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MA 4203, Midterm Q. Paper

Midterm, MA 4203 (Differential Geometry)

3:30 — 5:00pm, February 21, 2019

The word smooth used below means C^∞, i.e., infinitely differentiable.

1. a) (3 points) Give the definition of a manifold.

   b) (7 points) Give example of a compact manifold. Prove that it is a compact manifold directly, by verifying all the properties of a manifold stated in (a).

2. a) (5 points) Let f be a smooth function on $\mathbf{R}^n$. If $f(0) = 0$, show that there exist smooth functions $g_1, \ldots, g_n$ on $\mathbf{R}^n$ such that

$$f = x_1 g_1 + \cdots + x_n g_n$$

   where x_i are the coordinate functions on $\mathbf{R}^n$.

   b) (5 points) If $f(x_1, x_2) = x_1^n x_2^m$ (here n and m are positive integers) then compute g_1 and g_2 using the method you used in (a). You get no credit if you merely give examples of such g_1, g_2.

3. a) (4 points) Let $f : M \to N$ be a smooth map between differentiable manifolds. If $p \in M$, then define the tangent space $T_p(M)$ of M at p and construct a linear map from $T_p(M)$ to $T_{f(p)}(N)$.

   b) (6 points) Let p, q be points on M and N. Construct an isomorphism (independent of choice of charts on any choice) from $T_{(p,q)}(M \times N)$ and $T_p(M) \oplus T_q(N)$. One way is to begin by constructing maps between $T_{(p,q)}(M \times N)$ and the spaces $T_p(M)$, $T_q(N)$.