Indian Institute of Science Education and Research Kolkata
Department of Mathematics and Statistics
Sobolev Space: Theory and Applications (MA5214)

Spring Sem
February 19, 2019

IISER KOLKATA

Time: **10:00–11:30** *Mid-Semester Exam* Marks: **20**

1. Let U be an open set in $\mathbb{R}^n$.

(i) Define Sobolev Spaces $W^{1,p}(U)$ for $p \in [1, \infty]$. [2]

(ii) Show that $W^{1,p}(U)$ is a Banach space. [3]

2. Let $U = (-\infty, 0) \cup (0, \infty)$ and $f \in L^1_{loc}(\mathbb{R}) \cap C^1(U)$. Show that the weak derivative of f exists if and only if

$$\lim_{x \to 0+} f(x) = \lim_{x \to 0-} f(x).$$

[4]

3. Let U be a bounded open set in $\mathbb{R}^n$ with smooth boundary.

(i) Let $u \in W^{1,2}(U)$. Show that

$$\int_U u \frac{\partial v}{\partial x_i} dx = -\int_U v \frac{\partial u}{\partial x_i} dx$$

for all $v \in W_0^{1,2}(U)$. [3]

(ii) Let u, $v \in W^{1,2}(U)$. Show that $uv \in W^{1,2}(U)$. [4]

3. Let U be a bounded open set in $\mathbb{R}^n$ with smooth boundary. Let $u \in W^{1,n}(U)$. Show that $u \in L^p(U)$ for any $p \in [1, \infty)$. [4]

Rajib Dutta
20/02/2019

End of exam