Indian Institute of Science Education Research Kolkata

End-semester Examination: **CH4205 (Statistical Thermodynamics)**

Time: 3 hours **Full Marks: 50**

1) Suppose you want to put two indistinguishable particles in a triply degenerate energy levels. Find out the number of ways one can put the particles if both the particles are (a) classical particles, (b) Bosons and (c) Fermions. **3 marks**

2) Show that entropy (S) of a system can be expressed by $S = \langle -k_\beta \ln \rho_i \rangle$, where all the symbols have their usual meaning. **3 marks**

3) Show that for a closed system

   (a) the energy fluctuation is related to the specific heat of the system.

   (b) the root mean square fluctuation in energy and pressure are negligible when N is very large. **(3+3+3) marks**

4) In case of Grand Canonical Ensemble (GCE), find out the relation between the total number of particles N and isothermal compressibility κ_T. **5 marks**

5) (a) What are the minimum and maximum contribution of the molar vibrational heat capacity to the molar heat capacity.

   (b) Plot vibrational molar heat capacity as a function of temperature.

   (3+2) marks

6) Prove the following relation for an ideal Fermi gas:

$$PV = NKT(1 + \frac{1}{2^{5/2}} e^{\mu B})$$

All the symbols have their usual meaning.

5 marks

8) Derive the conditions (limits) when classical statistics will be valid.

3 marks

9) Explain why energy of Fermion (E_{FD}), classical ideal gas (E_{cl}) and Boson (E_{BE}) follows the following trend : $E_{FD} > E_{cl} > E_{BE}$

2 marks

10) Find out relation between the two equilibrium constants (K_p *and* K_c) in terms of molecular partition function. **3 marks**

11) Show that at $\lim_{T \to 0} S = k_B \ln(m)$ where m is the degeneracy in the ground state (All the symbols have their usual meaning) **4 marks**

12) Show that probability of a state having energy E_i in canonical ensemble is

$\rho = \frac{e^{\frac{-E_i}{KT}}}{\sum e^{\frac{-E_i}{KT}}}$, where K is the Boltzmann constant and T is the temperature. **3 marks**

13) Consider three Ising spins ($s \pm 1$) sitting on the corners of an equilateral triangle. The Hamiltonian of the system is given by $H = -J(s_1 s_2 + s_1 s_3 + s_2 s_3)$.

(a) Deduce the partition function and calculate the average energy of the system. **3 marks**

(b) Determine the magnetization and the probability that one corner of the triangle has spin up (s = 1) on it. **2 marks**