the questions. The maximum possible points is 20.
No marks will be given without justification!

---

1 (a). Show that there is no non-trivial ring homomorphism from $\mathbb{C}$ to $\mathbb{R}$. [3]
(b). Let I, J be two co-maximal ideals of a commutative ring R and $m, n \in \mathbb{N}$. Is it true that I^m and J^n are also co-maximal? [2.5]

2 (a). Let R be a commutative ring with 1 and $I, J \subset R$ be two ideals such that $I + J = R$. Prove that $IJ = I \cap J$. [3]
(b). Is the converse true? (i.e., $IJ = I \cap J \stackrel{?}{\Rightarrow} I + J = R$) [2.5]

3 (a). Let R be a commutative ring with 1 and $\mathcal{M} = (a)$ be a maximal ideal of R. Show that a is irreducible. [3]
(b). Is the converse true? (i.e., a is irreducible $\stackrel{?}{\Rightarrow}$ (a) is maximal) [2.5]

4 (a). Let R be a UFD. Show that every non-zero prime ideal of R contains a prime element. [3]
(b). Let R be a ring such that $R[X]$ is a PID. Is $R[X]$ a Euclidean domain? [2.5]

You can use the following result without a proof.
If K is a field then $K[X]$ is a Euclidean domain.

Md. Aliz imna
22/02/2019