

Department of Mathematics and Statistics, IISER Kolkata  
Mathematics-II (MA1201)  
Mid Semester Examination, Total Marks: 20, Time: 1 hr.

Instructor: Shirshendu Chowdhury

Date: 23rd February, 2019.

Question 1.

[5+3=8]

(i) If  $0 < a < b$ , then show that  $\lim_{n \rightarrow \infty} (a^n + b^n)^{\frac{1}{n}} = b$  and determine  $\lim_{n \rightarrow \infty} \frac{(a^{n+1} + b^{n+1})}{(a^n + b^n)}$ .

(ii) Check the convergence of the series  $\sum_{n=1}^{\infty} \frac{1}{(\sqrt{n} - \frac{3}{2})}$ .

Question 2.

[5]

State Fundamental theorem of Integral calculus and use it to evaluate

$$\int_{-3}^3 f \text{ where } f(x) = 2x \sin\left(\frac{\pi}{x}\right) - \pi \cos\left(\frac{\pi}{x}\right), x \in [-3, 3] - \{0\}, f(0) = 0.$$

Question 3.

[7]

Justify True or False. If it is true prove it, if not give counter example or justify with proper reason.

(a) A convergent sequence is bounded.

(b) If  $a_n > 0$  and  $\lim_{n \rightarrow \infty} n^2 a_n$  exists then  $\sum_{n=1}^{\infty} a_n$  is convergent.

(c) Let  $f, g : [0, 1] \mapsto \mathbb{R}$  such that  $f$  and composition function  $f \circ g$  are Riemann Integrable. Then  $g$  is Riemann Integrable.

THE END.

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