

IISER KOLKATA

MA 2203: End-semester exam

Spring 2019

May 05, 2019

Maximum marks: 50

Duration: $2\frac{1}{2}$ hours

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- (1) Describe all the isometries of \mathbb{R}^2 which maps the set $\{(x, 0) \in \mathbb{R}^2 : x \in \mathbb{R}\}$ to y -axis. (10 marks)
- (2) Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be an isometry and let P be a point such that $f(P) = Q$, where $P \neq Q$. Let X be a fixed point of f . Show that X lies on the bisector of PQ . (5 marks)
- (3) Identify the surface $4x^2 + 9y^2 + 4z^2 - 4xy + 8yz + 12xz + 9z - 3 = 0$ with proper justification (10 marks)
- (4) Consider the set $S : \{(x, y) \in \mathbb{R}^2 : ax^2 + 2hxy + by^2 = 1\}$. Suppose S contains at least two points. Prove the following assertions.
- i) If $ab - h^2 > 0$, then S is an ellipse.
 - ii) If $ab - h^2 < 0$, then S is a hyperbola
 - iii) If $ab - h^2 = 0$, then S is a pair of parallel lines.
- (3 + 3 + 3 = 9 marks)
- (5) True or False: Justify your answers
- i) Image of a parabola under any isometry of \mathbb{R}^2 is again a parabola.
 - ii) Any linear map $f : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ with $f(S^2) = S^2$ is an isometry of \mathbb{R}^3 , where $S^2 := \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 1\}$
 - iii) Composition of two orientation reversing isometry is an orientation reversing isometry.
 - iv) There are only one non-identity rotation with an invariant line.

4 × 5 = 20 marks

Without proper justification you will not get any marks in the true-false questions


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