MA211 Class test 1

Name : **Roll No. :** **Time : 15 mins**

1. The modulus and argument of the complex number $\frac{1+i}{1-i}$ are

   A. 1 and $\frac{3\pi}{2}$, respectively.

   B. 1 and $\frac{\pi}{2}$, respectively.

   C. 2 and $-\frac{\pi}{2}$, respectively.

   D. 2 and $\frac{\pi}{2}$, respectively

Solution:

$$\left|\frac{1+i}{1-i}\right| = \frac{|1+i|}{|1-i|} = \frac{\sqrt{2}}{\sqrt{2}} = 1$$

$\mathrm{Arg}(1 \pm i) = \pm\frac{\pi}{4}$, so that

$$\mathrm{Arg}\left(\frac{1+i}{1-i}\right) = \mathrm{Arg}(1+i) - \mathrm{Arg}(1-i) = \frac{\pi}{2}$$

2. The real part of $(2+3i)(3-2i)$ is

   A. 12

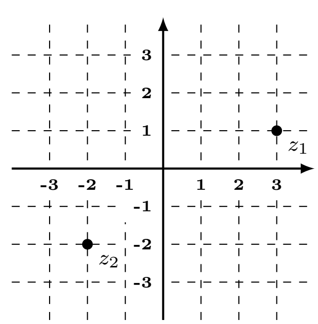
   B. 0

   C. 13

   D. -12

Solution: $\mathrm{Re}[(2+3i)(3-2i)] = 2 \times 3 - 3 \times (-2) = 12$

3. The complex numbers z_1 and z_2 are shown as points in an Argand diagram in the figure alongside. What is $z_1\,\overline{z_2}$?



A. $-8+4i$

B. $-8-4i$

C. $8+4i$

D. $4-8i$

Solution: $z_1 = 3+i, \quad z_2 = -2-2i$. So that

$$z_1\bar{z_2} = (3+i)(-2+2i) = -8+4i$$

4. Given two non-zero complex numbers z_1 and z_2, consider the four combinations (i) $\dfrac{z_1\overline{z_2}}{\overline{z_1}z_2}$, (ii) $\dfrac{z_1\overline{z_2}}{z_1+\overline{z_2}}$, (iii) $\dfrac{z_1z_2}{\overline{z_1}\ \overline{z_2}}$ and (iv) $\dfrac{z_1\overline{z_1}}{z_2\overline{z_2}}$. In general, the following have unit modulus :

A. (i) and (iii) only.

B. (i) only.

C. (ii) only.

D. (i), (iii) and (iv) only.

Solution: Use the facts that $\forall z, w \in \mathbb{C}$, we have $|z| = |\bar{z}|, \quad |zw| = |z||w|, \quad |z/w| = |z|/|w|$.

5. The function $f(z) = e^{iz}$ can be written in the form of two real functions, $u(x, y)$ and $v(x, y) :\ f(z) = u(x, y) + iv(x, y)$, where x and y are the real and imaginary parts of z, respectively. Then

A. $u = e^{-y}\cos x$**,** $v = e^{-y}\sin x$**.**

B. $u = e^{x}\cos y,\ v = e^{x}\sin y$.

C. $u = e^{-y}\cos x,\ v = -e^{-y}\sin x$.

D. $u = e^{x}\cos y,\ v = e^{-x}\sin y$.

Solution:

$$e^{iz} = e^{ix-y} = e^{-y}e^{ix} = e^{-y}(\cos x + i\sin x)$$

6. Let 1, ω and ω^2 be the three cube roots of unity $\left(\omega = \dfrac{-1+\sqrt{3}i}{2}\right)$. Then $1+3\omega+\omega^2$ is

A. 2ω

B. -2ω

C. $1+\omega$

D. $1+\omega^2$

Solution: Since $\omega^3-1=0,\ \omega\neq 1$, we have $1+\omega+\omega^2=0$ So

$$1+3\omega+\omega^2=1+\omega+\omega^2+2\omega=2\omega$$

7. Let ζ be one of the 52-nd root of unity with $0<\text{Arg}\,\zeta<\dfrac{\pi}{20}$. Then Arg ζ^{108} is

A. $\dfrac{2\pi}{13}$

B. $-\dfrac{2\pi}{13}$

C. $\dfrac{\pi}{13}$

D. $-\dfrac{\pi}{13}$

Solution: The 52nd roots of unity are of the form

$$\exp\left(i\frac{2\pi m}{52}\right), \qquad m=0,1,2,\ldots 51$$

Since $0<\text{Arg}\zeta<\frac{\pi}{20}$ we must have $\zeta=\exp\left(i\frac{\pi}{26}\right)$. Again, since $\zeta^{52}=1$, we have

$$\zeta^{108}=\zeta^{2\times 52+4}=\zeta^4=\exp\left(i\frac{2\pi}{13}\right)$$

8. The equation $x^{24}+a=0$ has, for all real non-zero numbers a,

A. 24 distinct complex roots

B. one real root and 23 non-real complex roots.

C. 24 complex roots, not all distinct.

D. two real roots, and no other roots.

Solution: Note that the roots of the equations are the 24 24-th roots of the real number $-a$. This has a real root only when $a < 0$, in general all roots are complex. It is easy to see that they are also all distinct.

9. The set

$$\{z \in \mathbb{C} \;:\; |z - z_0| \geq a\}$$

   A. is open for all $a \in \mathbb{R}, a \leq 0$.

   B. is open for all $a \in \mathbb{R}, a > 0$.

   C. is open for all $a \in \mathbb{R}$.

   D. is never open for any $a \in \mathbb{R}$.

Solution: If $a \leq 0$, then the set is $\mathbb{C}$!

10. The function $f(z) = z\bar{z}$ is

   A. differentiable but not holomorphic at the origin.

   B. holomorphic at the origin.

   C. differentiable nowhere.

   D. differentiable on the coordinate axes.

Solution: For $f(z) = z\bar{z}$, we have $u(x, y) = x^2 + y^2$ and $v(x, y) = 0$.Thus

$$u_x = 2x, \quad u_y = 2y, \quad v_x = v_y = 0$$

so that the partial derivatives are continuous and the Cauchy-Riemann conditions $u_x = v_y$ and $u_y = -v_x$ is satisfied only for $z = 0$. So, it is differentiable only at the origin, and holomorphic nowhere.

11. The function $f(z)$ is known to be entire. You also know that its imaginary part is $x^2 - y^2$. Then

   A. its real part is $-2xy$.

   B. its real part is $2xy$.

   C. its real part is $x^2 + y^2$.

   D. it is impossible to have an entire function whose imaginary part is $x^2 - y^2$.

Solution: Although you can find the solution using the C-R conditions, it is actually a lot simpler to note that $x^2 - y^2$, being the *real* part of the entire function z^2, is the *imaginary* part of the entire function iz^2.