Indian Institute of Science Education and Research Kolkata

Department of Mathematics and Statistics

Subject: Analysis II (MA2201)

End-Spring Semester Examination Date of Exam : May 7, 2019

Max. Marks: 50 Max. Time: 2 Hours 30 Minutes

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Instructions: In order to get full credits, all the intermediate steps must be provided (including statement of theorem(s), if any) and the answers must be given in its most simplified forms. The notations have their usual meaning. The number inside the square brackets at the right of each question denotes the marks corresponding to this question.

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1. For $n \in \mathbb{N}$, consider the function $f_n(x) = \frac{x}{n+n^2x^2}$. Verify whether the series $\sum_{n=1}^{\infty} f_n(x)$ is uniformly convergent over $\mathbb{R}$. [5]

2. Let a function f be defined as $f(x) = \frac{x^3}{x^2-1}$. Evaluate $\frac{d^n f(0)}{dx^n}$. [5]

3. Let $\phi : [0,1] \to \mathbb{R}$ be defined as $\phi(x) = f(x) + f(1-x)$ where f is such that $f''(x) > 0$ for all $x \in [0,1]$. Find values of x where ϕ is increasing. [5]

4. Find Maclaurin's expansion of $\sin x$. Also find the set of values of x where the expansion is valid. [5]

5. Let $f : [1,2] \to \mathbb{R}$ be a function such that $\int_1^2 |f(x)|dx < \infty$. Since $f(x) \leq |f(x)|$ for all x, we conclude that $\int_1^2 f(x)dx$ also converges. You prove or disprove the above statement. [5]

6. Evaluate $\lim_{n\to\infty} \int_0^{10} \frac{n(11-x)}{1+n^2x^2}\, dx$. [5]

7. Can you give an example of a sequence of functions $f_n(x) : [0,1] \to \mathbb{R}$ such that each f_n is Riemann integrable over $[0,1]$ but the point-wise limit function f, *i.e*, $\lim_{n\to\infty} f_n(x) = f(x)$ which exists, is not Riemann integrable over $[0,1]$? Can such a sequence $\{f_n\}$ converge uniformly on $[0,1]$? [3+2]

8. Find the set of values where the function f given by

$$f(x) = xe^{|x|+x^2} + (x-1)|x^2-1|$$

is not differentiable. Also, if possible, differentiate $e^{\sin^{-1}(x)}$ with respect to $e^{-\cos^{-1}(x)}$. [3+2]

9. A man can walk twice as fast as he can swim. To go from a point on the edge of a circular pool to a point diametrically opposite he may walk around the edge, swim straight across, or walk part way around and swim the rest of the way in a straight line. How shall he proceed if he is to make the trip in the least time? [5]

10. Find the radius of convergence of the power series [3+2]

$$1 + \frac{1}{9}x^2 + \frac{1\cdot 4}{9\cdot 18}x^4 + \frac{1\cdot 4\cdot 7}{9\cdot 18\cdot 27}x^6 + \ldots \infty$$

Give a specific choice of a, b and f such that $\int_a^b f(x)dx$ converges but $\int_a^b f^2(x)dx$ does not.

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