Mid-term Exam – PH-4206 (Many Body Physics) 19 Feb. 2019

1. Consider spinless fermions on a two-dimensional square lattice (consider lattice spacing $a = 1$) with N sites, described by the Hamiltonian:

$$\mathcal{H} = \sum_{i,\delta=\pm\hat{x},\pm\hat{y}} \left(-t\hat{c}_i^\dagger \hat{c}_{i+\delta} + \frac{V_0}{2}\hat{n}_i\hat{n}_{i+\delta} \right)$$

where the notations carry the usual meaning. We will drop the *hat* from the operators and the *vector-signs* in the following just for clarity, without distorting their meaning!

(a) Express $\mathcal{H}$ in momentum space and cast it in the following form:

$$\mathcal{H} = \sum_k \varepsilon(k) c_k^\dagger c_k + \sum_{k,k',q} V(k,k',q) c_k^\dagger c_{k-q} c_{k'}^\dagger c_{k'+q}$$

What are the explicit dependencies of $\varepsilon(k)$ and $V(k,k',q)$ on their arguments?

(b) Take the special case of $\vec{q} = (\pi,\pi)$, conventionally noted as $\vec{Q}$, i.e. $\vec{Q} = (\pi,\pi)$. Noting that k and k' lie within first Brillouin zone, i.e. $k, k' \in [0, 2\pi]$ (or alternatively, $k, k' \in [-\pi,\pi]$), show that we can write:

$$\mathcal{H} = \sum_k \varepsilon(k) c_k^\dagger c_k + \tilde{C} \sum_{k,k'} c_k^\dagger c_{k+Q} c_{k'}^\dagger c_{k'+Q}.$$

What is the constant $\tilde{C}$ in above in terms of the original model parameters?

(c) Obtain the mean-field Hamiltonian, $\mathcal{H}_{MF}$, corresponding to $\mathcal{H}$ in (b), in terms of the (mean-field) order parameter:

$$\Delta = \frac{1}{N} \sum_k \langle c_k^\dagger c_{k+Q} \rangle$$

(d) Please express above Δ as a position space summation (with corresponding $\langle \cdots \rangle$ expressed in position space), and interpret your result physically. Can you comment on the symmetry that is broken by the introduction of such order parameter (Considering a system with an average density of one electron per site might be helpful, though not necessary).

(points: 5+2.5+4.5+3)

2. Considering Bogoluibov quasiparticle operators (discussed in class) for two independent specias of spinless fermions:

$$\eta_{\mathbf{k}} = u_{\mathbf{k}} c_{\mathbf{k}\uparrow} + v_{\mathbf{k}} c_{-\mathbf{k}\downarrow}^\dagger \;\; and \;\; \gamma_{\mathbf{k}} = u_{\mathbf{k}} c_{-\mathbf{k}\downarrow}^\dagger - v_{\mathbf{k}} c_{\mathbf{k}\uparrow},$$

show that they satisfy Fermionic anti-commutation rules.

(points: 5)

19/02/2018