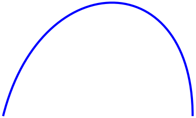
Complex numbers
Topology

Ananda Dasgupta

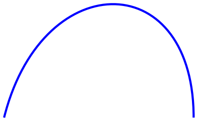
MA211, Lecture 3

Curve a.k.a. contour



- Intutively, a piece of string meandering on a flat surface!

Curve a.k.a. contour



- A curve C is defined as the range of a complex valued function $z : [a, b] \subset \mathbb{R} \to \mathbb{C}$:

$$C = \{z(t) = x(t) + iy(t) \;:\; a \leq t \leq b\}$$

where $x(t)$ and $y(t)$ are continuous real valued functions.

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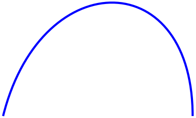

The function $z(t)$ is called a *parametrization* of C.

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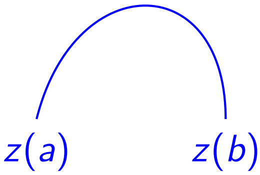
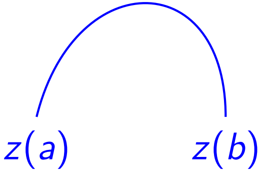


The parametrization also defines a direction for the curve : we say that the curve goes from $z(a) = x(a) + \imath y(a)$ to $z(b) = x(b) + \imath y(b)$.

The topology of complex numbers
Curve a.k.a. contour



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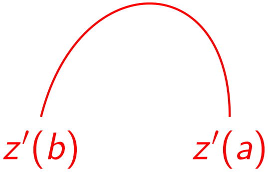
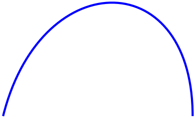


The curve $-C$ is one which is parametrized by a function $z'(t)$ which has the same range as $z(t)$, but with the initial and final points switched :

$$z'(a) = z(b), \qquad z'(b) = z(a)$$

Curve a.k.a. contour



- If both $x(t)$ and $y(t)$ are differentiable, then the curve C is called **smooth**!

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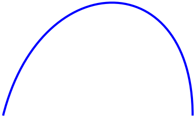

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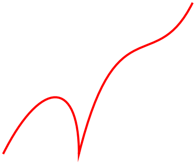
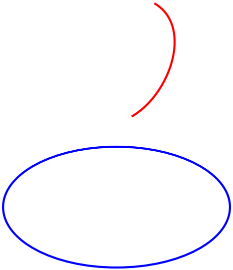


- If $x(t)$ and $y(t)$ are differentiable except at a finite number of points, then C is called **piecewise smooth**.

Simple and non-simple curves

- A curve is called **simple** if it does not cross itself.

Simple and non-simple curves



- A curve is called **simple** if it does not cross itself.
- This means that $z(t_1) \neq z(t_2)$ whenever $t_1 \neq t_2$, except possibly when $t_1 = a$ and $t_2 = b$.

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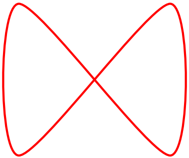

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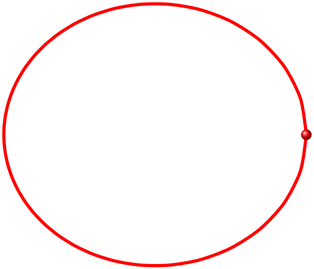
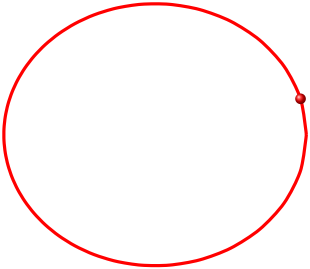
- This means that $z(t_1) \neq z(t_2)$ whenever $t_1 \neq t_2$, except possibly when $t_1 = a$ and $t_2 = b$.
- A self-intersecting curve is called non-simple.

Closed curves



A curve parametrized by $z : [a, b] \to \mathbb{C}$ is called **closed** if $z(a) = z(b)$.

Closed curves



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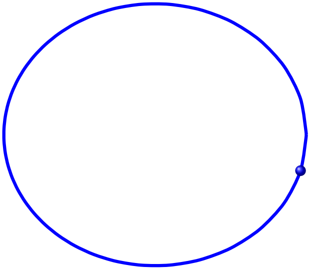

The curve $-C$ is the same curve traversed in the opposite direction!

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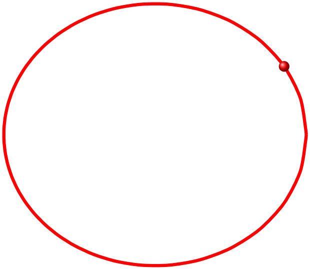

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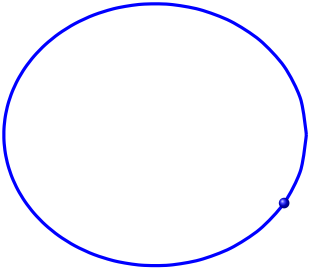

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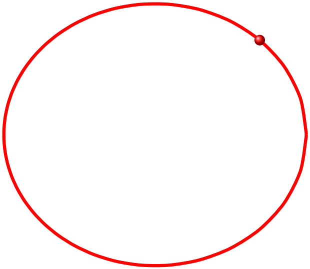

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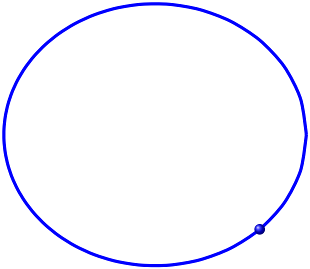

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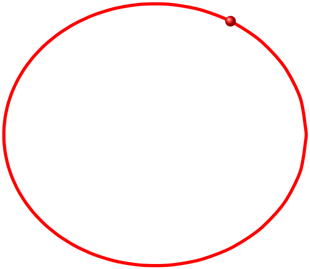

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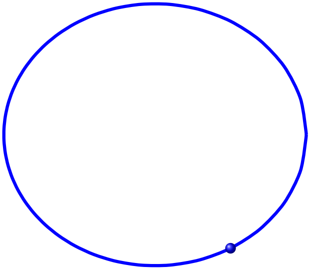

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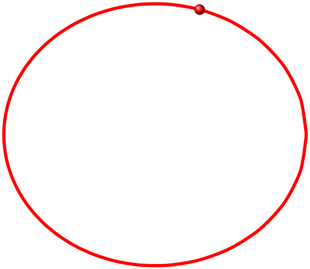

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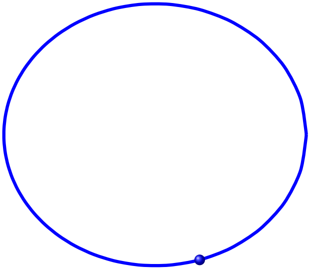

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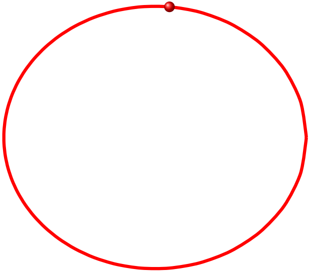

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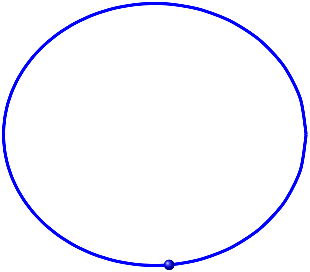

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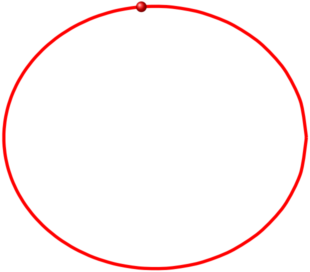

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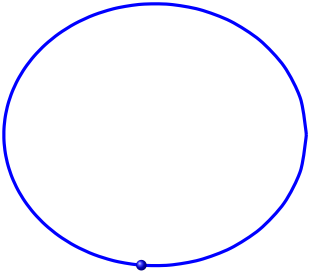

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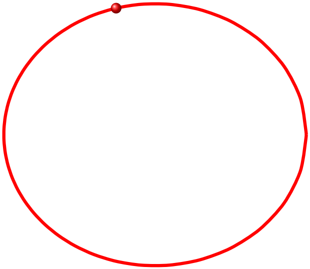

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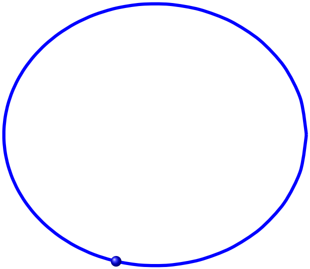

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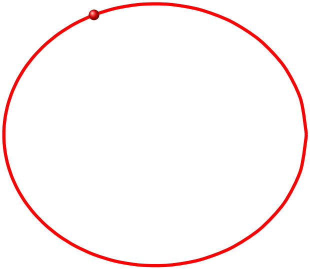

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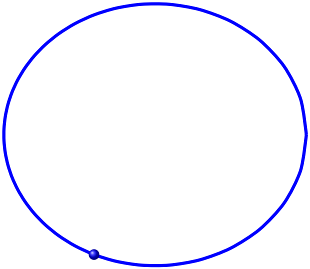

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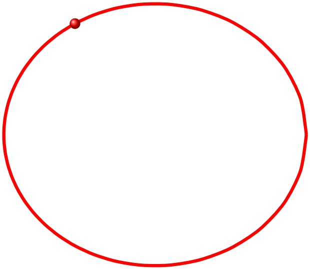

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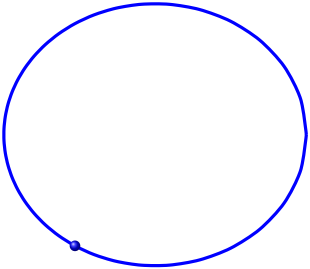

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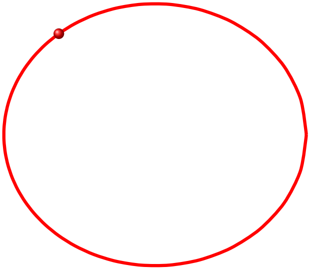

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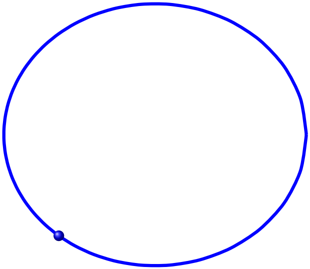

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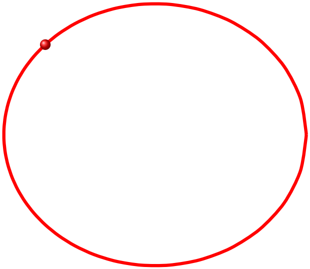

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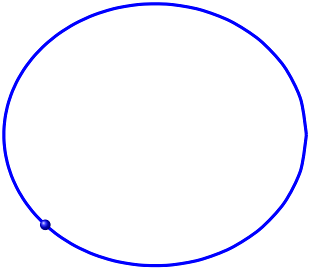

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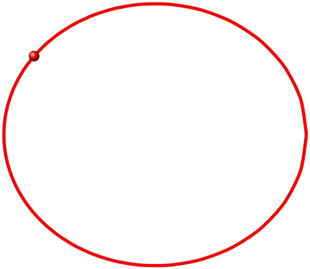

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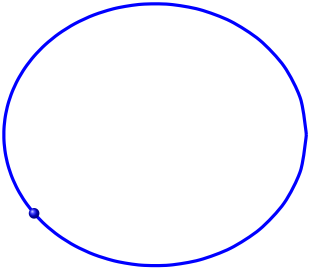

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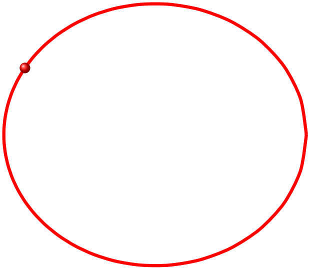

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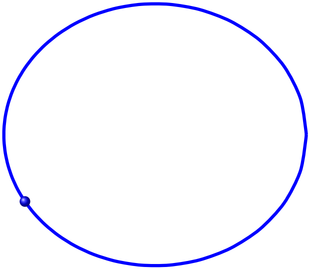

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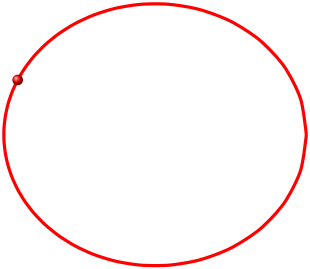

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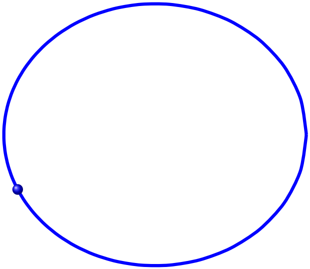

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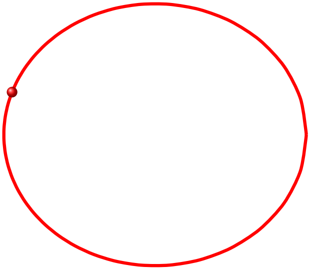

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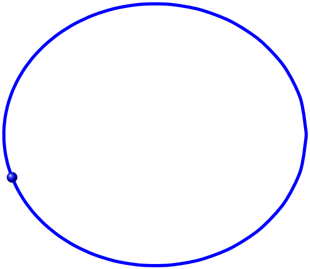

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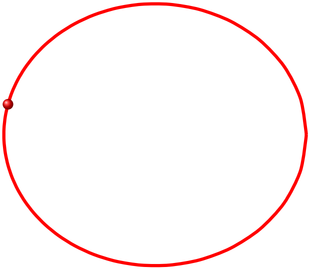

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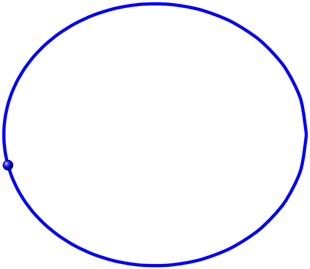

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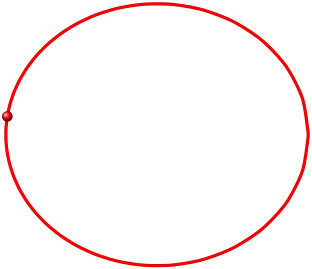

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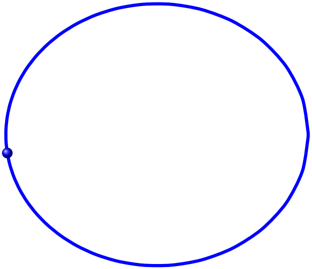

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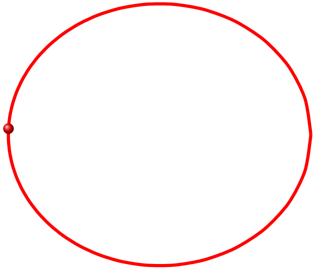

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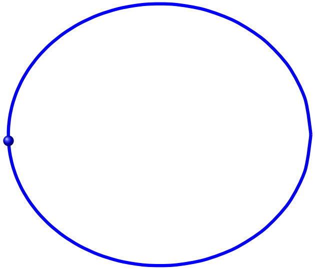

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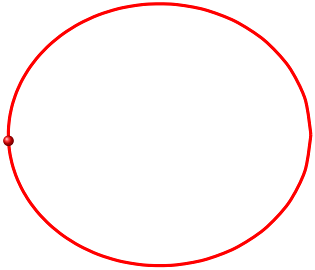
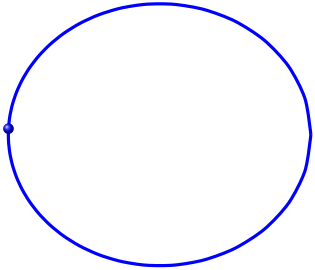


A simple closed curve is called positively oriented if its parametrization is such that when it is traversed, the interior is to the left.



If C is positively oriented, then $-C$ is negatively oriented.

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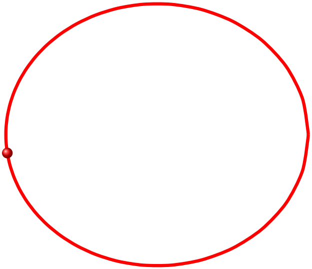

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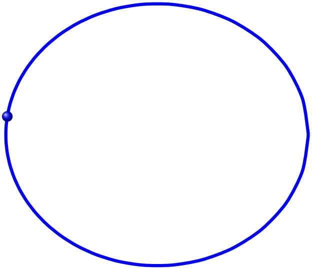

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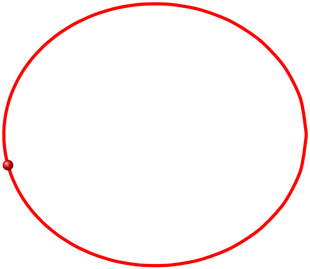

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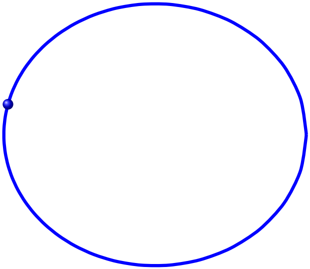

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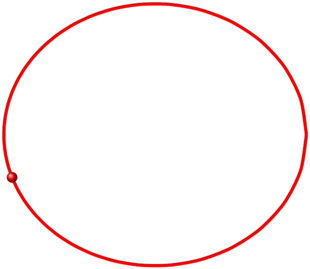

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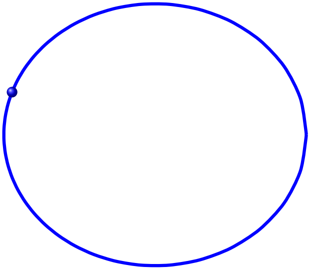

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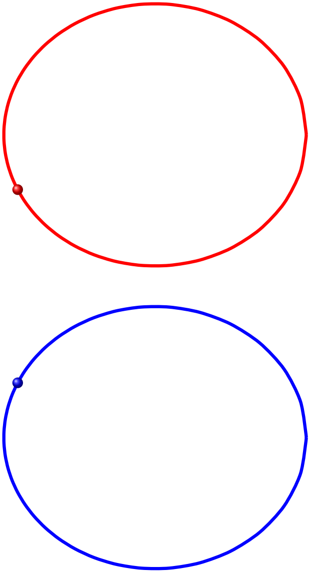

A simple closed curve is called positively oriented if its parametrization is such that when it is traversed, the interior is to the left.

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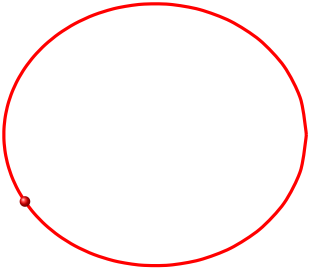

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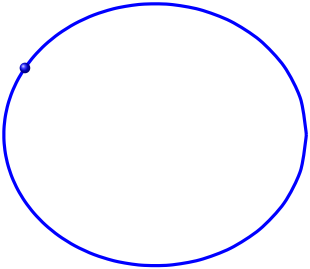

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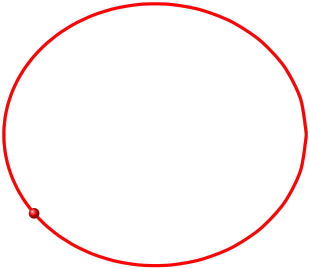

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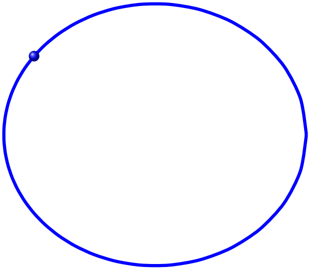

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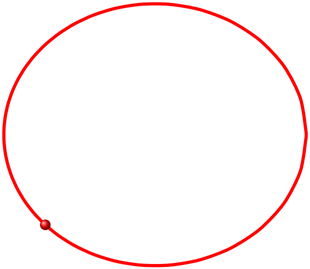

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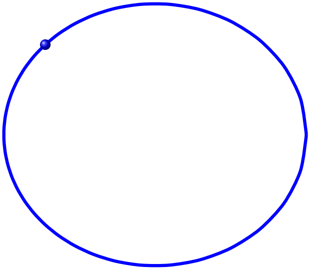

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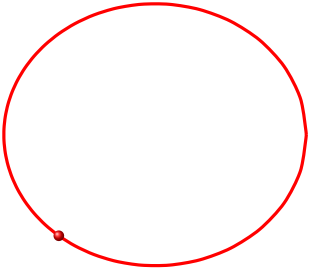

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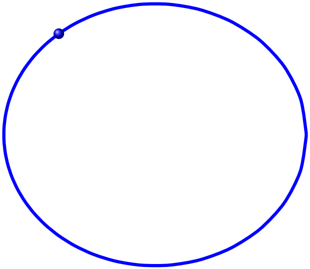

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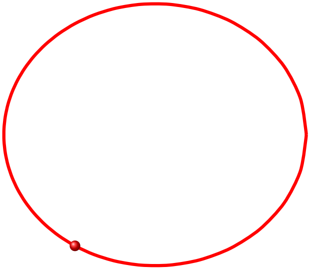

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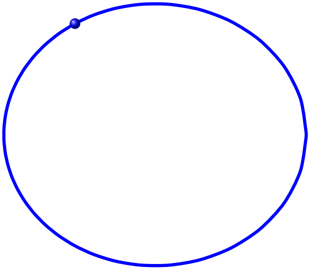

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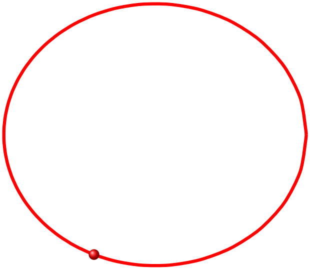

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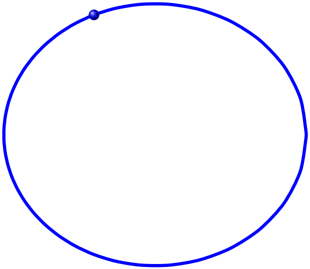

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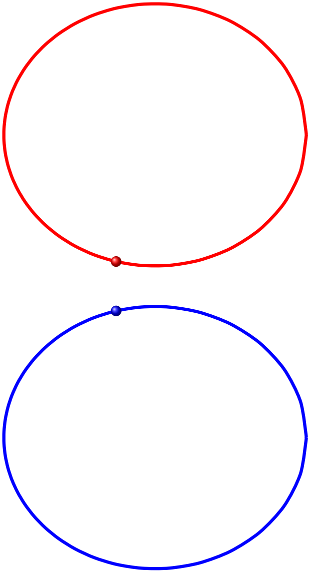

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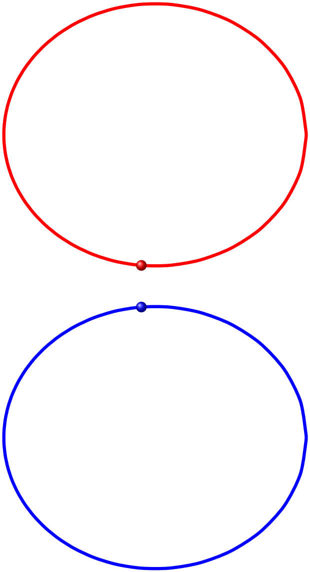

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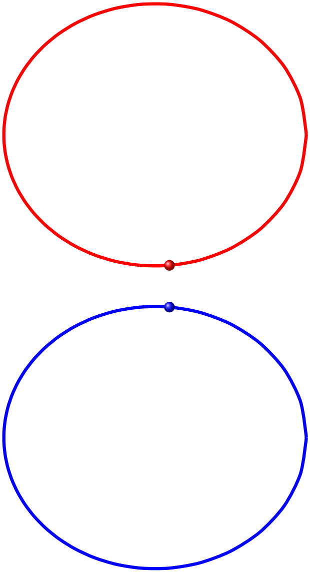

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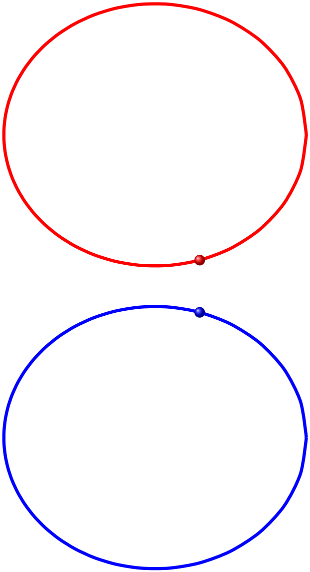

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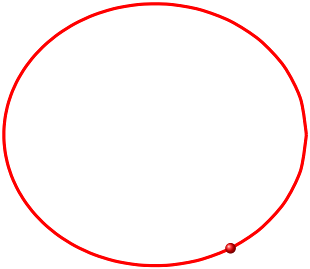

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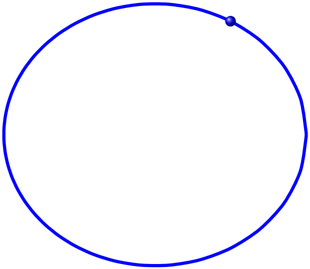

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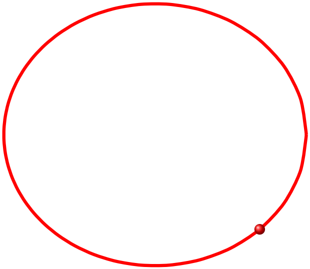

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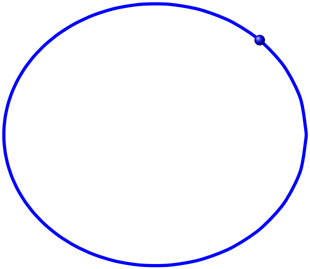

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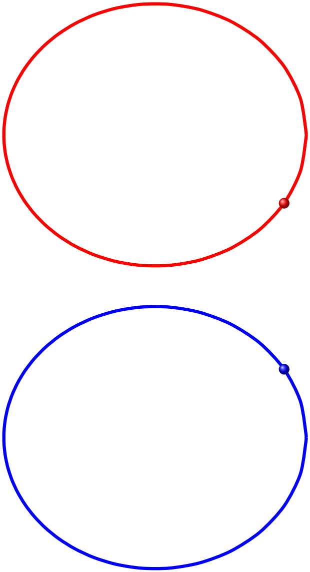

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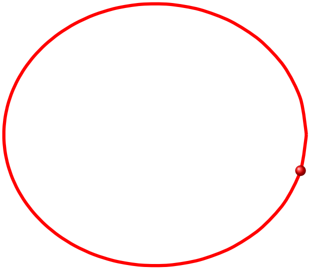

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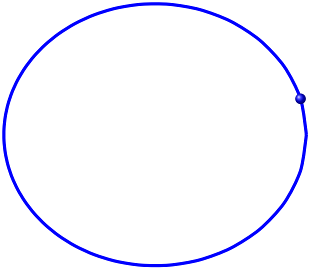

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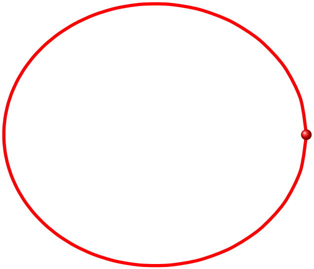

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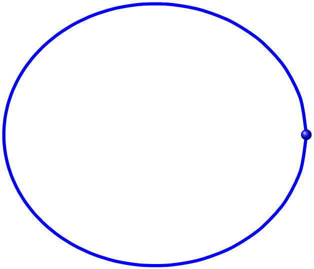

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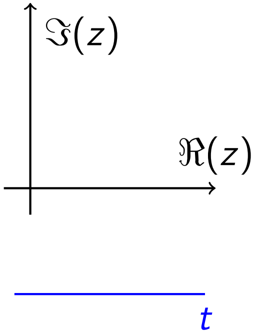
Curves vs. Paths



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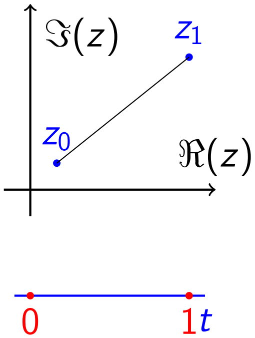
Curves vs. Paths



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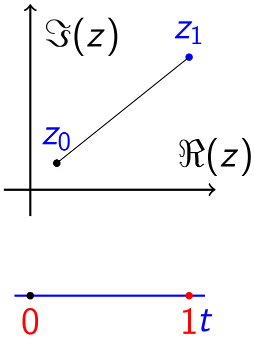
$$z(t) = z_0 + (z_1 - z_0)\, t, \quad 0 \leq t \leq 1$$

Curves vs. Paths



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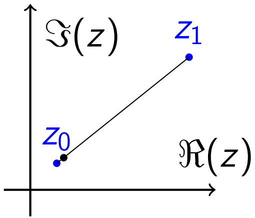

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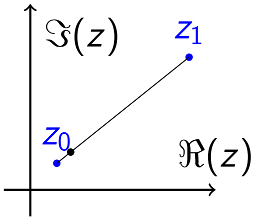

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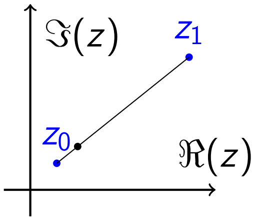

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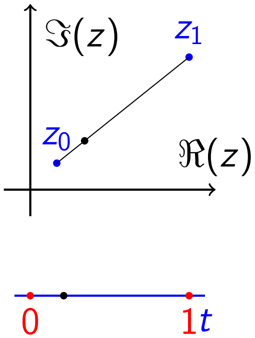

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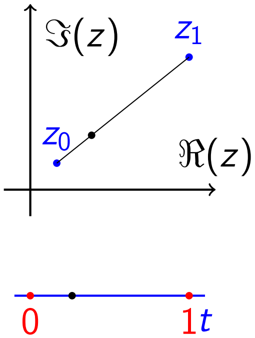

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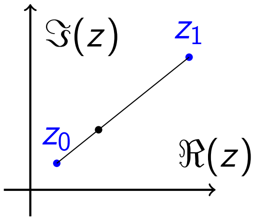

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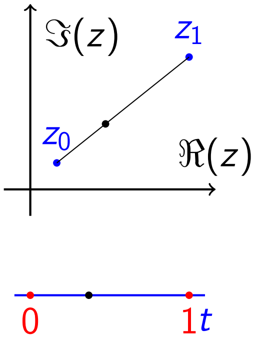

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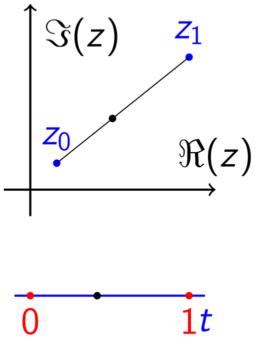

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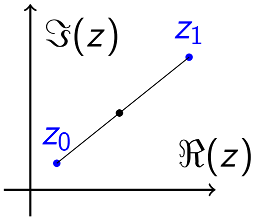

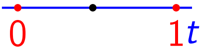

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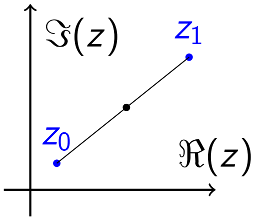

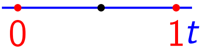

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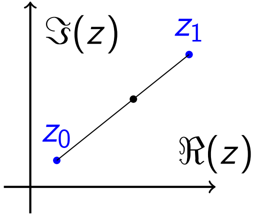

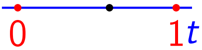

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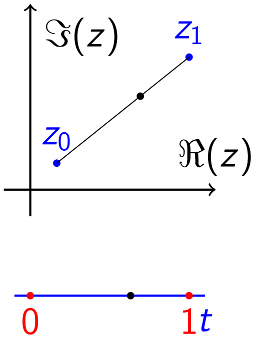

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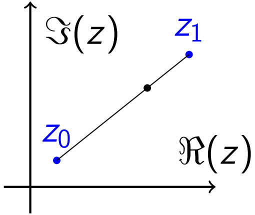

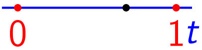

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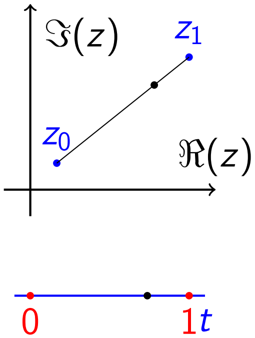

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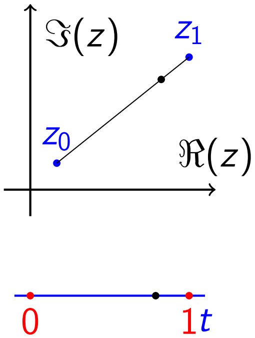

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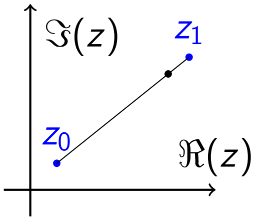

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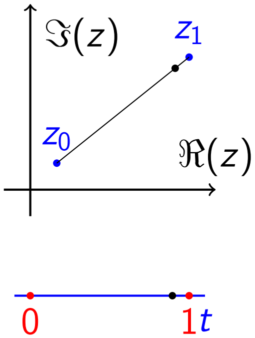

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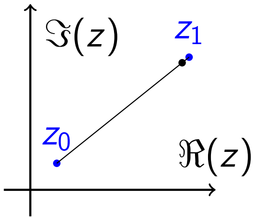

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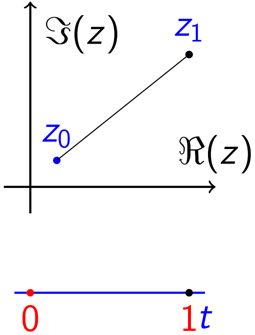

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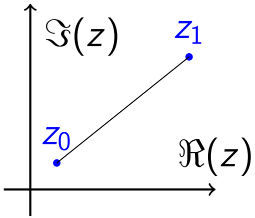

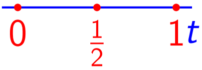

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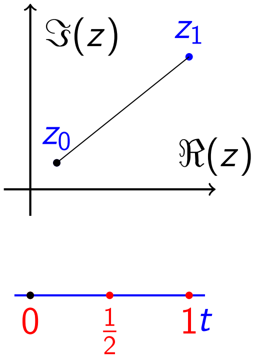

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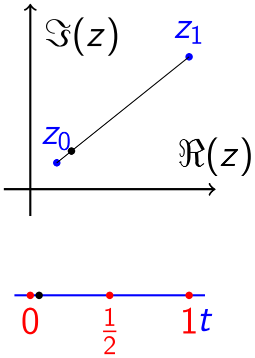

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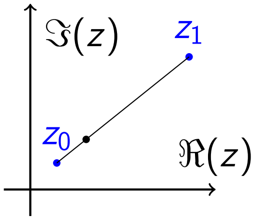

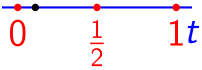

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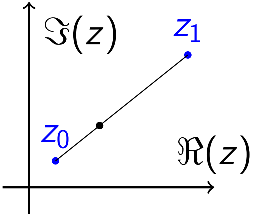

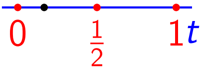

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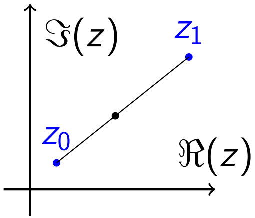

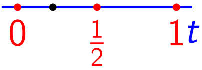

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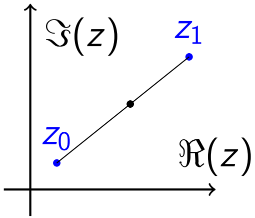

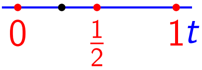

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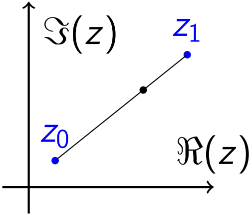

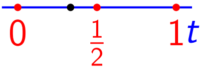

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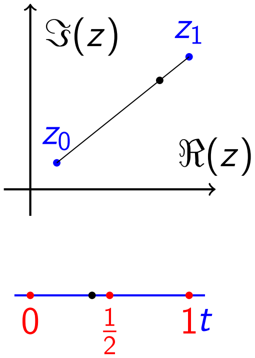

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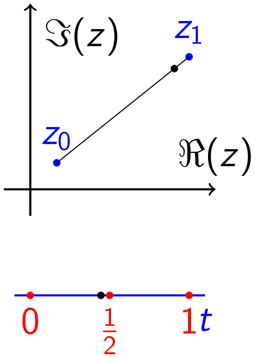

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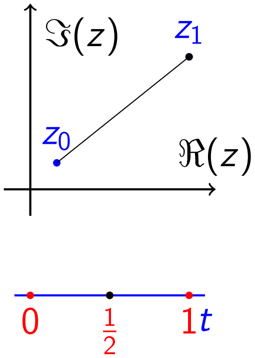

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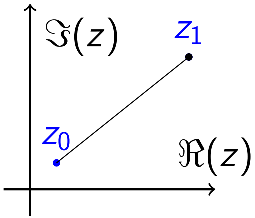

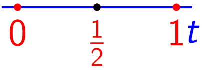

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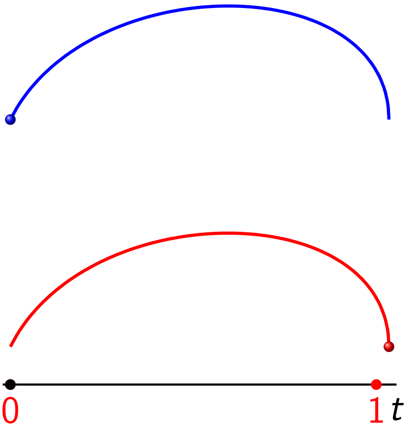
- Different parametrizations lead to different *paths* - but the same curve.

Operations on curves



If a curve C is parametrized by

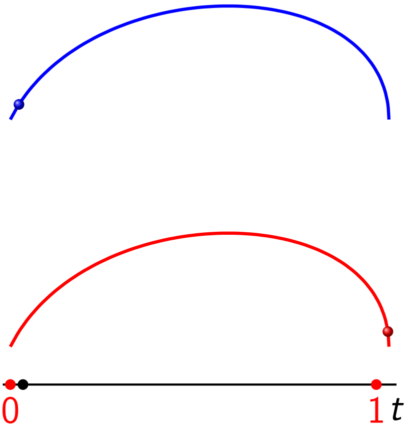
$$z : [0,1] \to \mathbb{C}, \quad t \mapsto z(t)$$

then $-C$ can be parametrized by

$$z' : [0,1] \to \mathbb{C},$$

$$t \mapsto z'(t) = z(1-t)$$

Operations on curves



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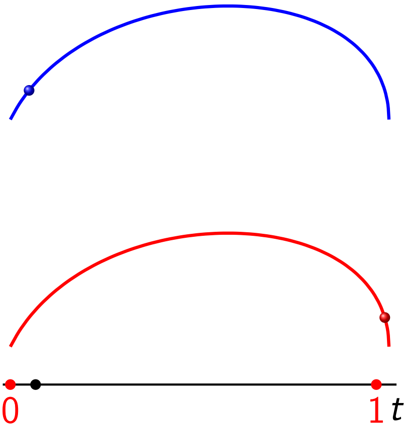

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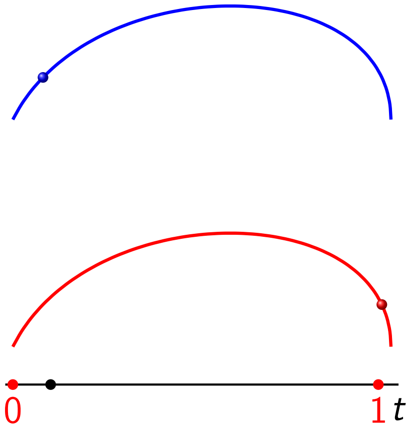

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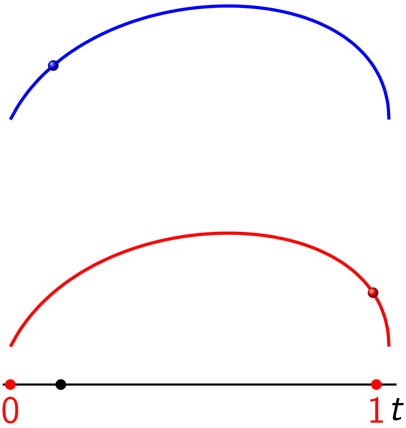

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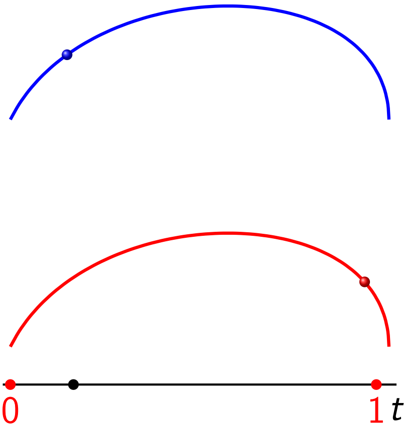

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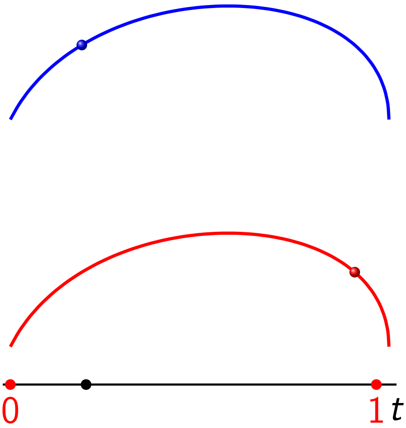

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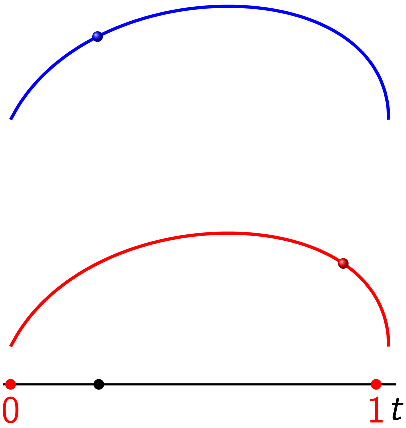

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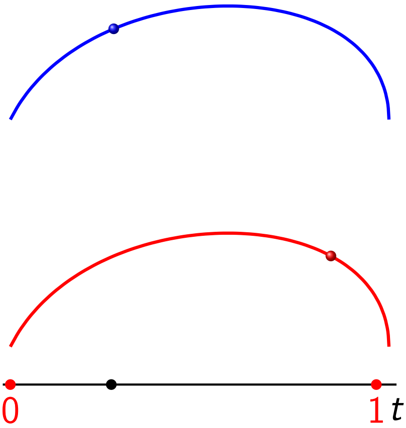

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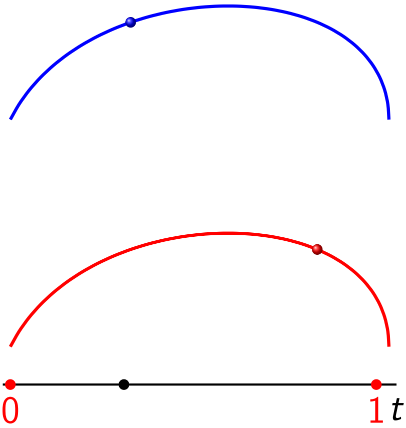

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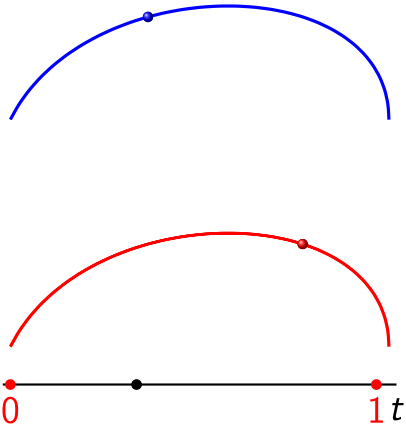

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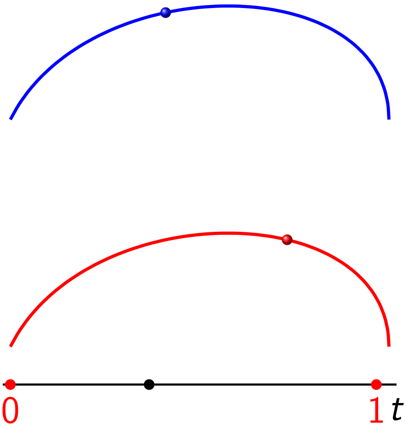

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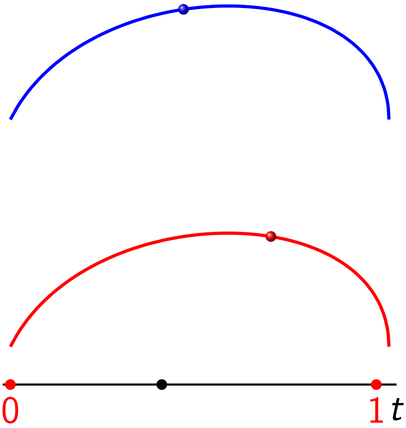

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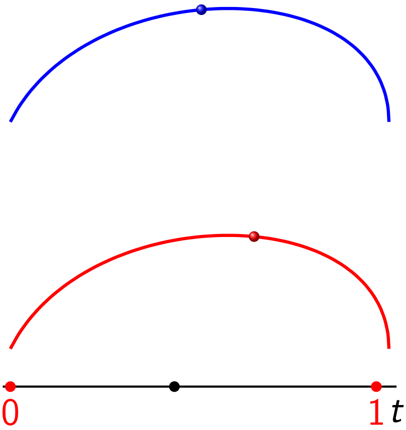

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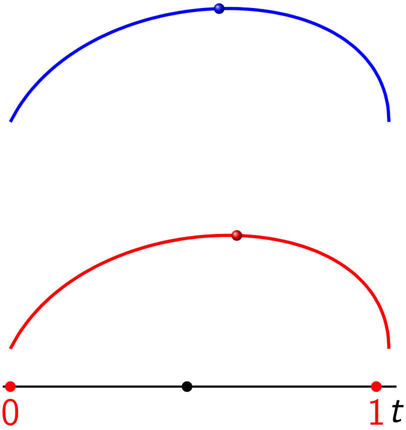

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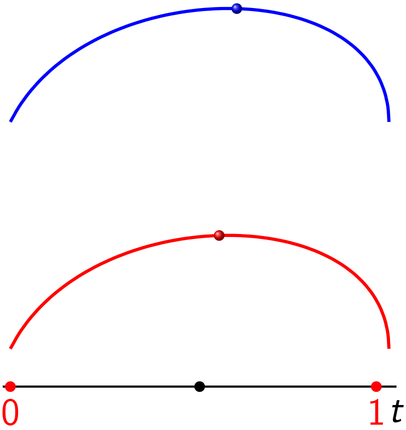

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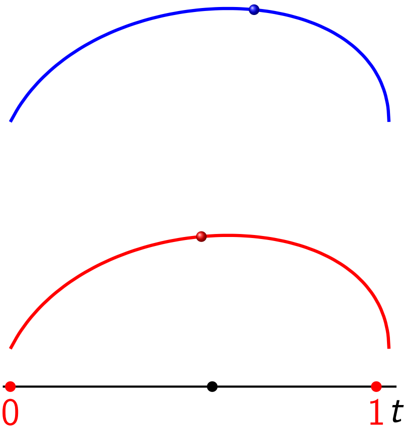

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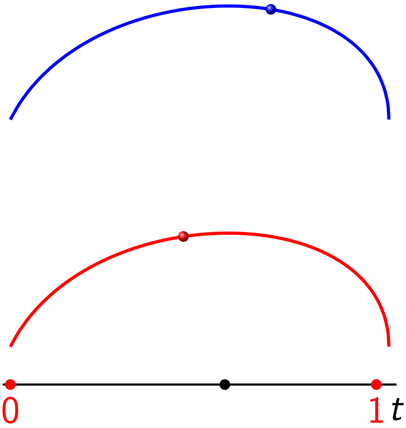

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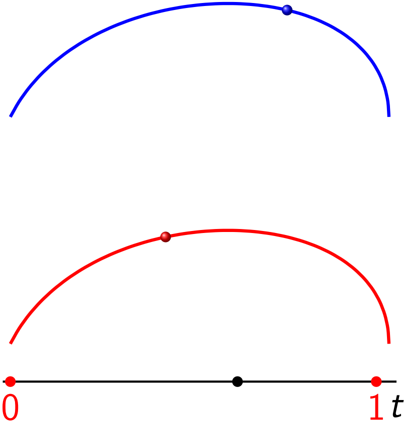

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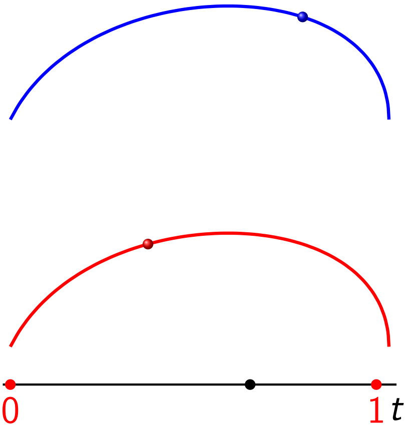

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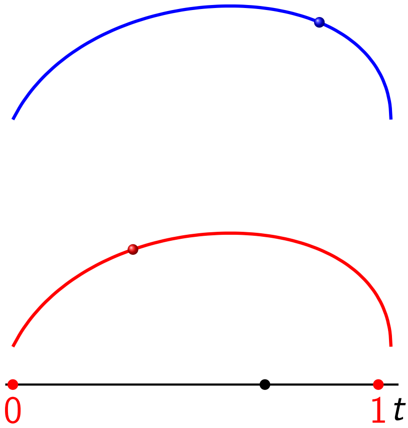

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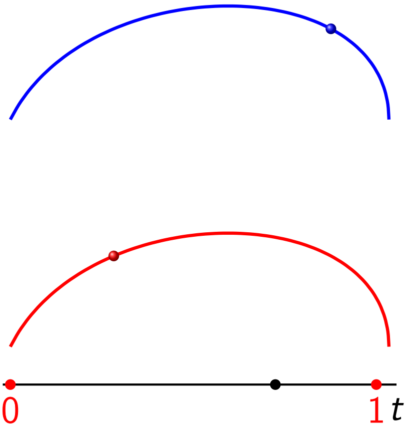

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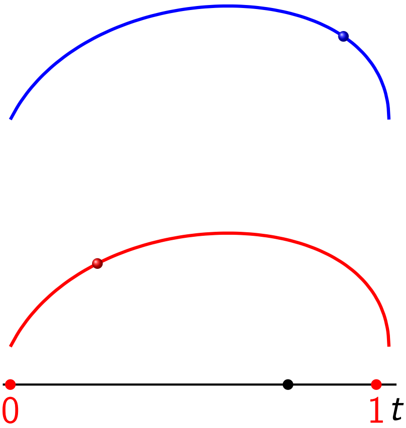
Operations on curves



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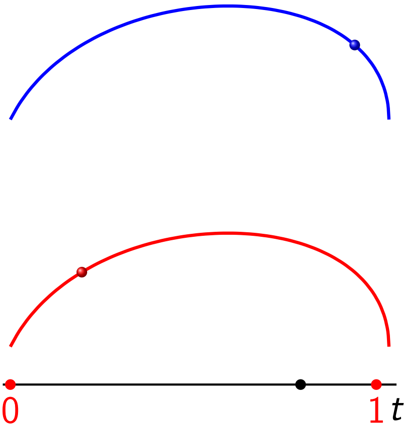

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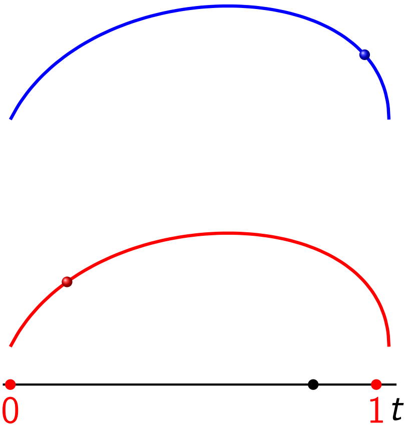

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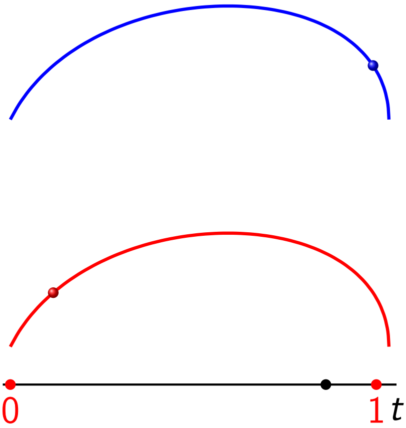

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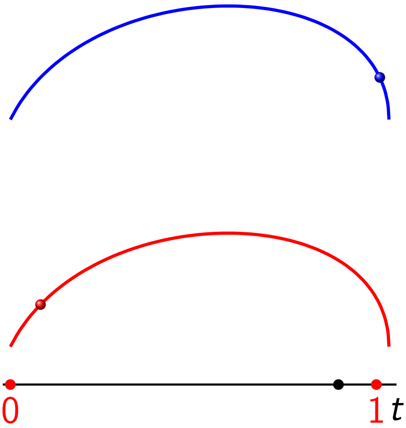

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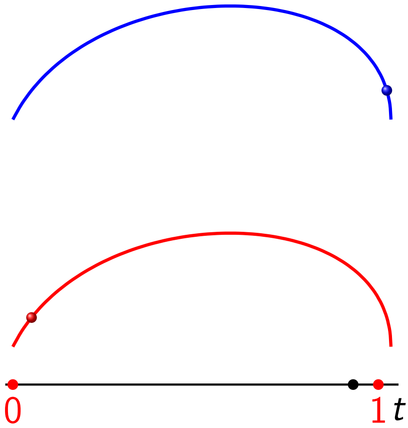

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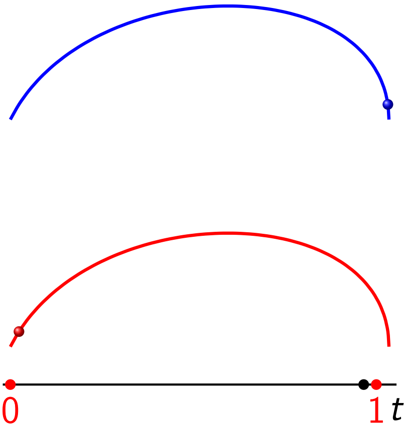

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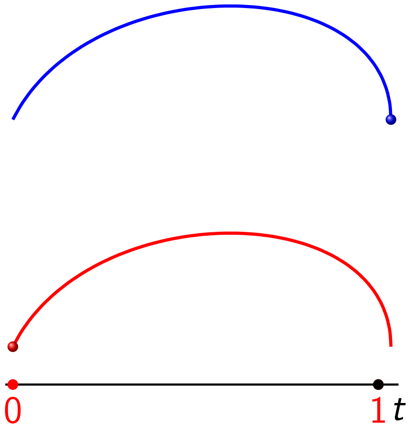

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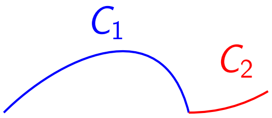
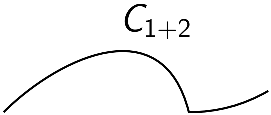


If two curves C_1, C_2, parametrized respectively by $z_1, z_2 \; : \; [0,1] \to \mathbb{C}$ are such that the final point of C_1 is the same as the initial point of C_2

$$z_1(1) = z_2(0)$$

then $C_{1+2} = C_1 \cup C_2$ is a curve.

Operations on curves

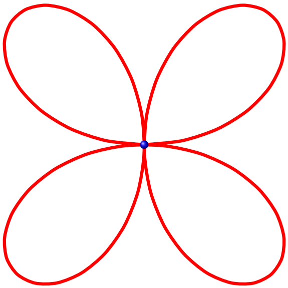


A possible parametrization of C_{1+2} is

$$z(t) = \begin{cases} z_1(2t) & 0 \leq t < \frac{1}{2} \\ z_2(2t-1) & \frac{1}{2} \leq t \leq 1 \end{cases}$$

Some examples

The four-leaved clover



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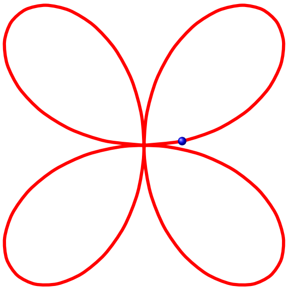

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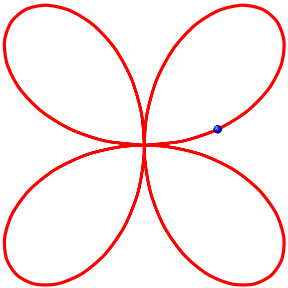

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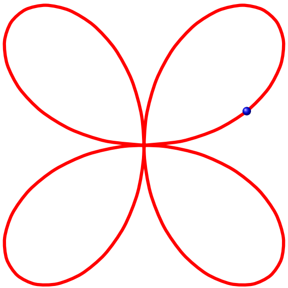

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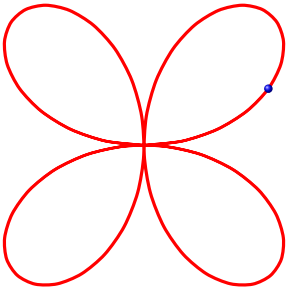

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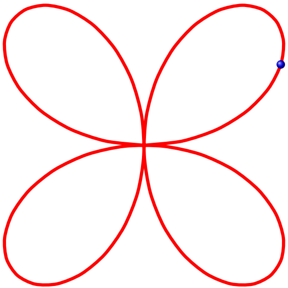

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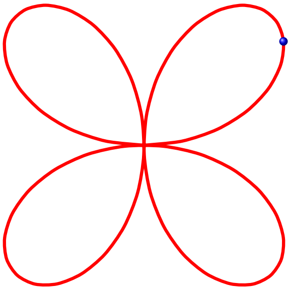

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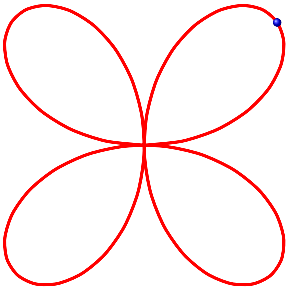

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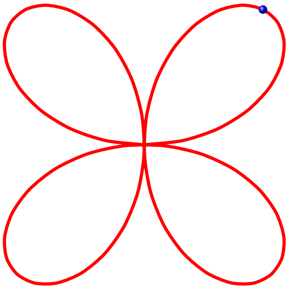

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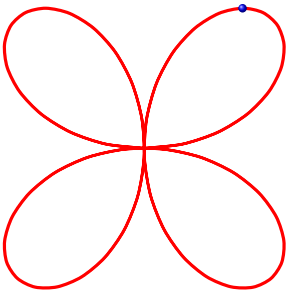

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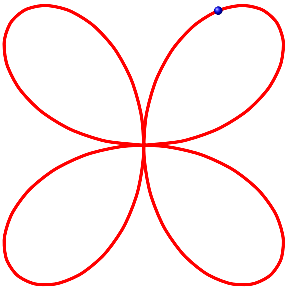

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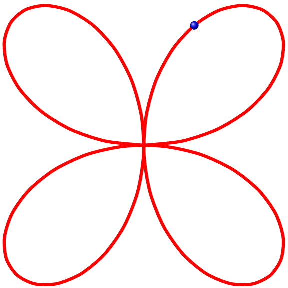

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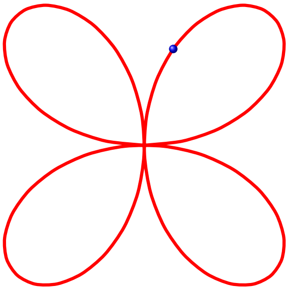

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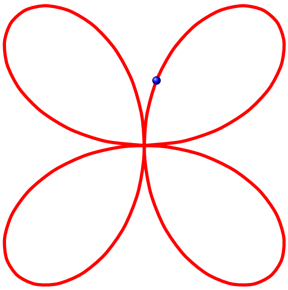

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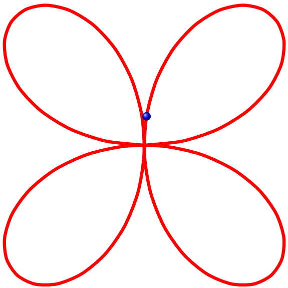

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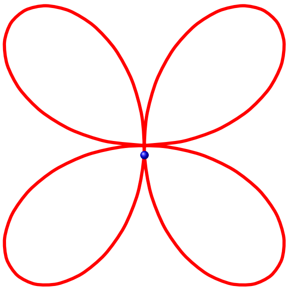

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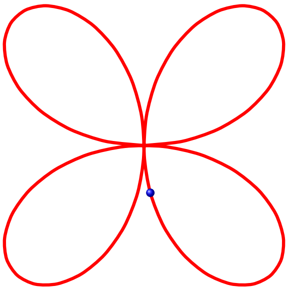

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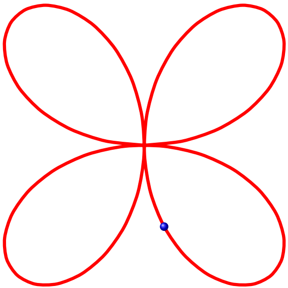

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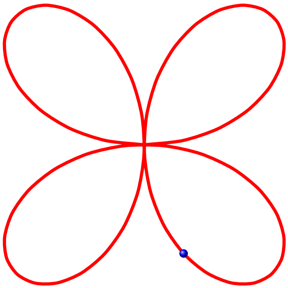

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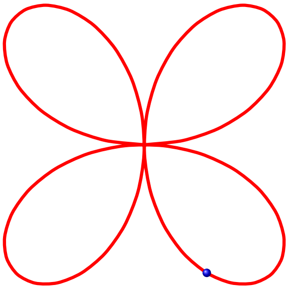

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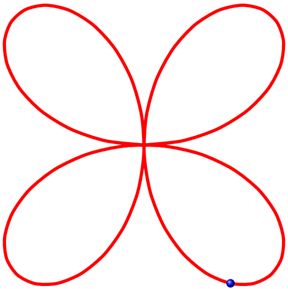

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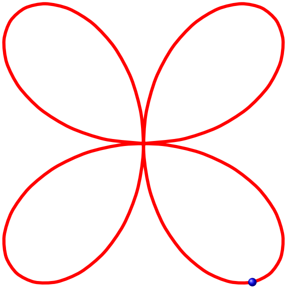

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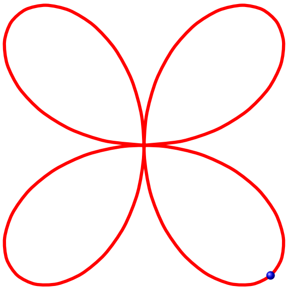

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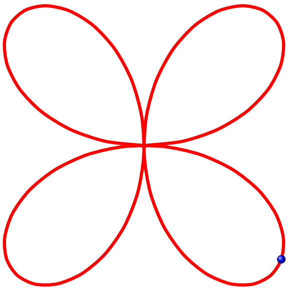

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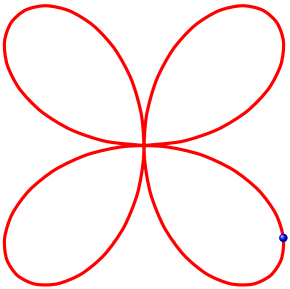

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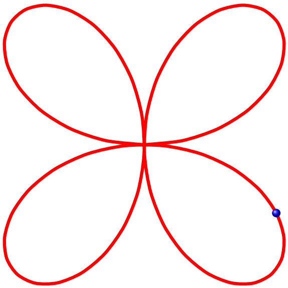

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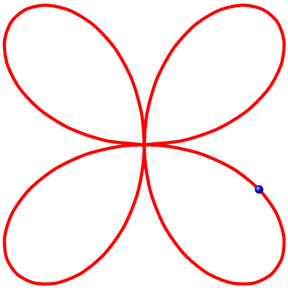

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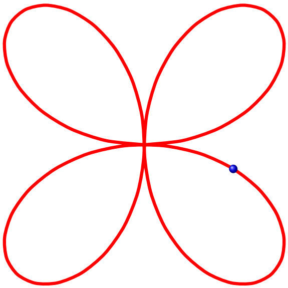

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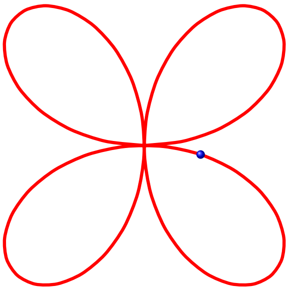

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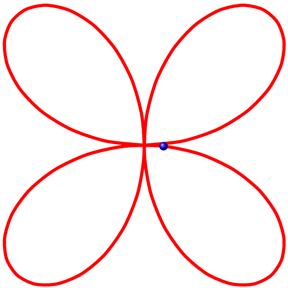

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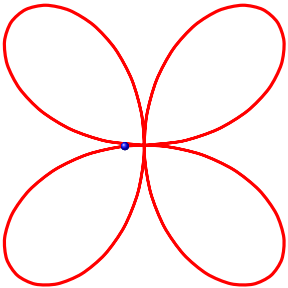

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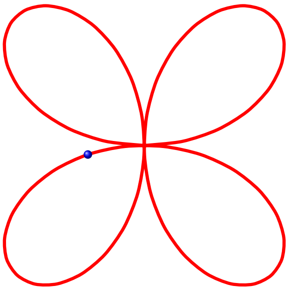

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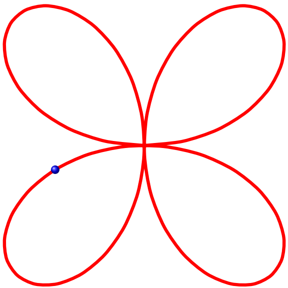

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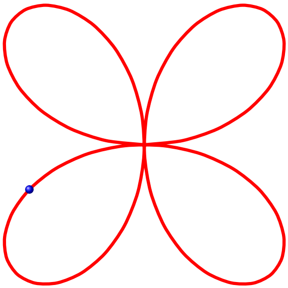

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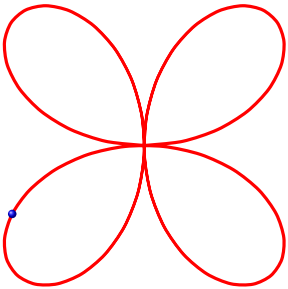

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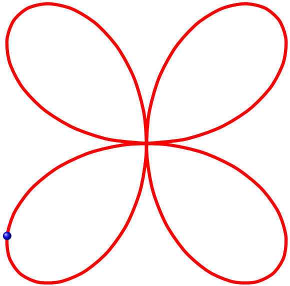

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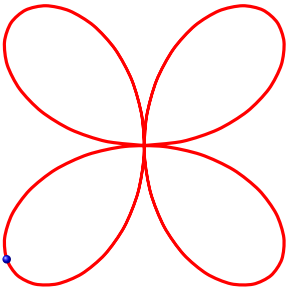

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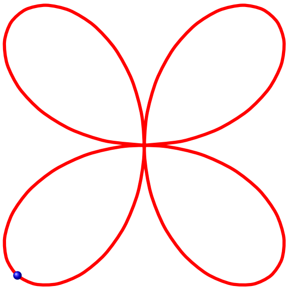

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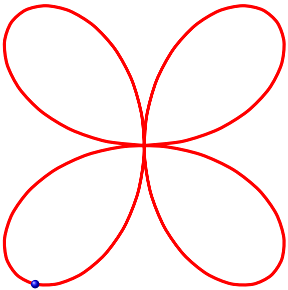

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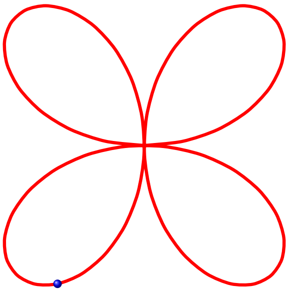

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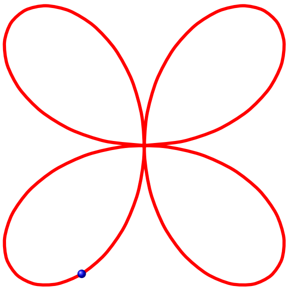

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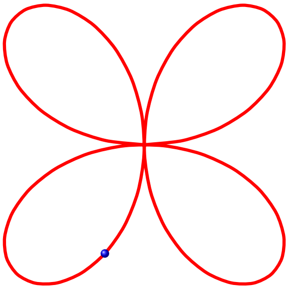

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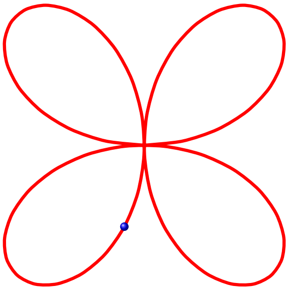

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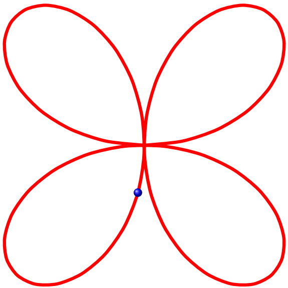

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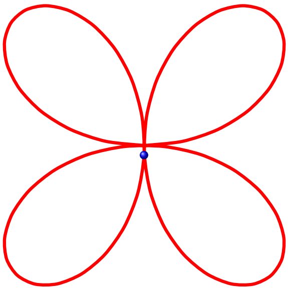

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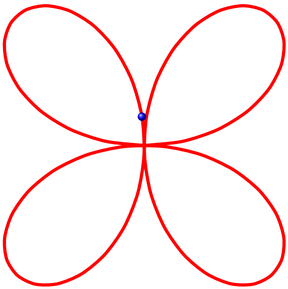

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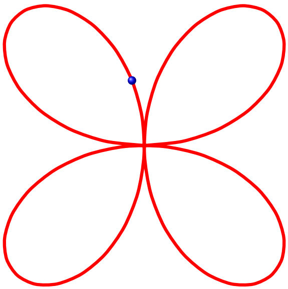

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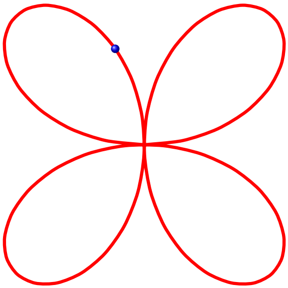

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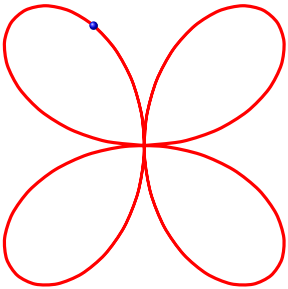

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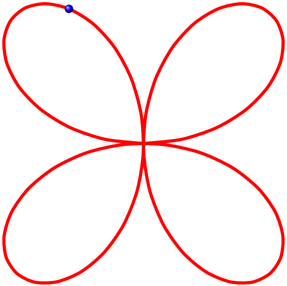

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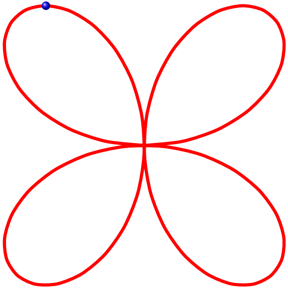

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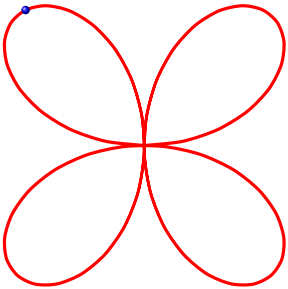

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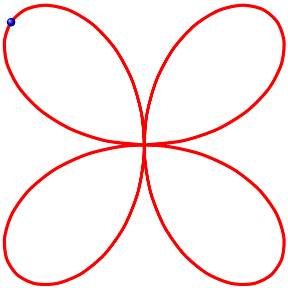

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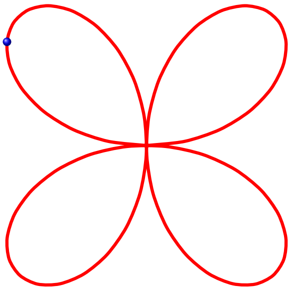

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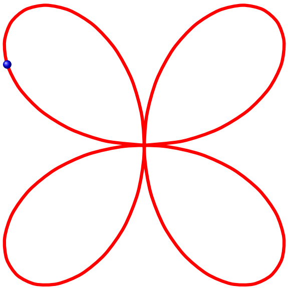

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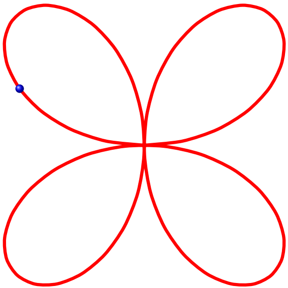

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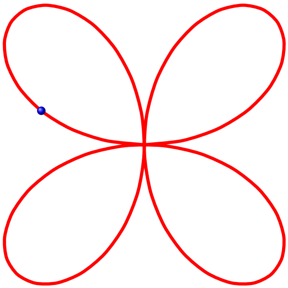

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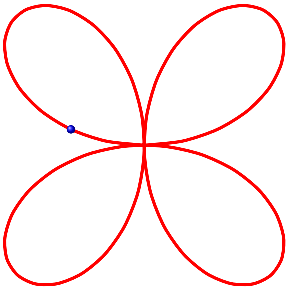

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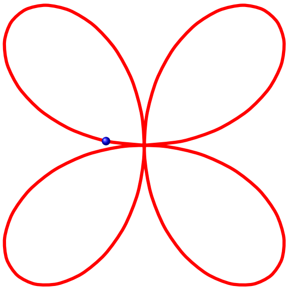

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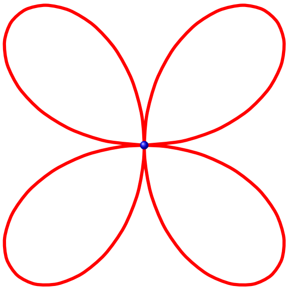

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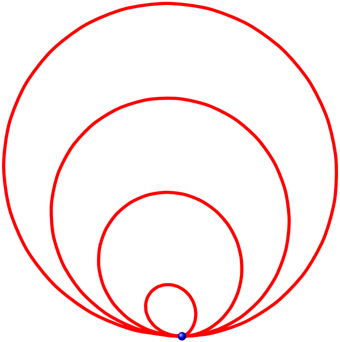
A more esoteric curve



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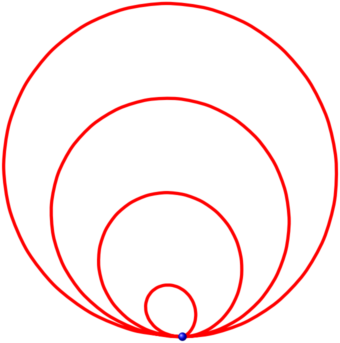

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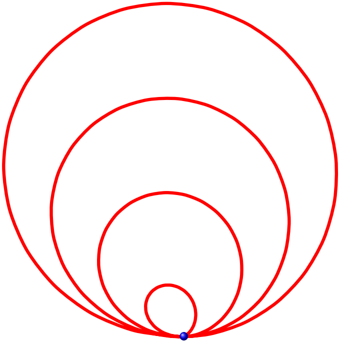

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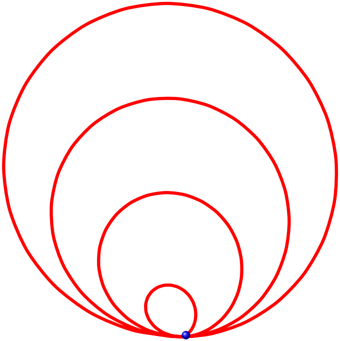

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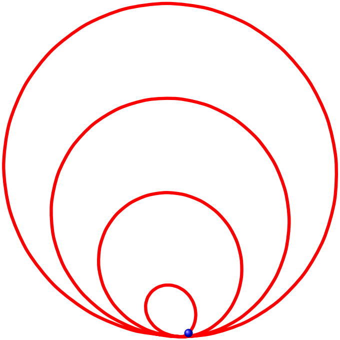

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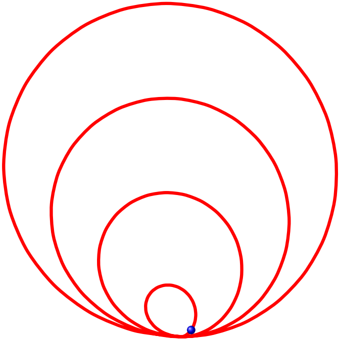

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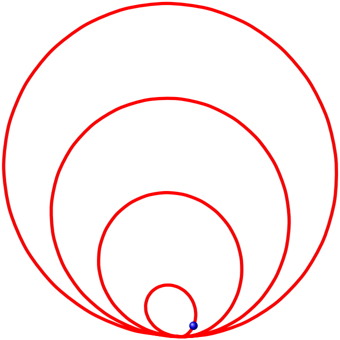

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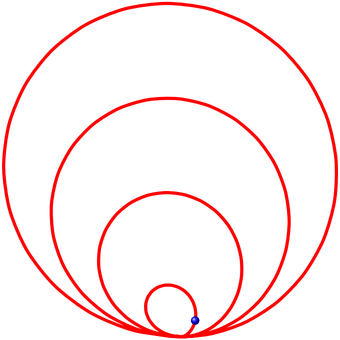

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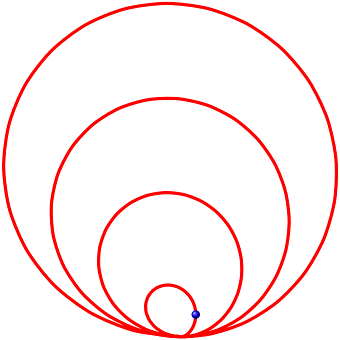

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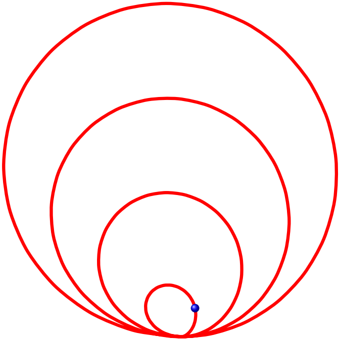

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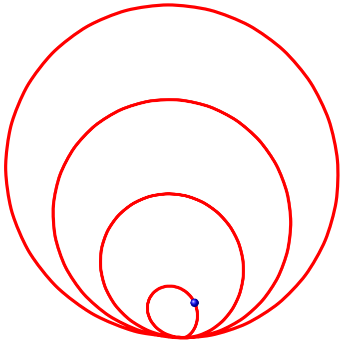

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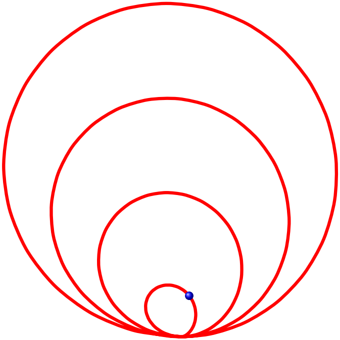

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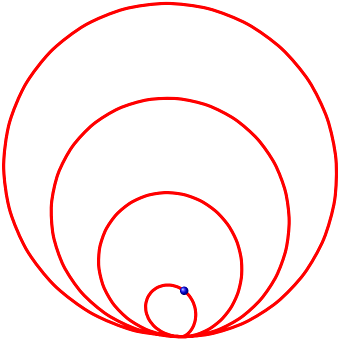

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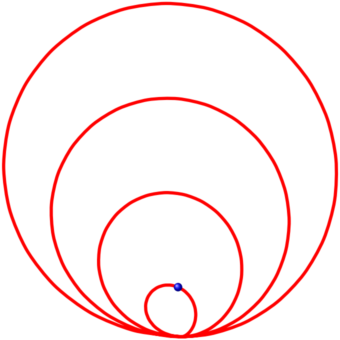

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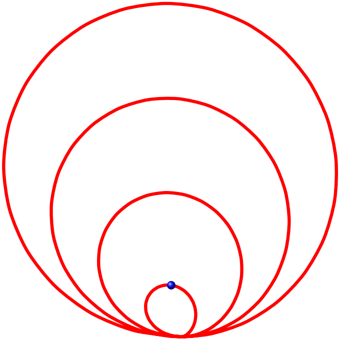

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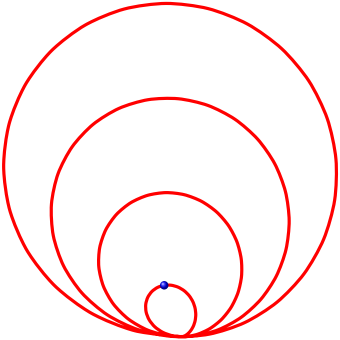

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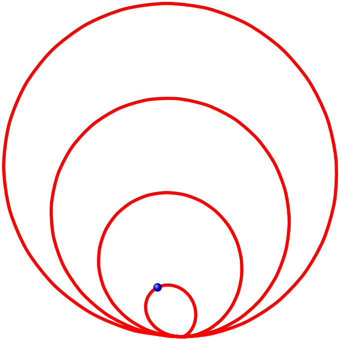

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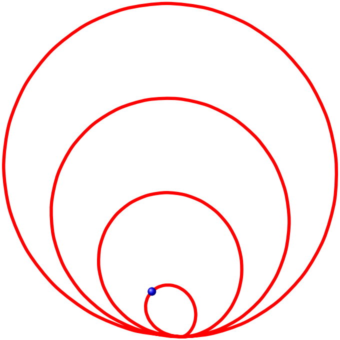

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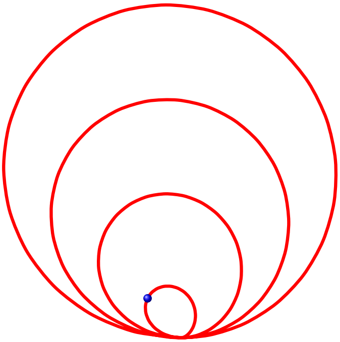

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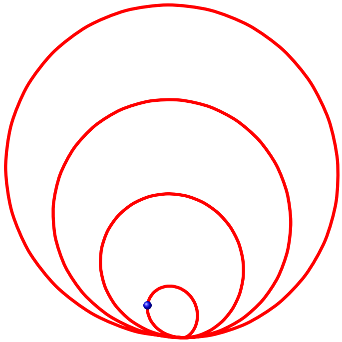

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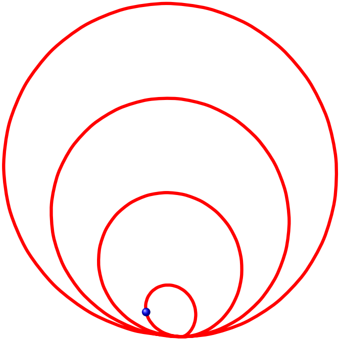

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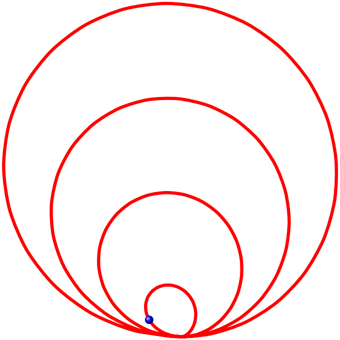

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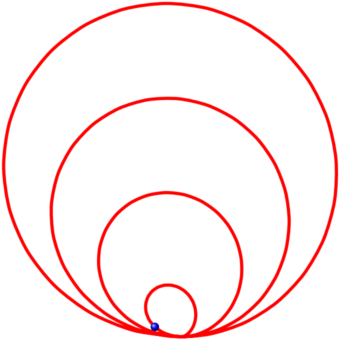

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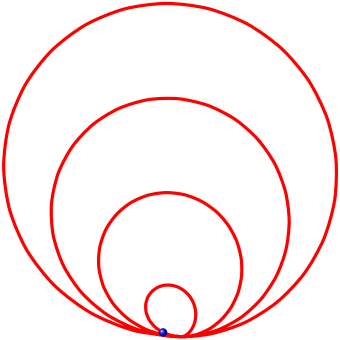

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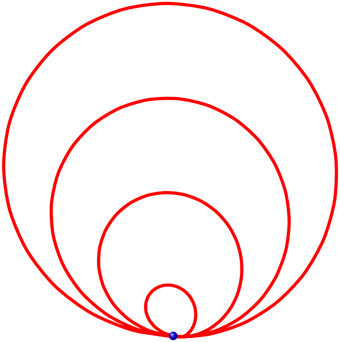

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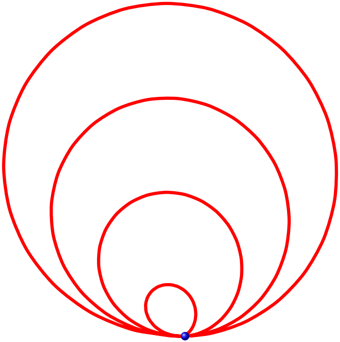

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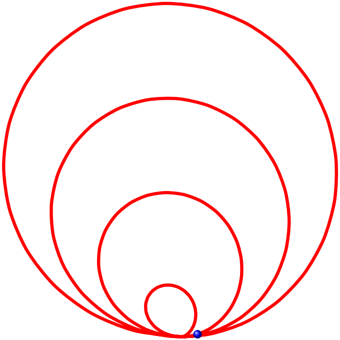

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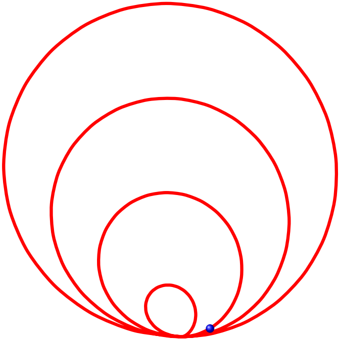

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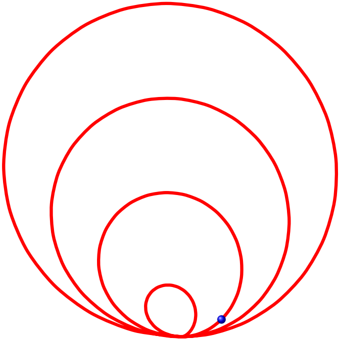

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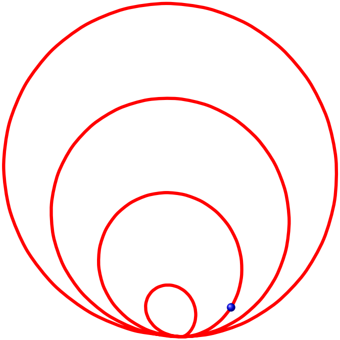

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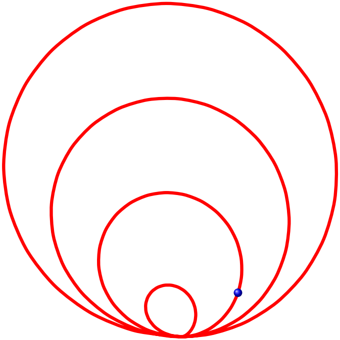

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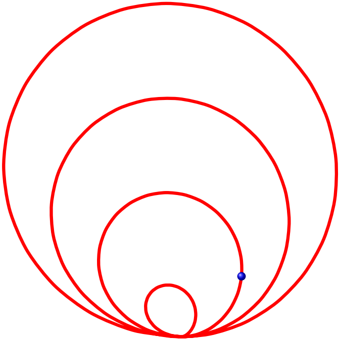

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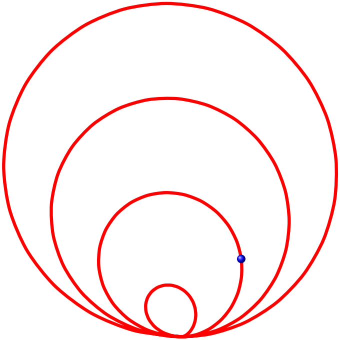

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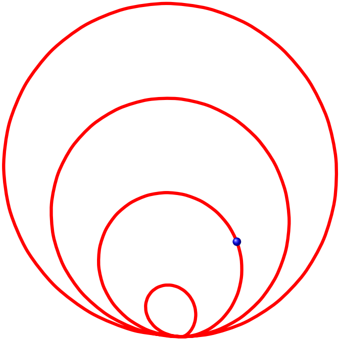

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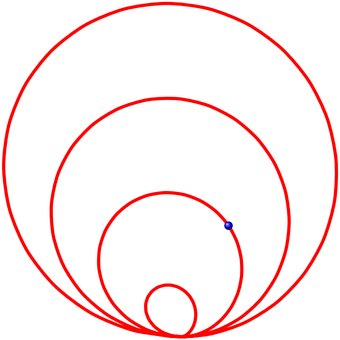

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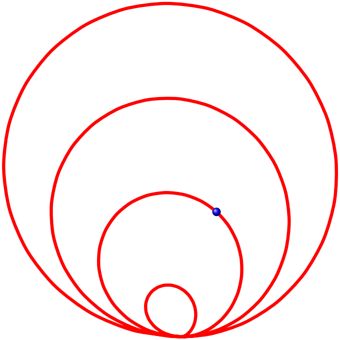

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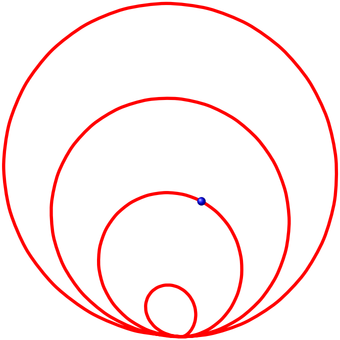

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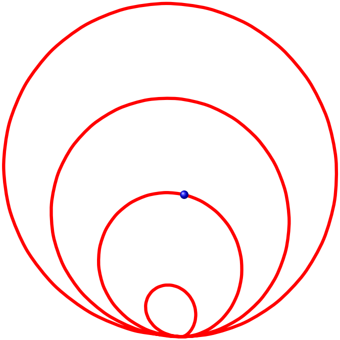

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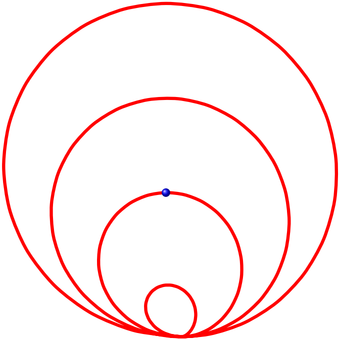

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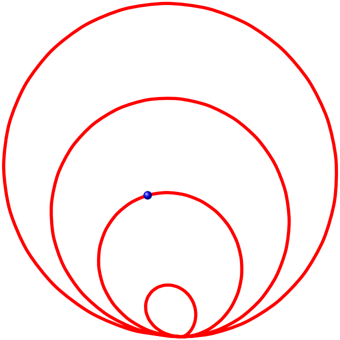

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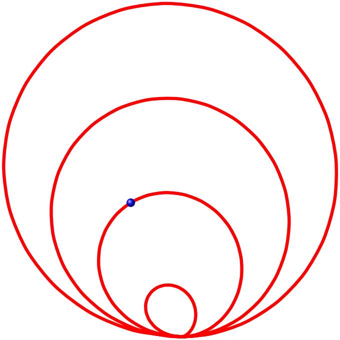

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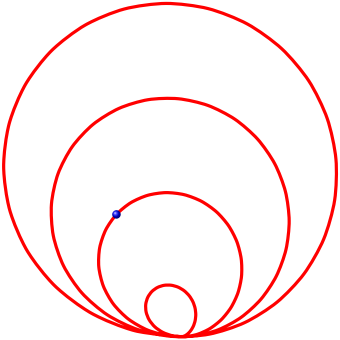

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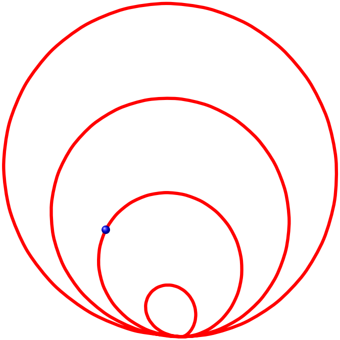

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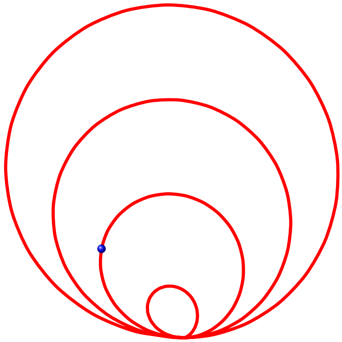

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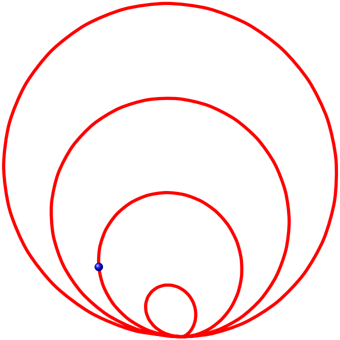

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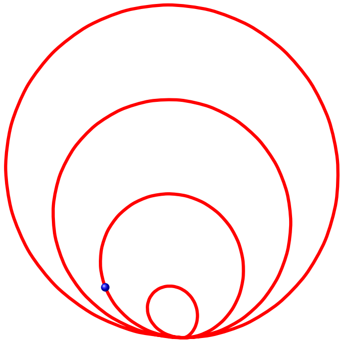

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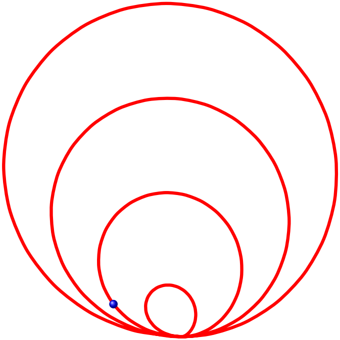

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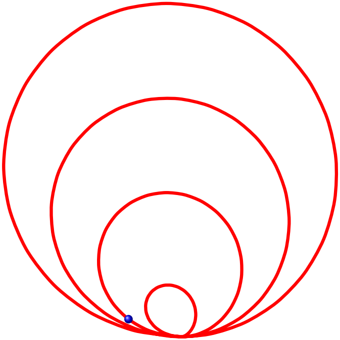

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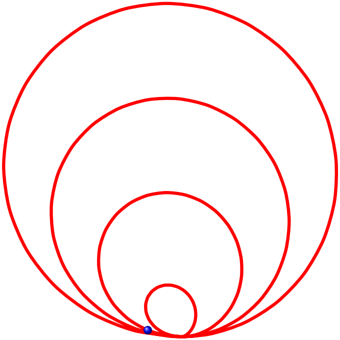

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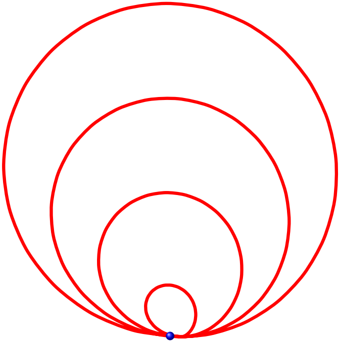

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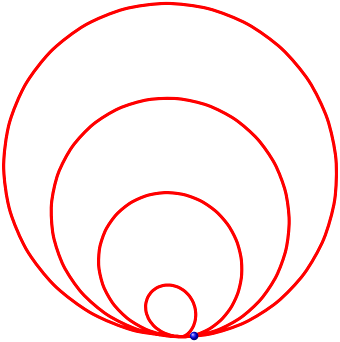

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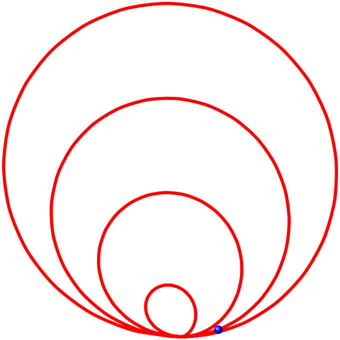

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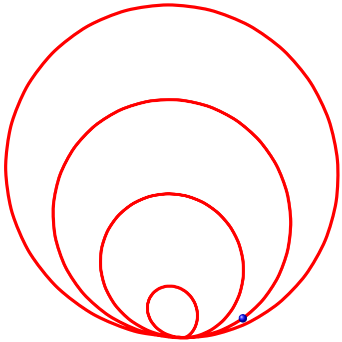

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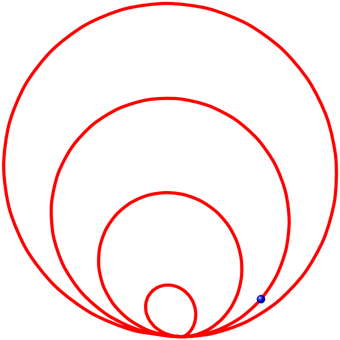

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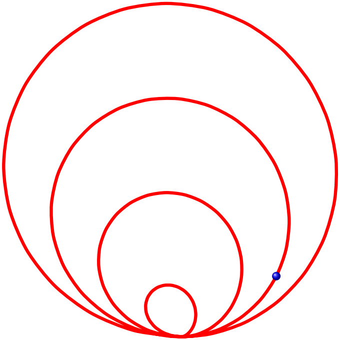

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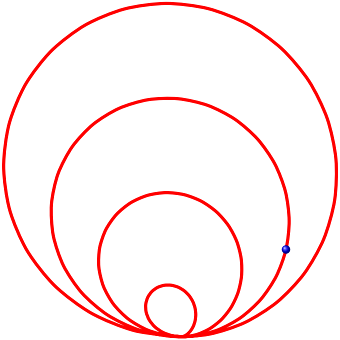

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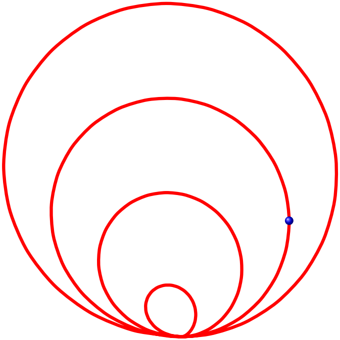

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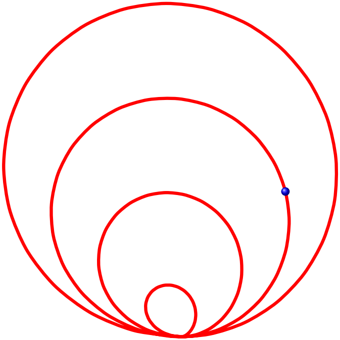

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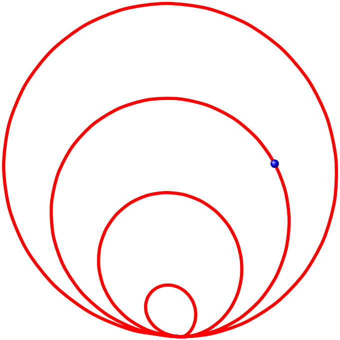

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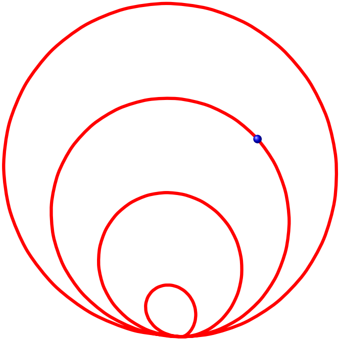

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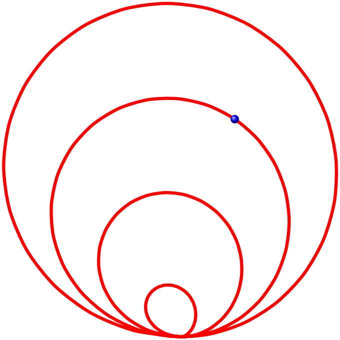

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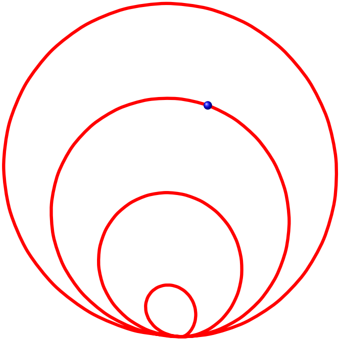

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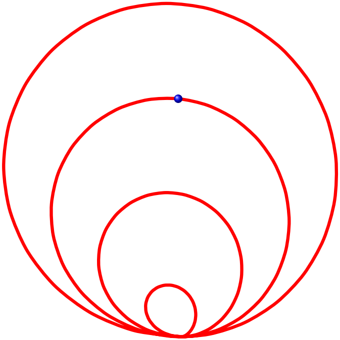

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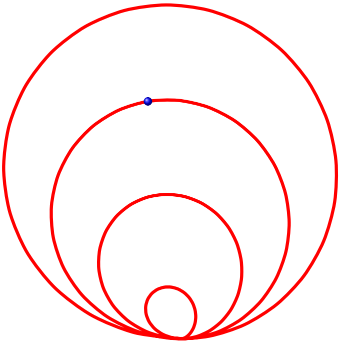

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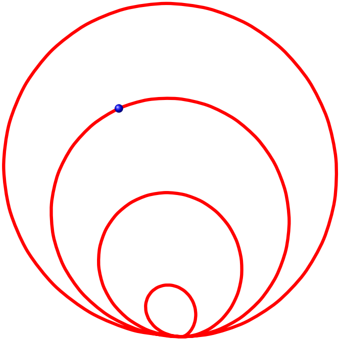

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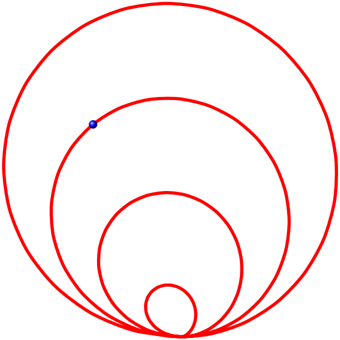

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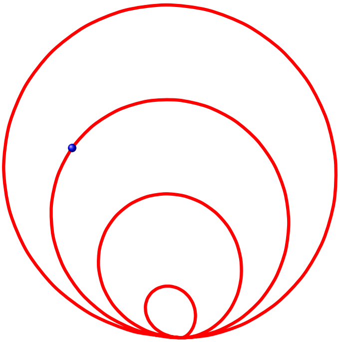

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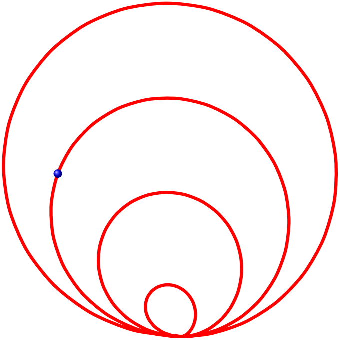

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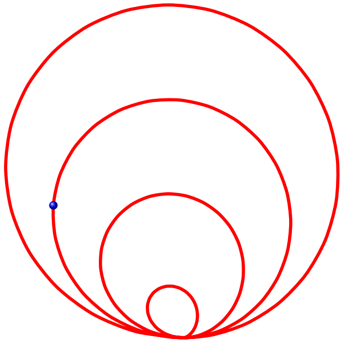

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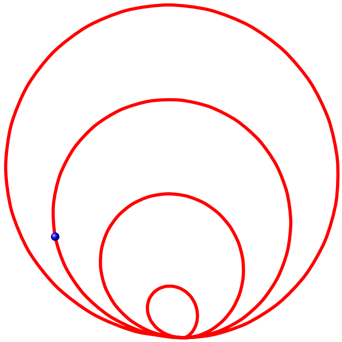

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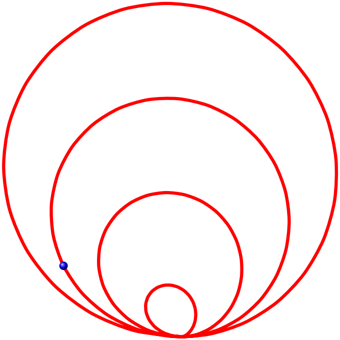

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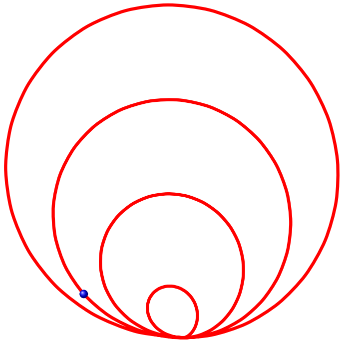

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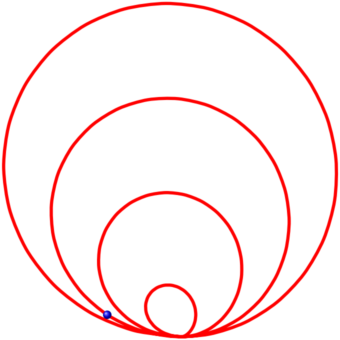

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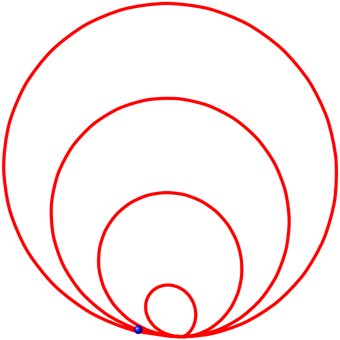

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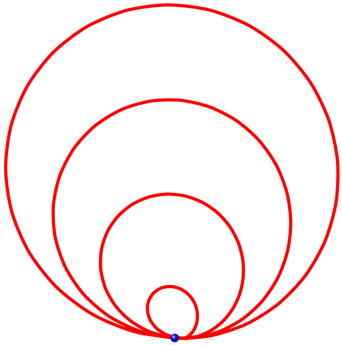

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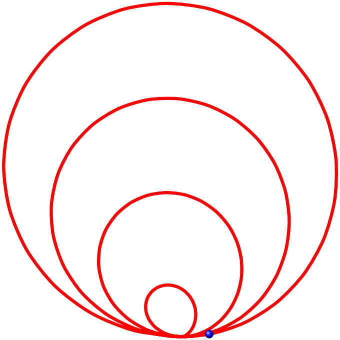

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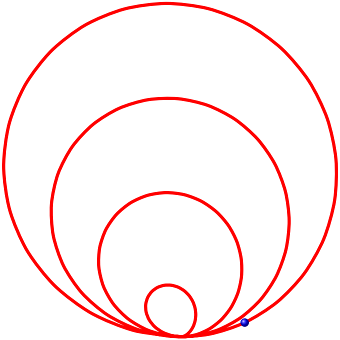

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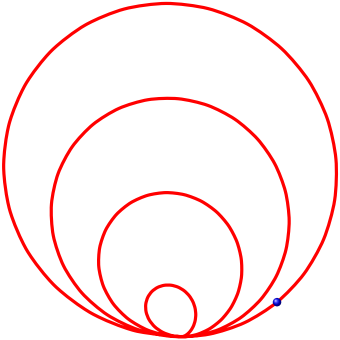

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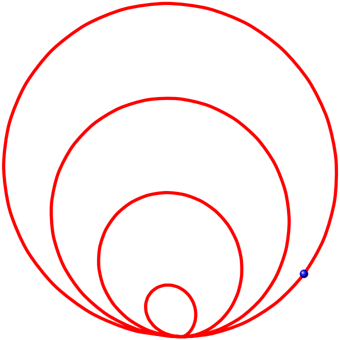

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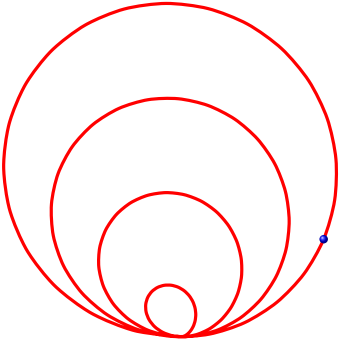

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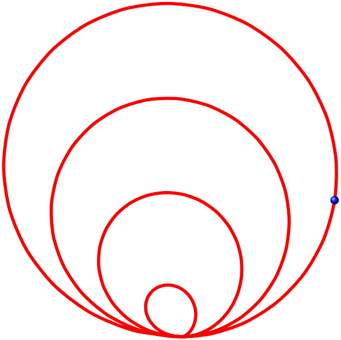

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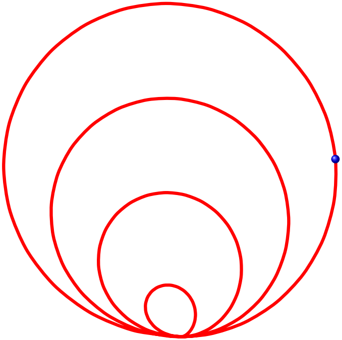

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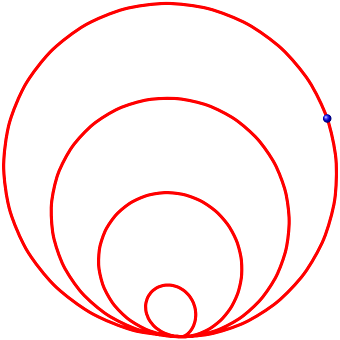

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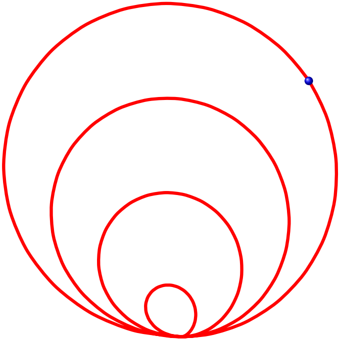

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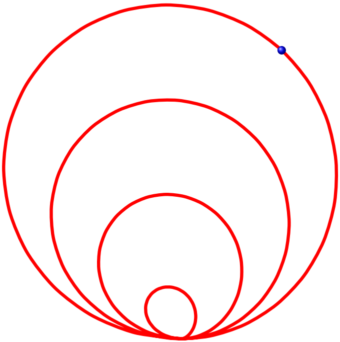

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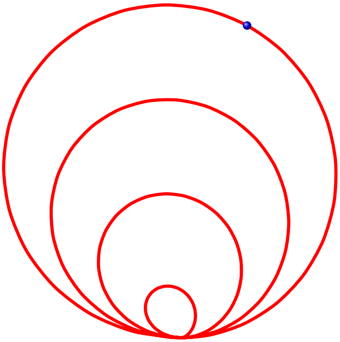

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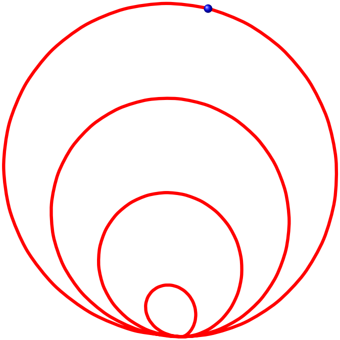

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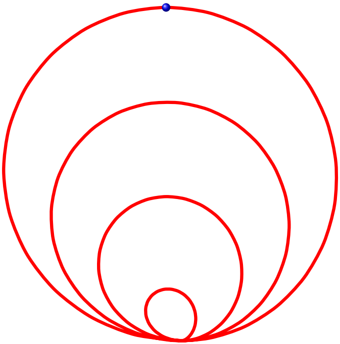

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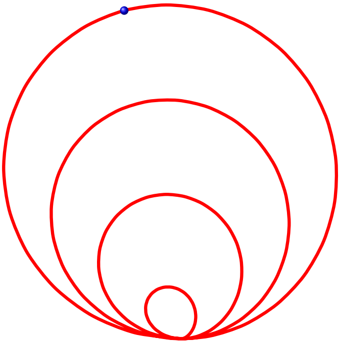

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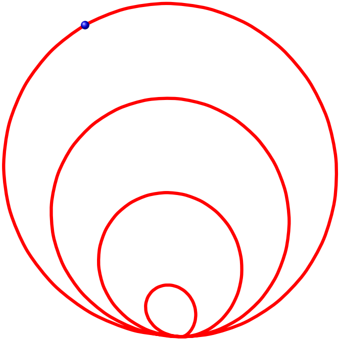

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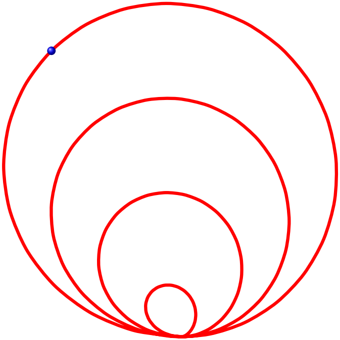

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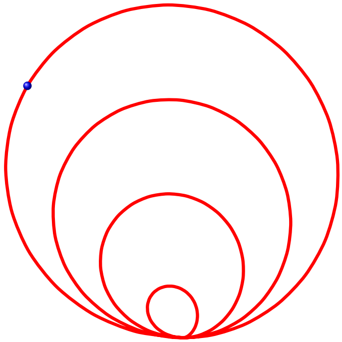

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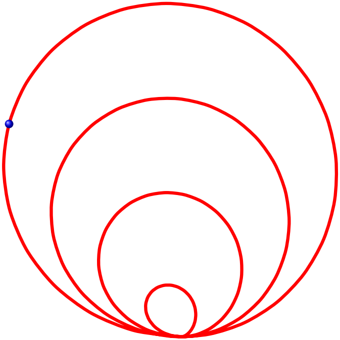

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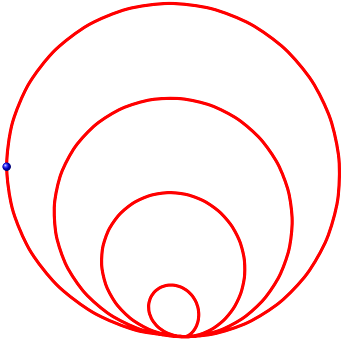

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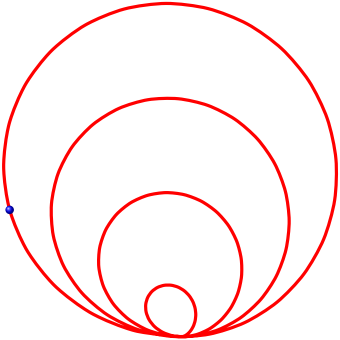

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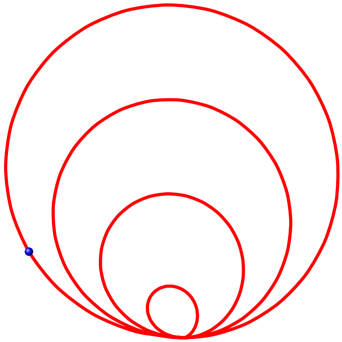

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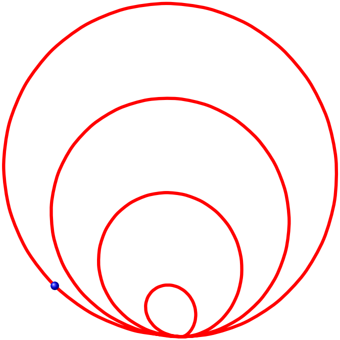

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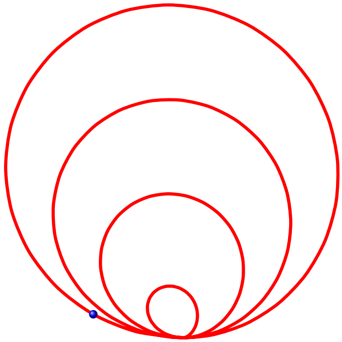

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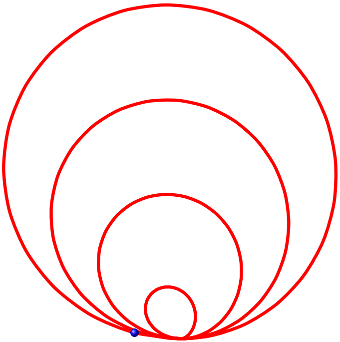

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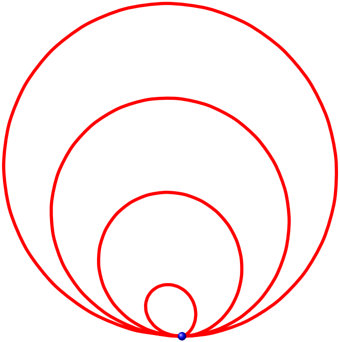

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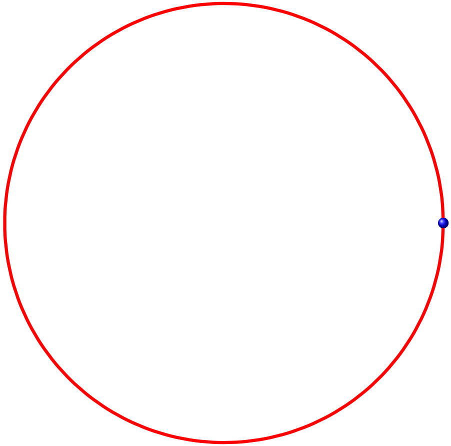
Some examples

The unit circle



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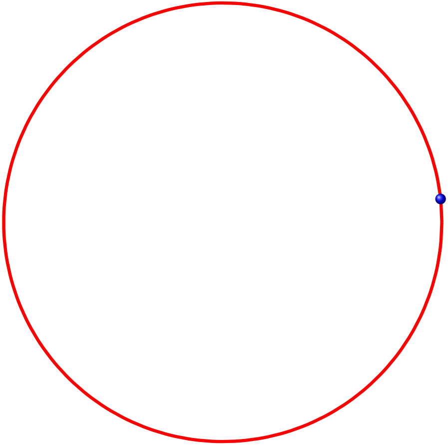

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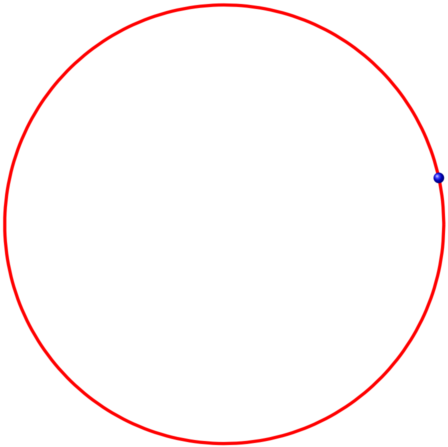

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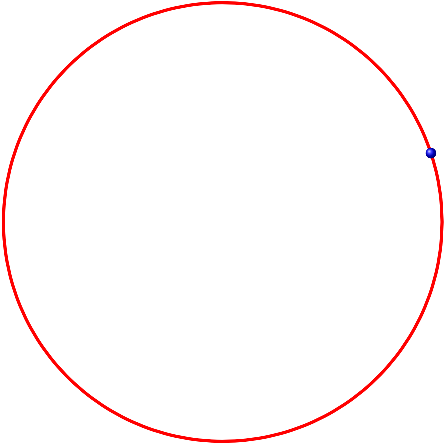

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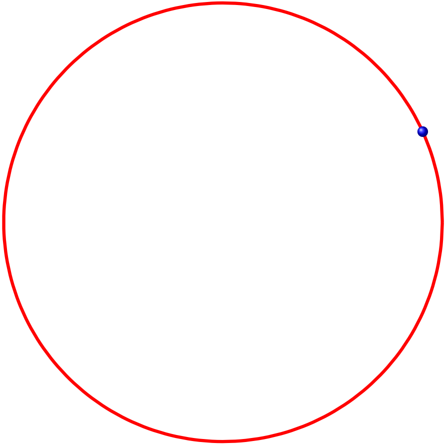

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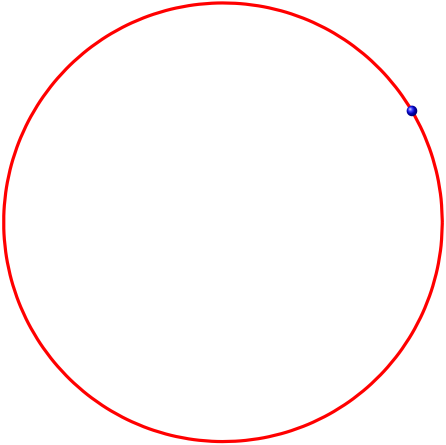

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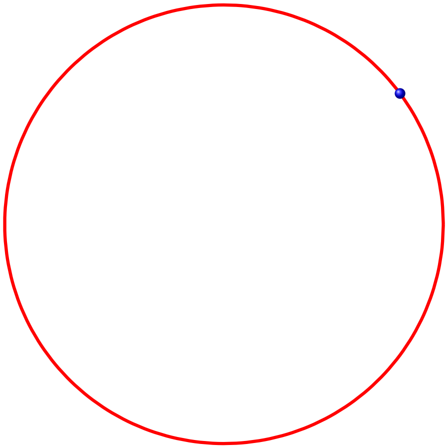

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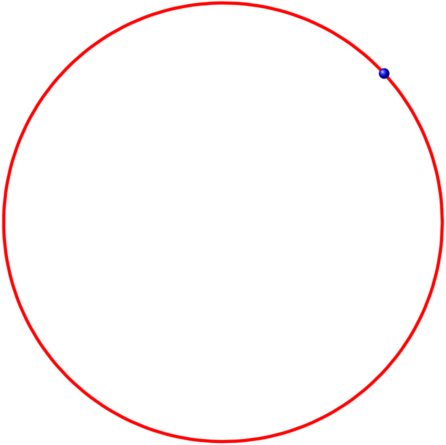

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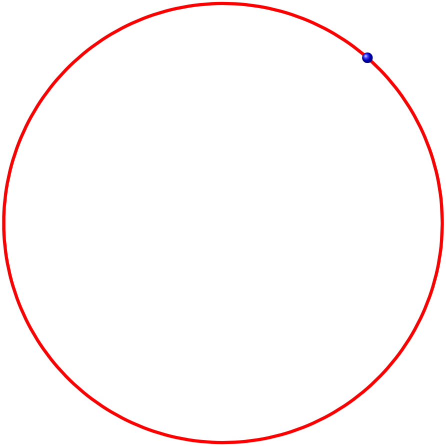

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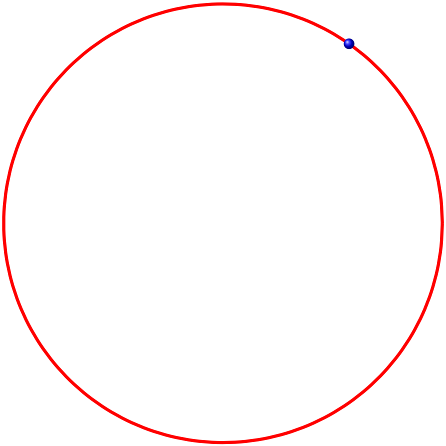

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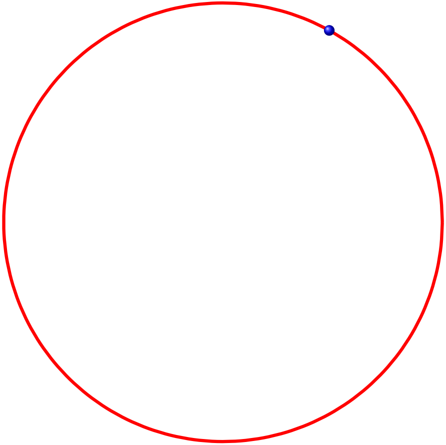

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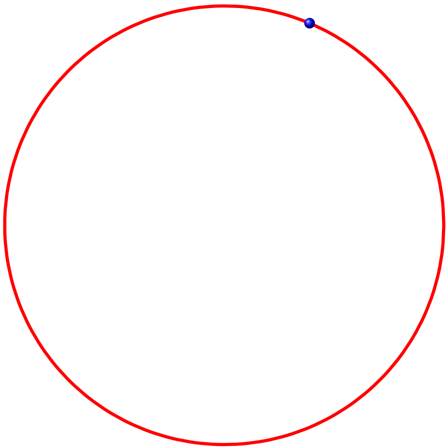

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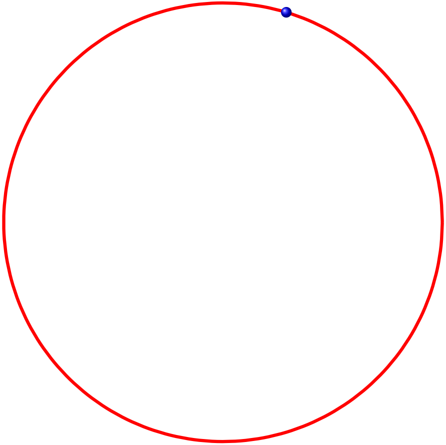

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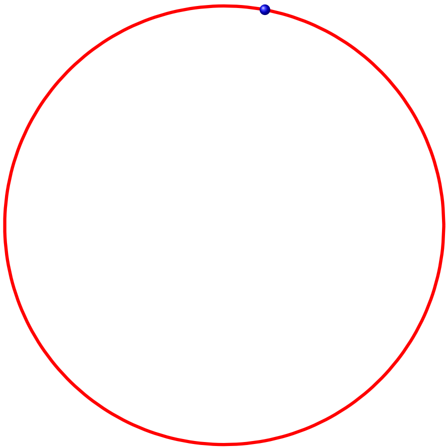

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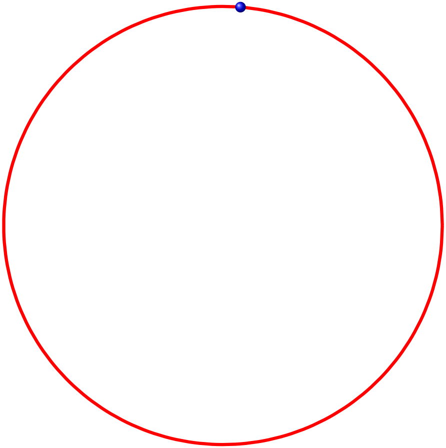

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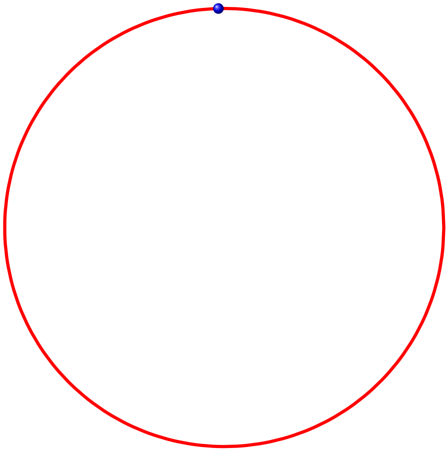

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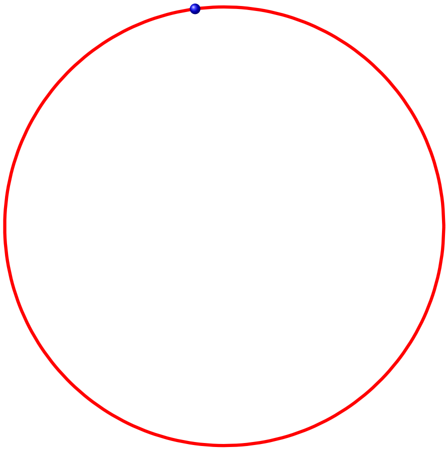

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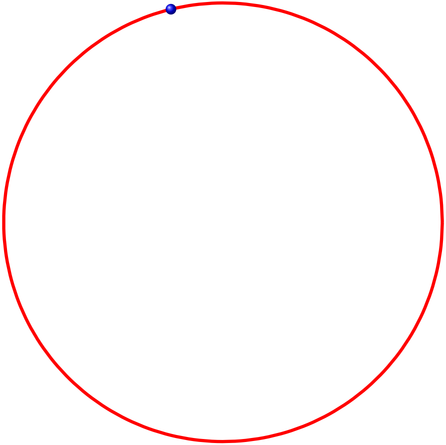

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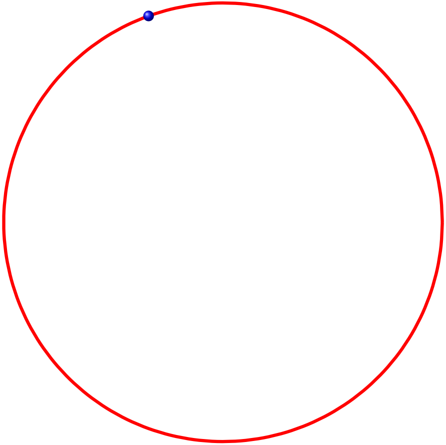

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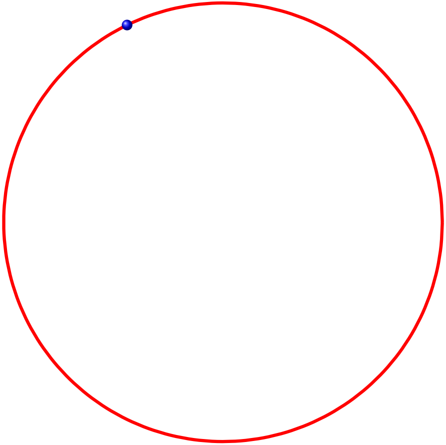

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$$z(t) = e^{it}$$

Some examples

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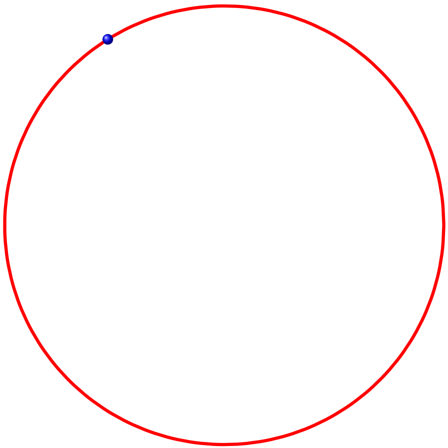

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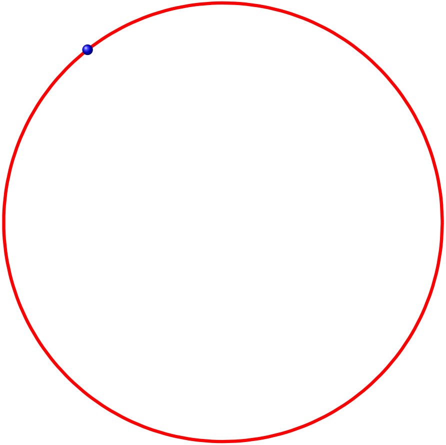

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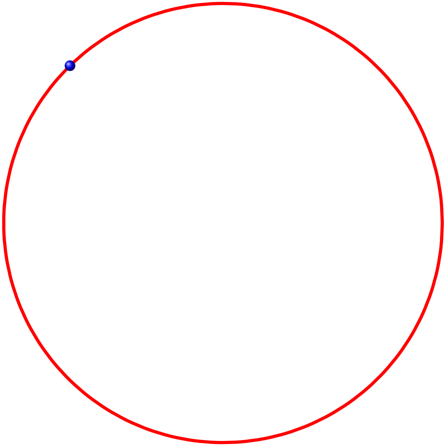

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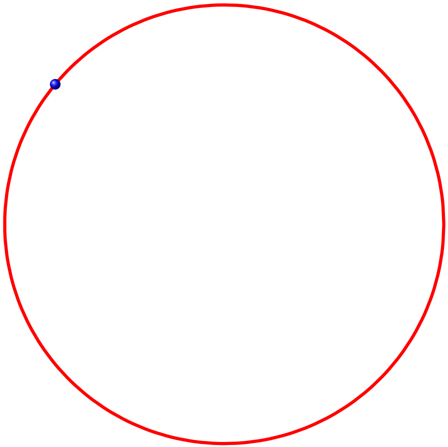

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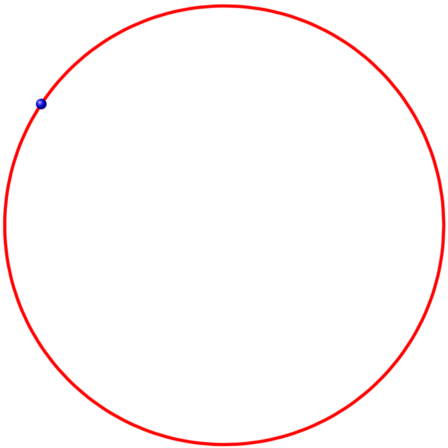

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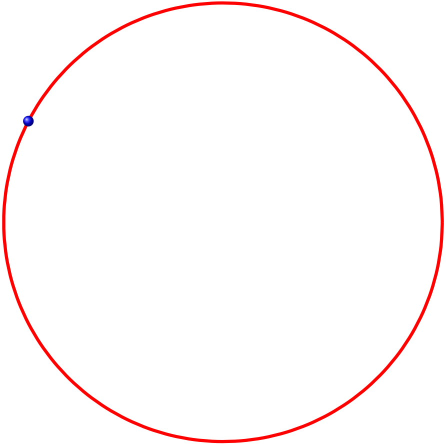

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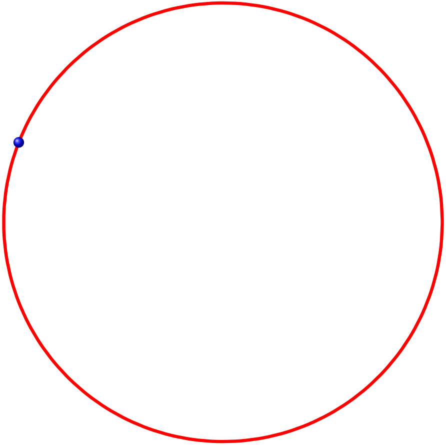

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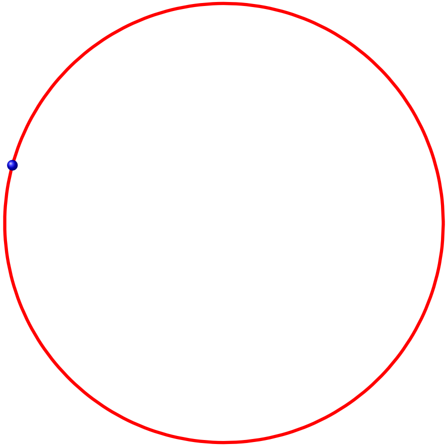

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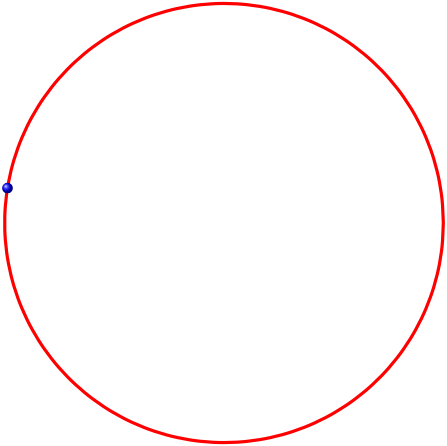

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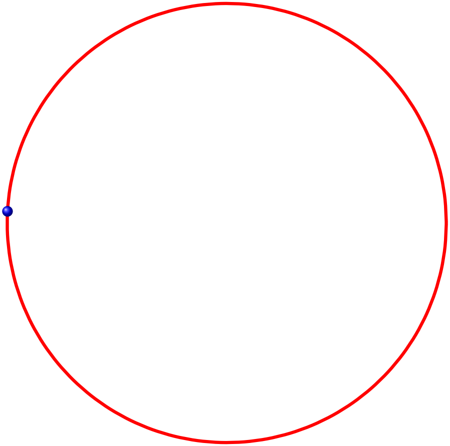

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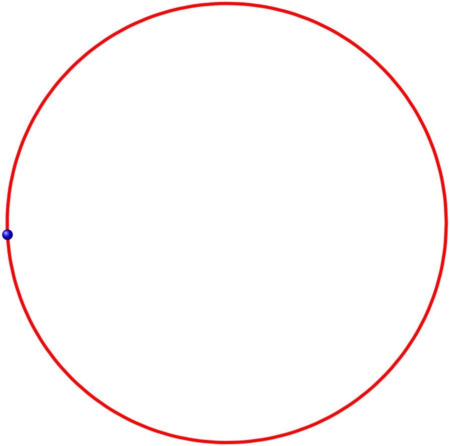

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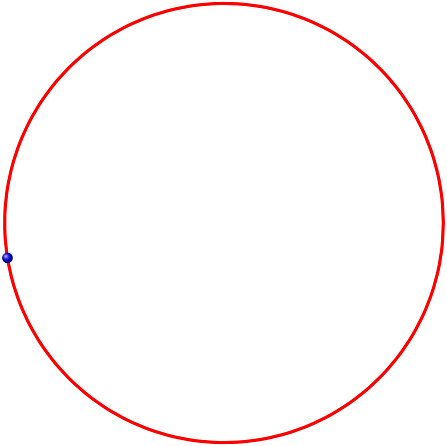

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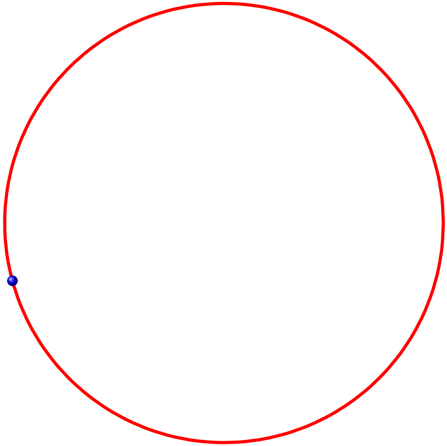

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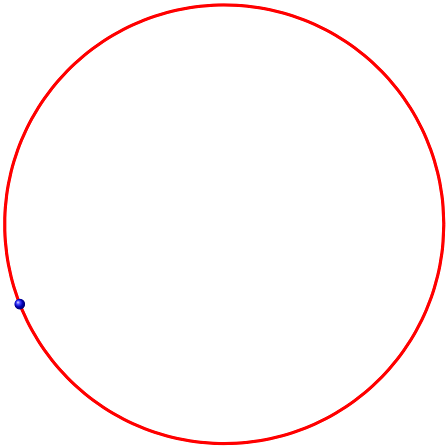

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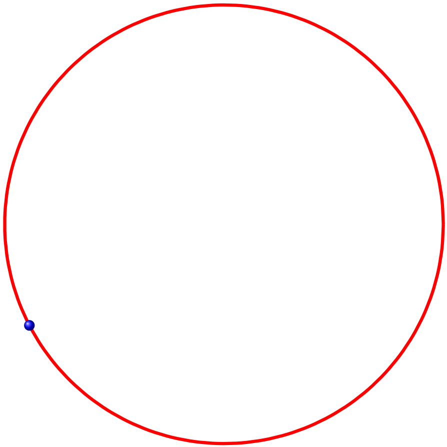

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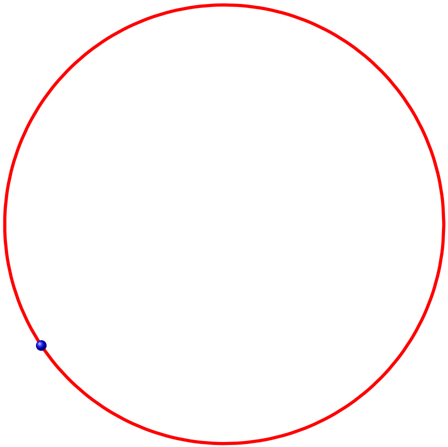

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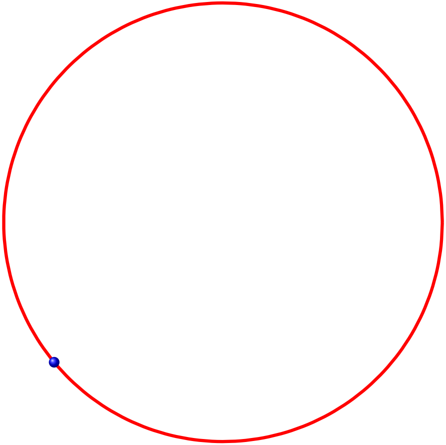

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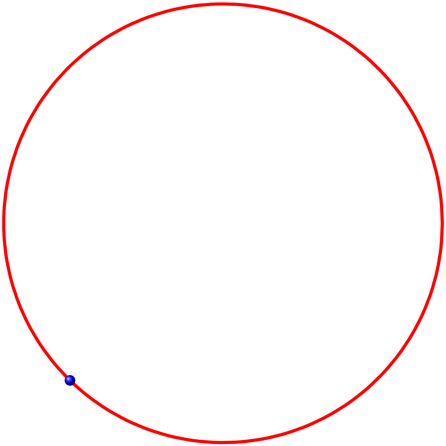

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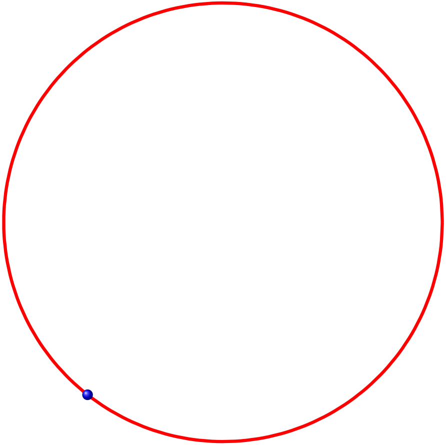

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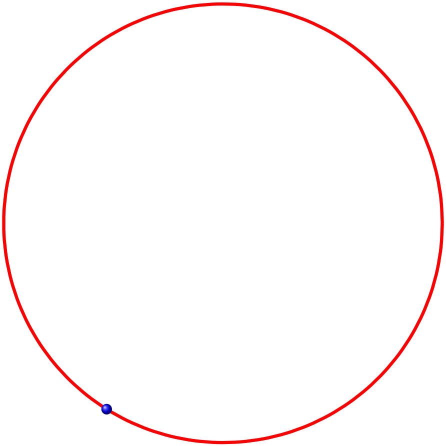

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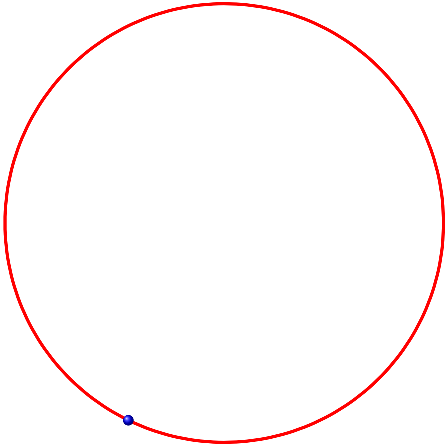

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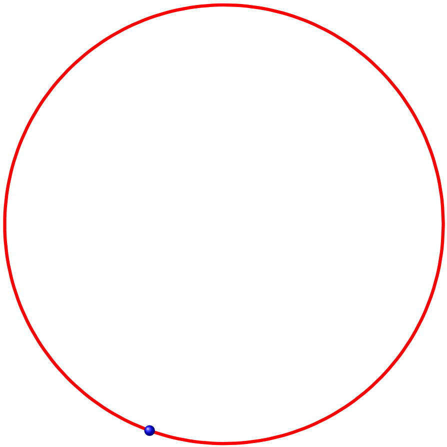

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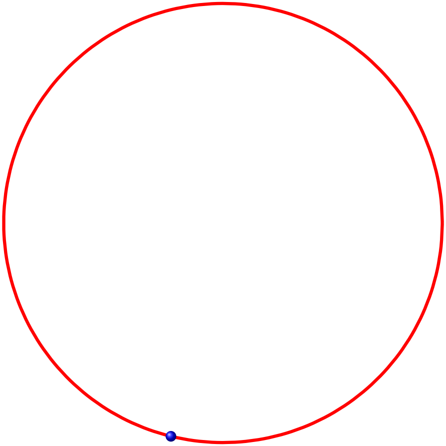

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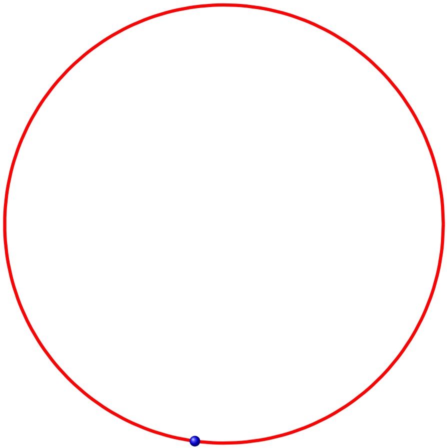

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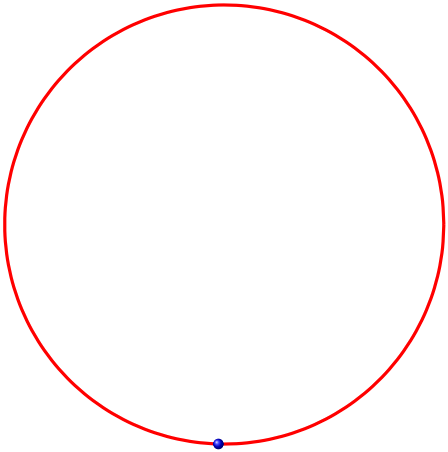

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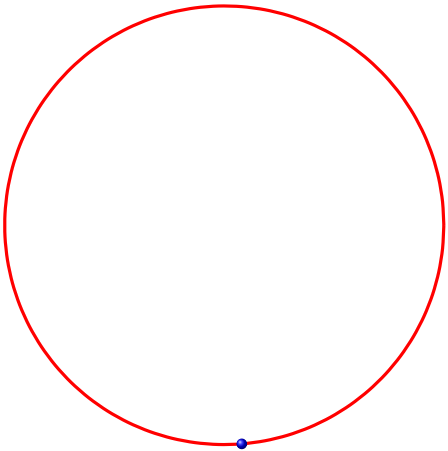

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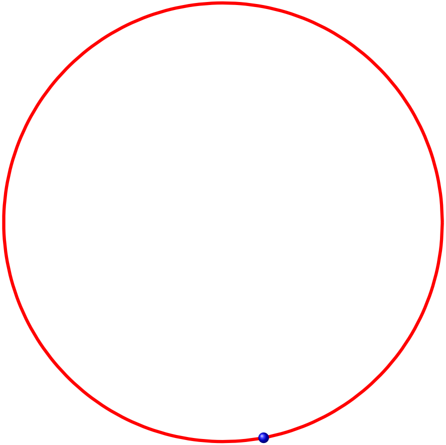

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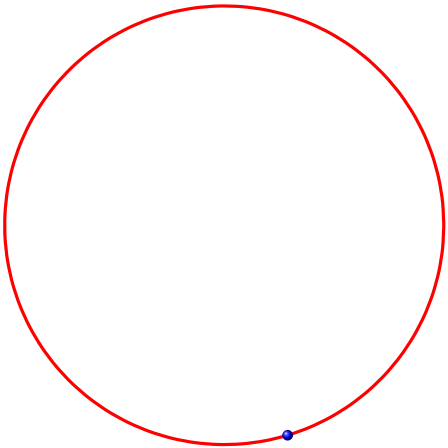

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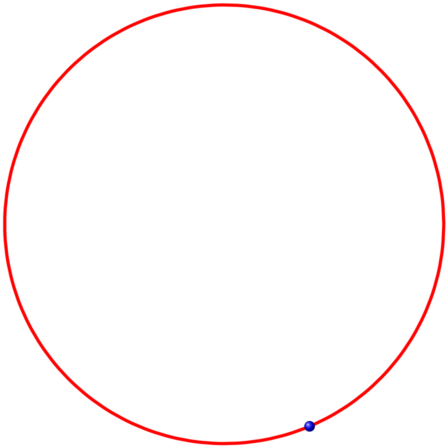

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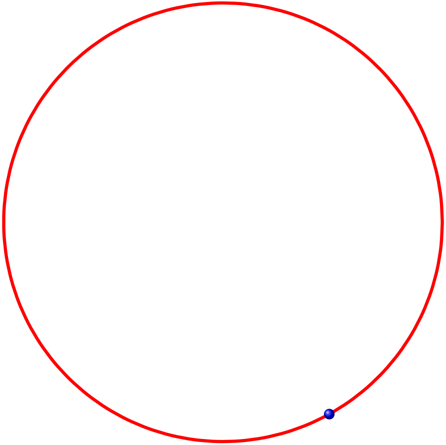

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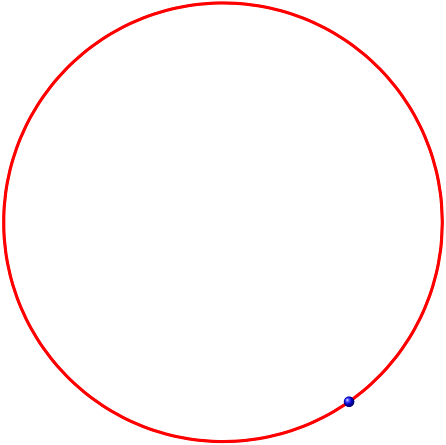

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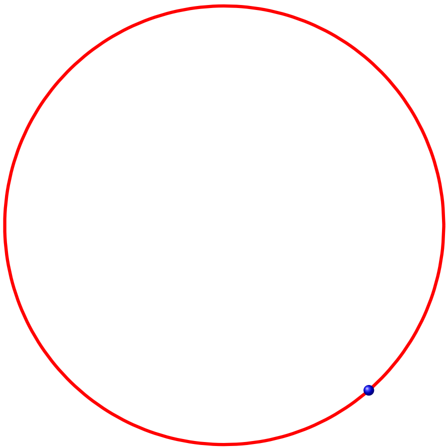

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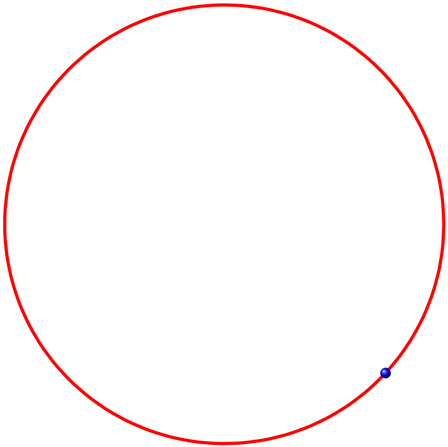

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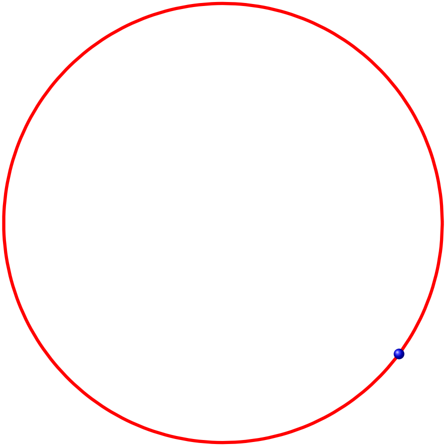

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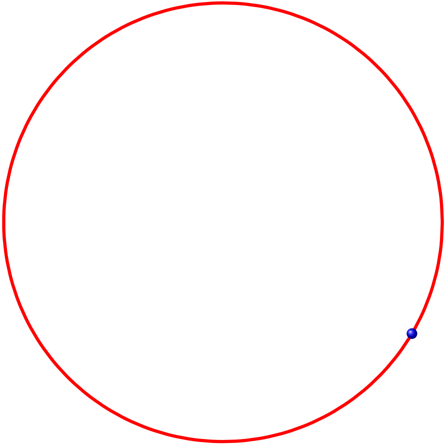

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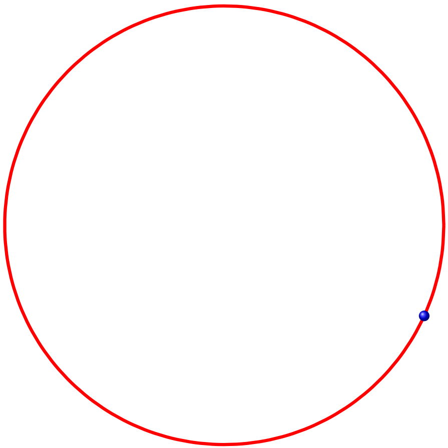

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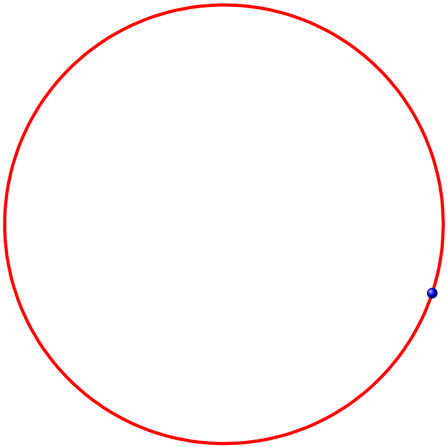

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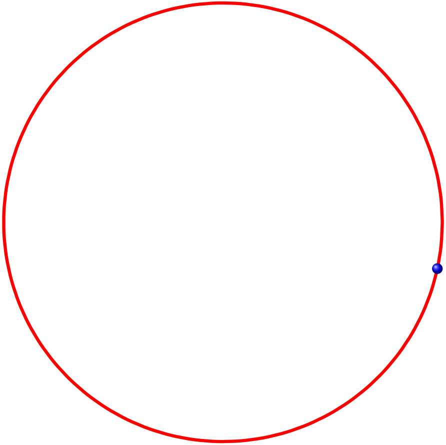

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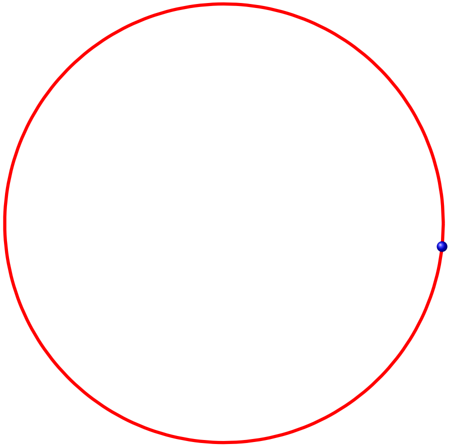

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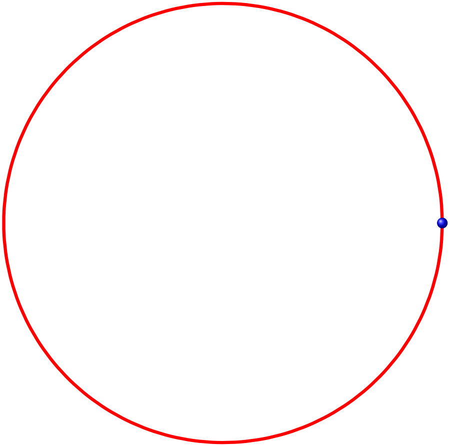

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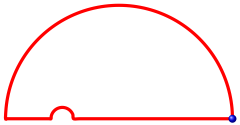
Some examples

A curve we will meet later



$$z(t) = \begin{cases} e^{i2\pi t} & 0 \leq t < \frac{1}{2} \\ 1.6t - 1.8 & \frac{1}{2} \leq t < \frac{3}{4} \\ \frac{e^{i\pi(7-8t)} - 5}{10} & \frac{3}{4} \leq t < \frac{7}{8} \\ 11.2t - 10.2 & \frac{7}{8} \leq t \leq 1 \end{cases}$$

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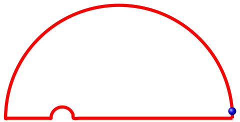

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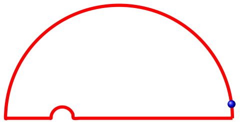

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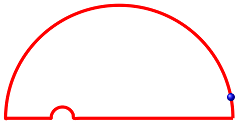

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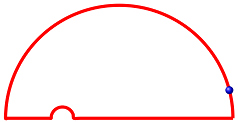

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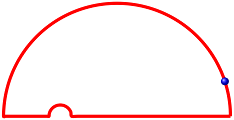

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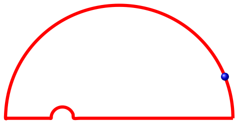

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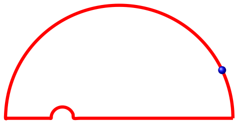

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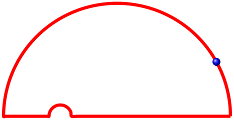

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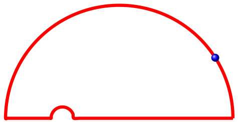

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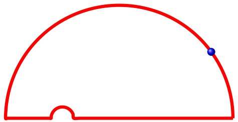

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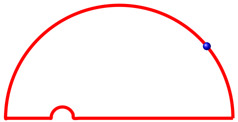

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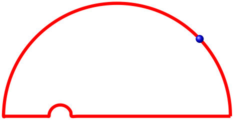

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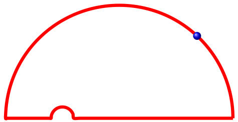

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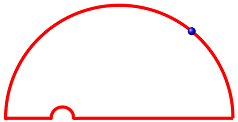

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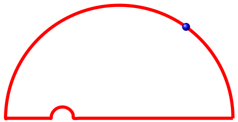

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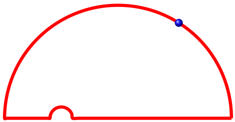

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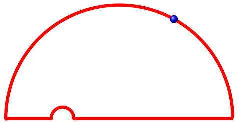

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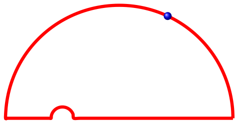

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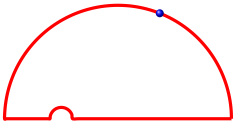

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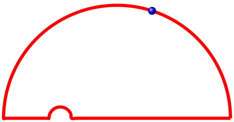

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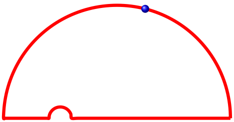

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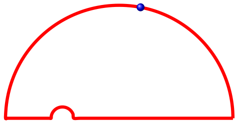

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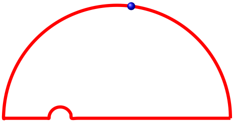

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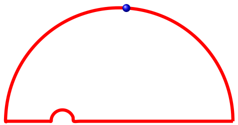

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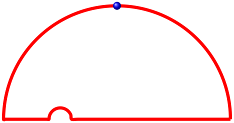

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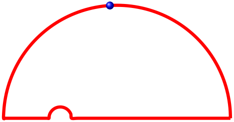

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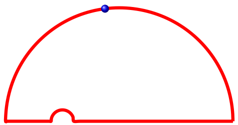

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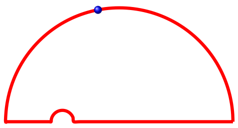

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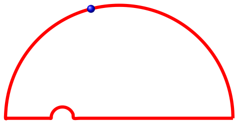

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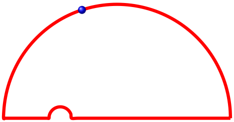

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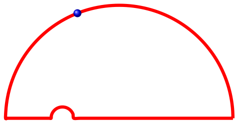

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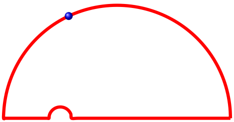

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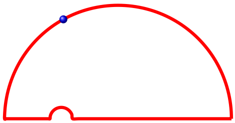

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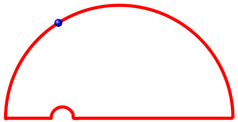

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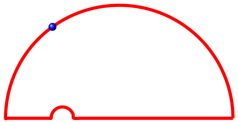

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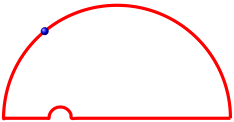

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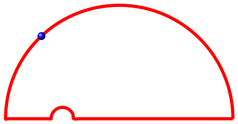

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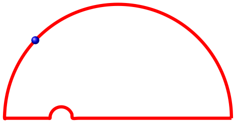

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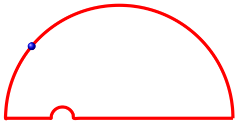

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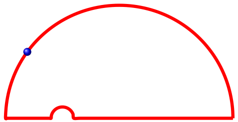

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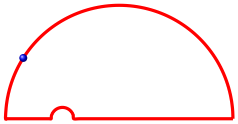

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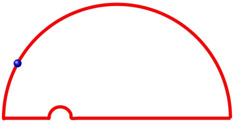

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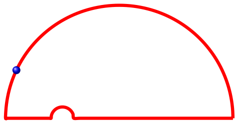

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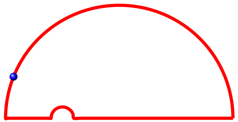

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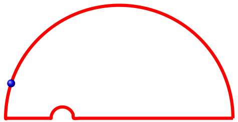

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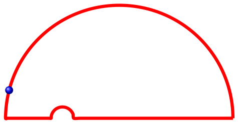

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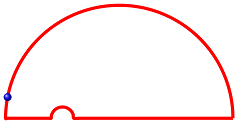

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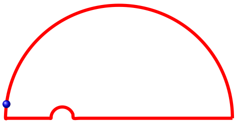

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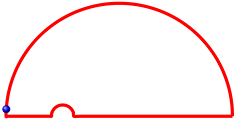

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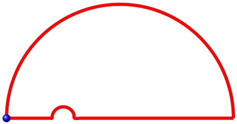

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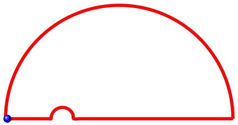

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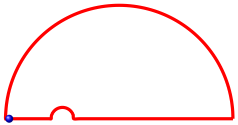

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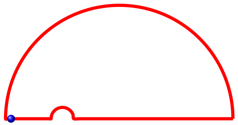

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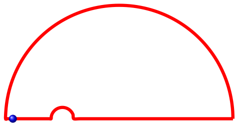

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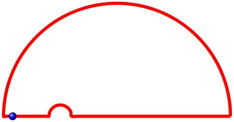

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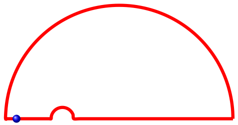

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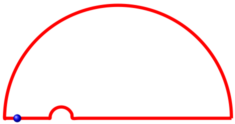

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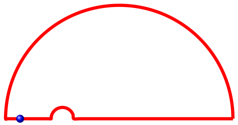

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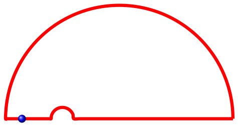

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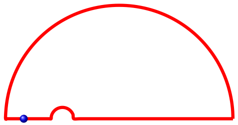

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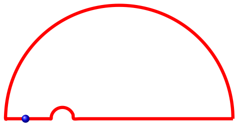

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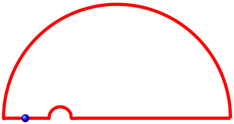

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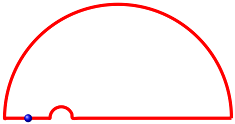

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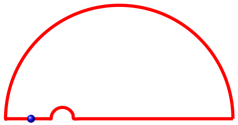

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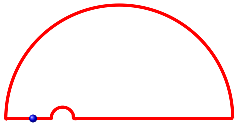

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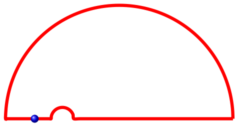

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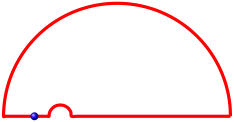

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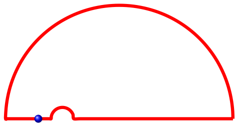

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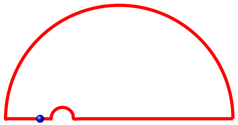

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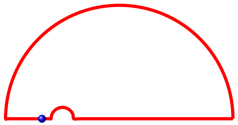

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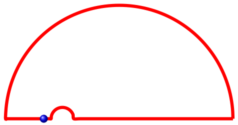

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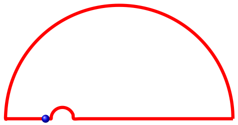

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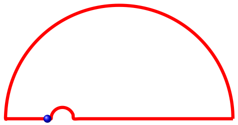

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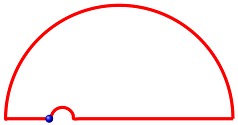

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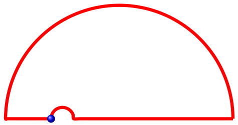

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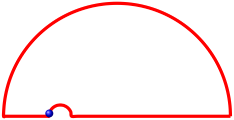

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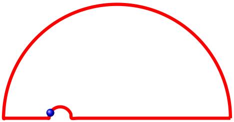

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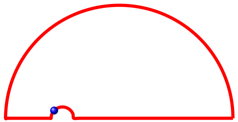

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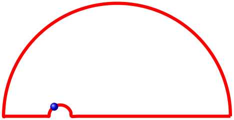

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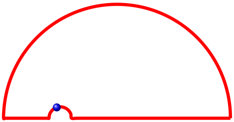

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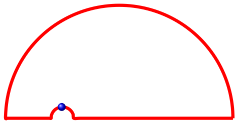

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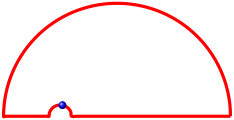

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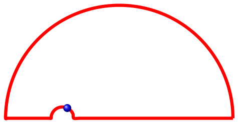

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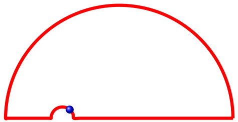

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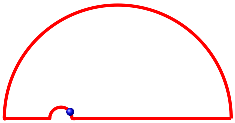

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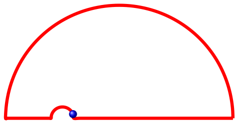

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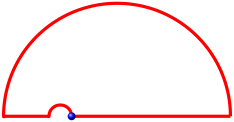

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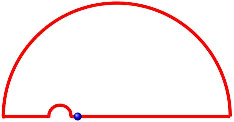

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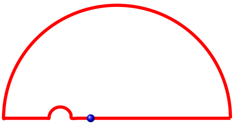

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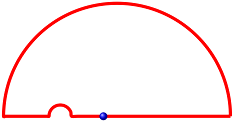

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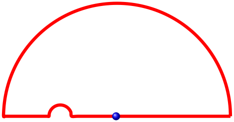

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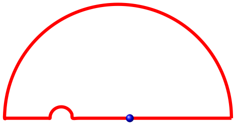

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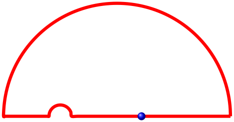

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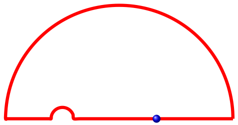

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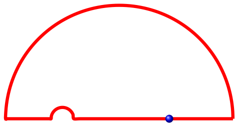

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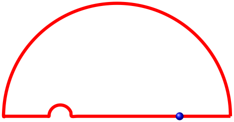

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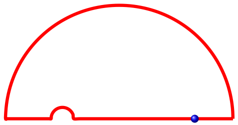

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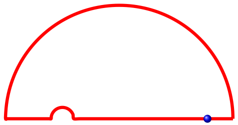

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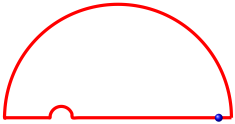

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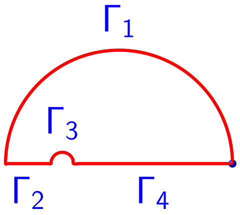

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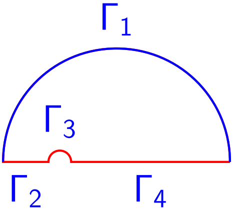

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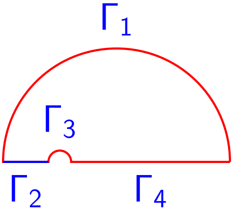

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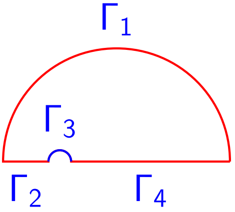

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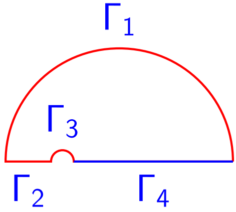

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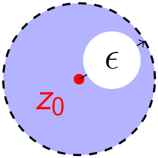
Some useful vocabulary



The ϵ-neighbourhood of the point z_0 is the set of all points z closer to the point z_0 than ϵ :

$$D_\epsilon(z_0) = \{z \in \mathbb{C} \ : \ |z - z_0| < \epsilon\}$$

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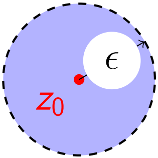

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This is also called the **open ϵ-disk** centered at z_0.

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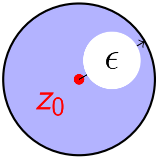

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The **closed disk** of radius ϵ centered at z_0 is defined as :

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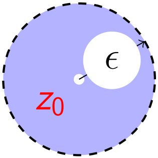

The ϵ-neighbourhood of the point z_0 is the set of all points z closer to the point z_0 than ϵ :

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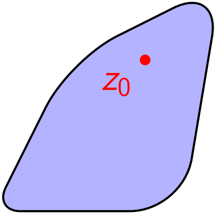
Given a set $S \subset \mathbb{C}$:

• The point $z_0 \in \mathbb{C}$ is an **interior point** of S if:



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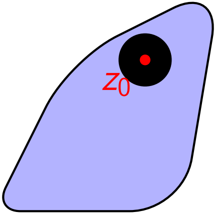

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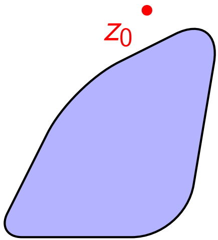

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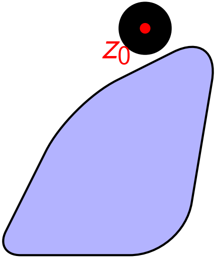

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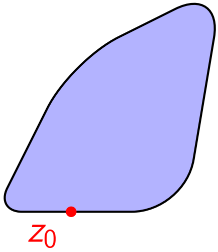

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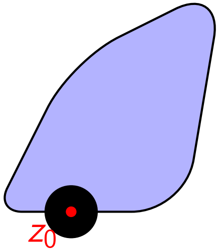

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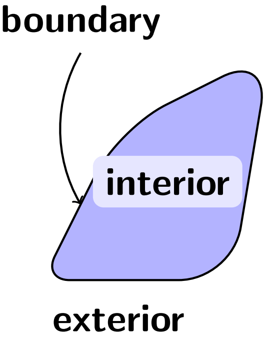

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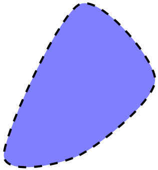

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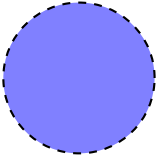

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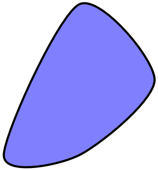

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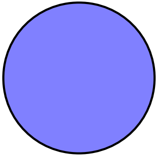

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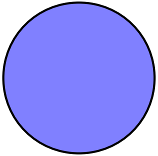

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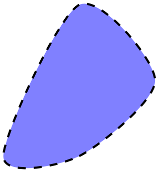

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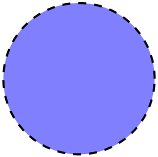

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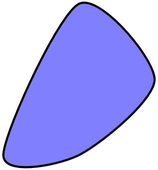

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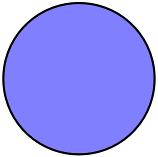

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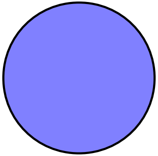

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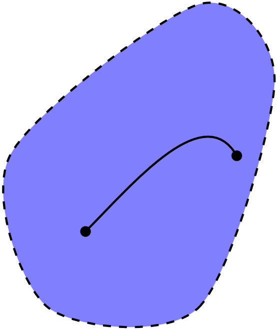

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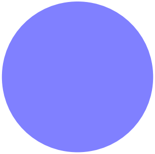

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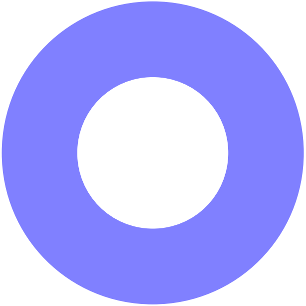

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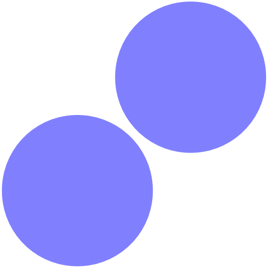

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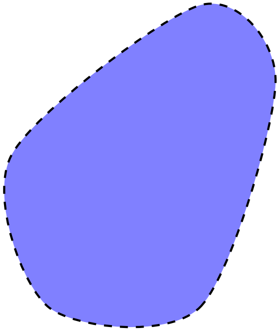

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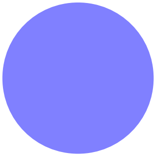

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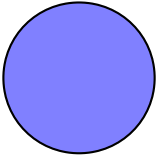

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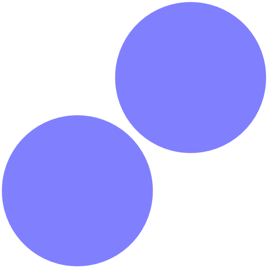

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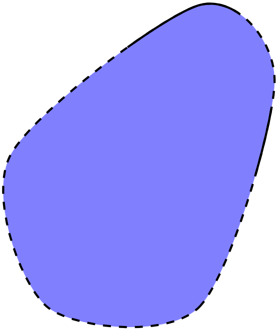

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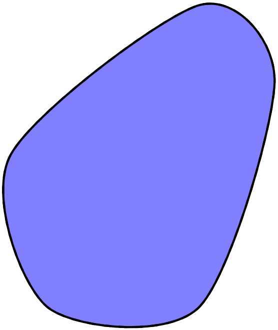

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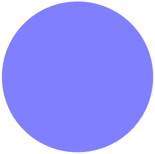

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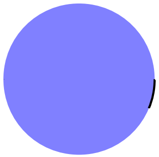

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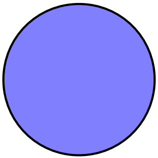

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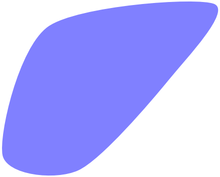

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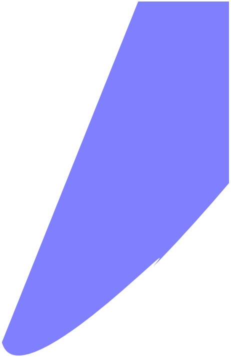

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