* **Question paper to be submitted with answer sheets**

Time : 2.5 hour
Instructor : Dr. Sourin Das

Name :

Roll No. :

INSTRUCTIONS

This question paper contains three Parts A, B and C. Part A contains 11 multiple choice questions(MCQs), each carrying 2 marks, all compulsory.
Part B and Part C has 3 questions each out of which 2 questions are to be answered from each of these Parts. Each questions in Part B and Part C carry a total of 7 marks. Part B and Part C are to be answered in regular answer sheet.
Clearly mention the Parts in your answer sheet.

INSTRUCTIONS for Part A (MCQs)

Circle only one of the four choices (A, B, C, D) for each question in the MCQ question paper itself. Answers should be circled using a PEN only.
A correct answer will fetch 2 marks, a wrong answer or no answer 0 mark.
All rough calculation for the MCQ should be done on the answer sheet. Clearly mention "PART A" on top of the answer sheet on which the rough calculation is done. These calculations will not be marked.
NO WRITING OR MARKING ON THE MCQ QUESTION PAPER IS PERMITTED AT ANY STAGE OF THE EXAMINATION. ANY WRITING OR MARKING ON THE MCQ QUESTION PAPER WILL BE PENALIZED. For every question that is marked in any fashion in the MCQ question paper , 1 marks will be deducted from the overall score, up to a maximum deduction of 3 marks.
Please tie the three sheets together with a tag (MCQ question paper, the white answer sheet for rough work corresponding to part A, the white answer sheet corresponding to Part B and Part C: in this order) and hand the bunch over to the invigilators when you leave.

Part A

1. Ohm's law stated as $\vec{J} = \sigma\vec{E}$, where $\vec{J}$ is the current density, $\vec{E}$ is the applied external electric field and σ is the conductivity, implies that the drift velocity (v_d) ,

   A. $v_d \propto |\vec{E}|$

   B. $v_d =$ constant

   C. $v_d \propto \tau^2$

   D. none of the above

   Here τ is the mean free time between successive collisions.

2. The total work done in establishing a steady current I in a current loop of self inductance L is given by

   A. $\frac{1}{2\mu_0}\int_{\text{all space}} B^2 dV$

   B. LI^2

   C. It is independent of I

   D. None of the above

3. For two current carrying circular loops labelled 1 and 2 of radius r_1 and r_2 $(r_1 = 2r_2)$, the magnetic flux (ϕ_1) through 1 is related to the current (I_2) through 2 as $\phi_1 = M_{12}I_2$ and similarly the magnetic flux (ϕ_2) through 2 is related to the current (I_1) through 1 as $\phi_2 = M_{21}I_1$, then

   A. $M_{12} = M_{21}$

   B. $M_{12} = \frac{1}{2}M_{21}$

   C. $M_{12} = 2M_{21}$

   D. $M_{12} = C\sqrt{M_{21}}$, where C is a dimensionful constant

4. $\vec{\nabla}.\vec{B} = 0$ implies that the normal component of the magnetic field $(B^{\perp})$ across a surface with current density $\vec{K}$ is such that

   A. $B^{\perp}_{\text{above}} - B^{\perp}_{\text{below}} = \mu_0 K$

   B. $B^{\perp}_{\text{above}} + B^{\perp}_{\text{below}} = \mu_0 K$

   C. $B^{\perp}_{\text{above}} - B^{\perp}_{\text{below}} = 0$

   D. None of the above

$\vec{\nabla}.\vec{E} = \rho/\epsilon_0$, where $\vec{E}$ is the electric field and ρ is the charge density, implies that the normal component of the electric field ($E^{\perp}$) across a two dimensional surface charge density σ is such that

A. $E^{\perp}_{\text{above}} - E^{\perp}_{\text{below}} = \sigma/\epsilon_0$

B. $E^{\perp}_{\text{above}} - E^{\perp}_{\text{below}} = 0$

C. $(E^{\perp}_{\text{above}} + E^{\perp}_{\text{below}})/2 = \sigma/\epsilon_0$

D. None of the above

6. The amount of work done to move a charge q once around a closed path defined by a circle of radius R lying in the xy-plane where an uniform electric field $\vec{E}$ is applied in the z direction is

A. $2\pi R|\vec{E}|$

B. $R|\vec{E}|$

C. Zero

D. None of the above

7. The monopole moment in the multi-pole expansion for the potential of a given charge distribution is

A. independent of coordinate system

B. depends explicitly on the coordinate system

C. is identically zero

D. none of the above

8. For an infinitely long wire with uniform line-density λ along the z-axis, the electric field at a point $(a, b, 0)$ away from the origin, where the origin lies on the wire, is ($\hat{e}_x, \hat{e}_y$ and $\hat{e}_z$ are unit vectors in Cartesian coordinate system)

A. $\dfrac{\lambda}{2\pi\epsilon_0\sqrt{a^2+b^2}}(\hat{e}_x + \hat{e}_y)$

B. $\dfrac{\lambda}{2\pi\epsilon_0(a^2+b^2)}(a\hat{e}_x + b\hat{e}_y)$

C. $\dfrac{\lambda}{2\pi\epsilon_0\sqrt{a^2+b^2}}\hat{e}_x$

D. $\dfrac{\lambda}{2\pi\epsilon_0\sqrt{a^2+b^2}}\hat{e}_y$

[Hint : $\vec{E} = \frac{\lambda}{2\pi\epsilon_0\rho}\hat{\rho}$]

9. A charged particle in a uniform magnetic field $\vec{B} = B_0\hat{e}_z$ starts moving from the origin with velocity $\vec{v} = (3\hat{e}_x + 2\hat{e}_z)$ m/s. The trajectory of the particle and the time time t at which it reaches 2 meters above the xy-plane are ($\hat{e}_x, \hat{e}_y$ and $\hat{e}_z$ are unit vectors in Cartesian coordinate system)

   A. Helical path; $t = 1$ s
   B. Helical path; $t = 2/3$ s
   C. Circular path; $t = 1$ s
   D. Circular path; $t = 2/3$ s

10. Let the electric field in some region R is given by $\vec{E} = e^{-y^2}\hat{\imath} + e^{-x^2}\hat{\jmath}$. From this we may conclude that

   A. R has a non-uniform charge distribution
   B. R has a time dependent magnetic field
   C. The energy flux in R is zero everywhere
   D. None of the above

11. If $\vec{E}_1 = xy\hat{\imath} + 2yz\hat{\jmath} + 3xz\hat{k}$ and $\vec{E}_2 = y^2\hat{\imath} + (2xy + z^2)\hat{\jmath} + 2yz\hat{k}$ then

   A. Both are impossible electrostatic fields
   B. Both are possible electrostatic fields
   C. Only $\vec{E}_1$ is a possible electrostatic field
   D. Only $\vec{E}_2$ is a possible electrostatic field

Part B

1. Two point charges q and λq respectively in vacuum are located at the points $(a, 0)$ and $(\mu a, 0)$ in the xy-plane. Assume throughout that $a > 0$ and $q > 0$.

   (a) If the sum of the charges is fixed, what value of λ will maximize the electric force between them ? Is the force attractive or repulsive? [1.5 Marks]

   (b) If the midpoint of the line joining the charges is fixed, what value of μ will maximize the electric force (amplitude) between them? Is the force attractive or repulsive? [1.5 Marks]

   (c) Show that the equation for the lines of force in the xy-plane is given by:

$$\frac{\lambda(x-\mu a)}{\sqrt{(x-\mu a)^2 + y^2}} + \frac{\lambda(x-a)}{\sqrt{(x-a)^2 + y^2}} = c$$

   where c is a const. (Hint : lines of force is given by $\frac{dy}{dx} = \frac{E_y}{E_x}$) [4 Marks]

2. (a) Find the electric field outside a uniformly charged solid sphere of radius R and total charge q using Gauss's law. [3 Marks]

   (b) Using Coulomb's law, show that the field inside a uniformly charged spherical shell at any point vanishes identically. [4 Marks]

3. (a) A point charge q is situated at a distance a from the center of a grounded conducting sphere of radius R. Use method of images to find the position and magnitude of the image charge expressed in terms of R, a and q. [3 Marks]

   (b) An electrostatic potential profile $V(\theta) = \frac{K}{2}(1-\cos\theta)$ is imposed on the surface of spherical shell whose center is taken to be the origin and θ represents the polar angle. Find the potential profile inside the sphere using the general solution of Laplace equation given by :

$$V(r,\theta) = \sum_{l=0}^{\infty} A_l \, r^l P_l(\cos\theta)$$

where r is the radial distance from the center of the sphere, A_l's are constant and $P_l(\cos\theta)$ represents Legendre polynomials which satisfy :

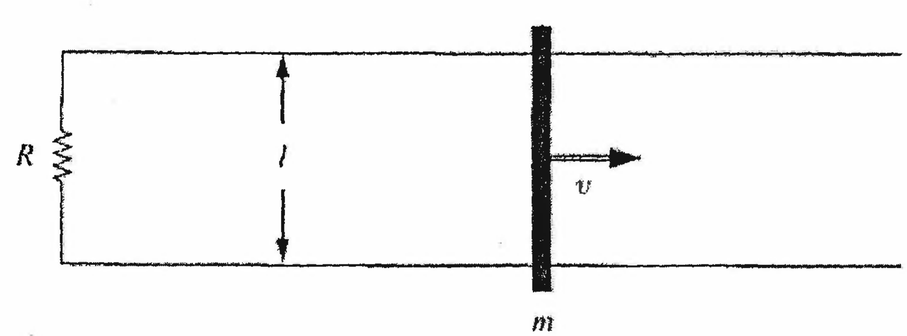
$$\int_0^{\pi} P_l(\cos\theta)\sin\theta d\theta = \begin{cases} 0 & \text{if } l' \neq l \\ \frac{2}{2l+1} & \text{if } l' = l \end{cases}$$

[4 Marks]

Part C

1. (a) Write down Faraday's law and state the Lenz's law in brief. [2 Marks]

   (b) A metal bar of mass m slides frictionlessly on two parallel conducting rails at a distance l apart (see figure). A resistor R is connected across the rails and a uniform magnetic field B, pointing into the page, fills the entire region.



1.b.1 If the bar moves to the right at speed v, what is the current in the resistor? In what direction does it flow? [1 Mark]

1.b.2 What is the magnetic force on the bar? In which direction does it point? [1 Mark]

1.b.3 If the bar starts out with speed v_0 at time $t = 0$, and is left to slide, what is its speed at a later time t? [1 Mark]

1.b.4 The initial kinetic energy of the bar was $\frac{1}{2}mv_0^2$. Check if the total energy dissipated at the resistor is exactly $\frac{1}{2}mv_0^2$. [2 Marks]

2. (a) Describe Ampere's circuital law? [2 Marks]

   (b) A steady current I flows along a long cylindrical wire of radius a. Find the magnetic field, both inside and outside the wire, if:

   2.b.1 The current is uniformly distributed over the outside surface of wire. [2.5 Marks]

   2.b.2 The current is distributed in such a way that J, the current density is proportional to s, the distance from the axis. [2.5 Marks]

3. (a) Write down the four Maxwell's equations. [2 Marks]

   (b) By taking divergence of the last two equations corresponding to Faraday's law and Ampere's law, show that they are consistent with the first two equations. [2 Marks]

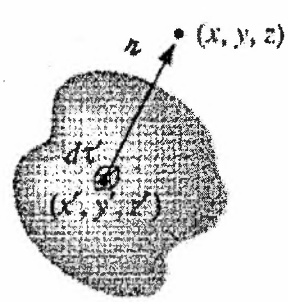
   (c) Show that Biot- Savart's law (for general case of a volume current density $\vec{J}$) which reads as:

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r'}) \times \hat{\mathscr{r}}\, d\tau'}{\mathscr{r}^2}$$

is consistent with

$$\vec{\nabla}.\vec{B} = 0$$

where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $\vec{r'} = x'\hat{i} + y'\hat{j} + z'\hat{k}$ and $d\tau' = dx'dy'dz'$ and $\mathscr{r} = \vec{r} - \vec{r'}$. [3



Marks]