

Indian Institute of Science Education and Research Kolkata
Department of Physical Sciences

PH4205 : General Theory of Relativity and Cosmology

Mid-semester Examination (Spring 2019)

Marks: 20, Duration: 90 minutes

1. Consider the coordinate transformation $\bar{x} \equiv (r, \psi) \rightarrow \bar{x}' \equiv (x, y)$ such that $x = \frac{1}{2}r \cos 2\psi$, $y = \frac{1}{2}r \sin 2\psi$. Compute elements of the transformation matrix Λ if $d\bar{x}' = \Lambda d\bar{x}$. If the invariant distance in \bar{x}' coordinate system is $ds^2 = dx^2 + dy^2$ then compute the metric tensor in \bar{x} coordinate system. [3+4]
2. Prove the following identity, if $\partial_\rho g = g g^{\mu\nu} g_{\mu\nu,\rho}$ [4]

$$\nabla_\mu V^\mu = \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} V^\mu) .$$

3. If $\nabla_\alpha g^{\mu\nu} = 0$ and $[\nabla_\alpha, \nabla_\beta]V^\sigma = R^\sigma{}_{\rho\alpha\beta}V^\rho$ then show that $R_{\rho\sigma\alpha\beta} = -R_{\sigma\rho\alpha\beta}$ [4]
4. Show that the geodesics on a two-cylinder with the metric $ds^2 = r^2 d\theta^2 + d\rho^2$ are helices. [5]

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