
MA 3201: Topology

Date: February 18, 2019

Duration: 1 hour 30 minutes

Maximum marks 20

1. Let F be a closed subset of a metric space X . Does there exist a continuous function $g : X \rightarrow \mathbb{R}$ such that $F = g^{-1}(\{0\})$? Prove your claim or give an example of such a function. 5
2. Let (X, d) be a metric space and E be a subset of X . For a point $x \in X$, define *the distance from x to E* by
$$\text{dist}(x, E) = \inf\{d(x, y) \mid y \in E\}.$$
Prove that the function $f : X \rightarrow \mathbb{R}$, given by $f(x) = \text{dist}(x, E)$ is uniformly continuous on X . 5
3. Let X be any topological space. Does there exist a topological space Y containing more than one point such that any function $f : X \rightarrow Y$ is continuous? Prove your claim or give an example of such a topological space. 3
4. Is it true that any one-to-one continuous mapping of a topological space onto another is a homeomorphism? Prove your claim or give a counterexample. 4
5. Is it true that any compact topological space is necessarily first countable? Prove your claim or give a counterexample. 6

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