Indian Institute of Science Education & Research

Mid Term Examination - MA4206 Algebraic Topology

Date : 18^{th} February 2019

INSTRUCTIONS

This is a **closed-book** exam.

You have 1.5 hours.

- The examination is scored out of 20 **points**.
- It has **THREE** problems with a total worth of 22 points.
- You must do **ALL** problems. Even if your score exceeds 20 points it will be counted as 20 points.

Good luck!

Problem 1 Let $\mathcal{P}_2$ denote the set of all polynomials (with real coefficients) of degree 2. Given $P_i(x) = a_i x^2 + b_i x + c_i$ for $i = 1, 2$ we define a metric

$$d(P_1, P_2) := \sqrt{(a_1 - a_2)^2 + (b_1 - b_2)^2 + (c_1 - c_2)^2}.$$

(i) [3 **points**] Show that under the map $\iota : \mathcal{P}_2 \to \mathbb{R}^3, \iota(ax^2 + bx + c) = (a, b, c)$, the space $\mathcal{P}_2$ is homeomorphic to its image.
(ii) [3 **points**] Let $\mathcal{P}_2^{m}$ denote the subspace of polynomials with top coefficient ± 1. Show that $\mathcal{P}_2$ deformation retracts to $\mathcal{P}_2^{m}$.
(iii) [3 **points**] Calculate $\pi_1(\mathcal{P}_2, x^2)$.

Problem 2 Let $\mathbb{R} \times [\pi, \pi]$ be given the topology induced from $\mathbb{R}^2 = \mathbb{C}$. Consider the map

$$\mathcal{E} : \mathbb{R} \times [-\pi, \pi] \longrightarrow \mathbb{C}^\times, \ (x, y) \mapsto e^{x+iy} = (e^x \cos y, e^x \sin y).$$

(i) [3 **points**] Let X be the quotient space obtained from $\mathbb{R} \times [-\pi, \pi]$ under the equivalence relation generated by $(x, \pi) \sim (x, -\pi)$. Show that there is an induced continuous map $\tilde{\mathcal{E}} : X \to \mathbb{C}^\times$.
(ii) [2 **points**] Show that $\tilde{\mathcal{E}}$ is a bijection.
(iii) [3 **points**] Prove or disprove: *The map above is a homeomorphism.*

Problem 3 [5 **points**] Let $X = S^1 \cup ([-1, 1] \times \{0\})$ and consider the relation $(\cos\theta, \sin\theta) \sim (-\cos\theta, -\sin\theta)$. Show that $X/\sim$ is homeomorphic to "figure eight" given by $(S^1 + (\frac{1}{2}, 0)) \cup (S^1 - (\frac{1}{2}, 0))$.