MARGINAL FERMI LIQUID BEHAVIOUR IN A COUPLED QUANTUM DOT SYSTEM

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PLAN

- Introduction
- The model

    Spectral properties

    Transport properties

- Conclusions

PD, EPL, 95 (2011)

Quantum impurity problems

Interaction of conduction electrons with local quantum degeneracies

Local degeneracy - physical spin,
two level systems
orbital quadrupolar degrees of freedom …

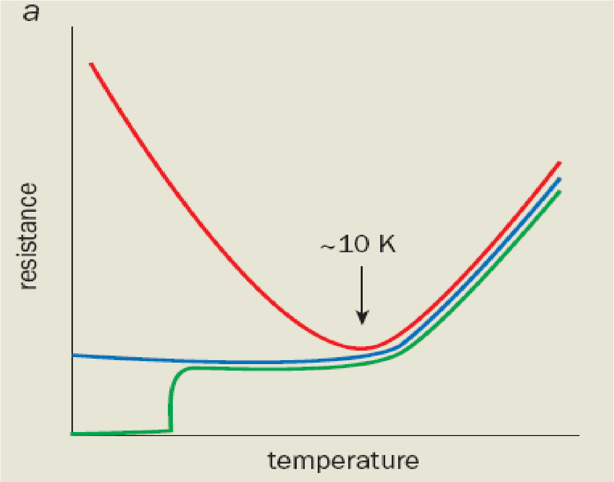
Non-Fermi Liquid physics, quantum phase transitions

Exact solutions - RG, Bethe Ansatz, Boundary Conformal Field Theory methods

Quantum dot systems - 'tunable' impurity physics

Kondo problem

$$H = \sum_{k,\sigma} \epsilon_k c^\dagger_{k,\sigma} c_{k,\sigma} + \sum_{k,k'} J_{k,k'}\, \vec{S}.\, c^\dagger_k \frac{\vec{\sigma}}{2} c_{k'}$$



Ref: Beenaker

Scaling/Renormalization Group

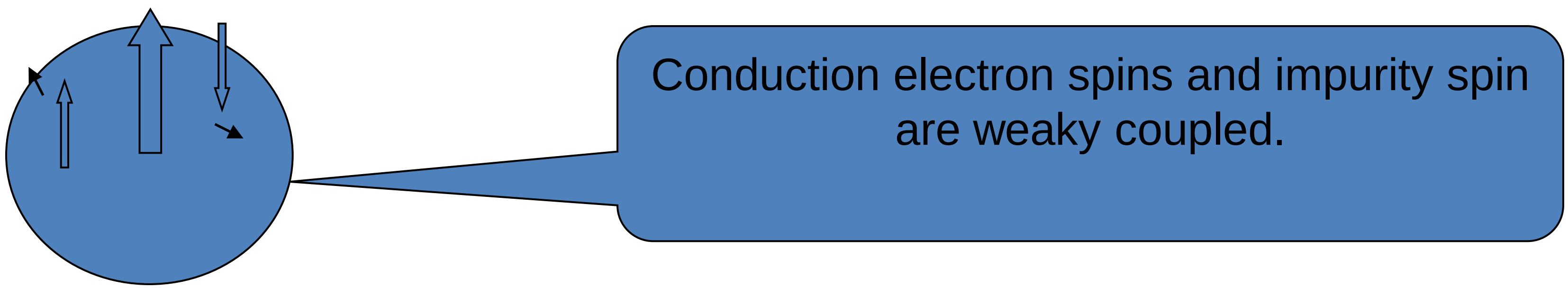
- Crossover from weak coupling to strong coupling.
- Universal scaling laws for physical quantities

$T >> T_K$ Free spin

$T << T_K$ Many body Bound state resonance

$k_B T_K \approx D e^{-1/2J\nu_0}$ Kondo temperature :

$T >> T_K$ $\Longrightarrow$ Weak coupling regime

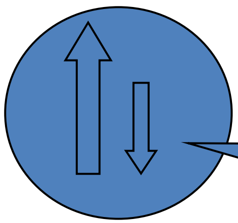


$$R_{imp}(T) = \frac{R_0 \pi^2 S(S+1)}{4(\ln(T/T_K))^2}[1 - \frac{3\pi^2 S(S+1)}{4(\ln(T/T_K))^2}...]$$

$$\chi_{imp}(T) = \frac{(g\mu_B)^2}{4k_B T}[1 - \frac{1}{\ln(T/T_K)} + ...]$$

Kondo behaviour

$T << T_K$ Strong coupling regime



The impurity spin is strongly bound to the conduction electron spin to form a singlet state – many body bound state –
Kondo resonance

$$\chi_{imp}(T) = \frac{(g\mu_B)^2 w}{4k_B T_K}[1 + O((\frac{T}{T_K})^2)]$$

$$R_{imp}(T) = R_0[1 - \frac{\pi^4 w^2}{16}(\frac{T}{T_K})^2 + O((\frac{T}{T_K})^4)]$$

Fermi liquid behaviour!

Generalized Kondo models

- Multi channels
- Multi Impurities

Different ground states possible due to competing interactions

Multichannel models with SU(N) channel symmetry

Nozieres and Blandin

$$H = \sum_{i=1}^{N} \sum_{k,\sigma} \epsilon(k)\, c^{\dagger}_{i\,k\,\sigma} c_{i\,k\,\sigma} + \sum_{i=1}^{N} \sum_{k,k'} J_{k,k'}\, \vec{S}.\, c^{\dagger}_{i\,k} \frac{\vec{\sigma}}{2} c_{i\,k'}$$

N=2S - Exactly screened Kondo $\longrightarrow$ Fermi liquid

N>2S - Overscreened Kondo $\longrightarrow$ Non Fermi Liquid

N<2S, Underscreened Kondo $\longrightarrow$ Singular Fermi liquid

N=2 Two channel Kondo model (2CK)

Affleck, Ludwig, ……

Non-Fermi-liquid ground state

$$S_{imp}(0) = \frac{1}{2}\ln 2 \qquad \chi(T) \propto \ln T$$

Spectral density $A(\omega) \propto |\omega|^{1/2}$

Conductance/resistivity $\rho(T) \propto T^{1/2}$

Channel Anisotropy - Fermi Liquid

Quantum Phase transition

Multi Impurities

Kondo interaction between the conduction electrons and the impurity spin

RKKY spin exchange interaction between the two impurities

Two impurities:

$$H = \sum_{k,\sigma} \epsilon_k c^\dagger_{k,\sigma} c_{k,\sigma} + J(\vec{S}_1.\vec{s}(\vec{R}_1) + \vec{S}_2.\vec{s}(\vec{R}_2)) + I\vec{S}_1.\vec{S}_2$$

Krishnamurthy, Wilkins, Varma, Jones,
Affleck, Ludwig,..

J>> I Kondo ground state

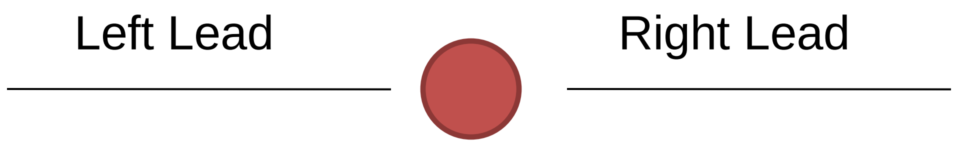
Fermi liquid

| I | >>J RKKY ground state

For AFM RKKY - intermediate unstable NFL FP

Competition between a state where each spin is Kondo screened
and a state where the two impurities form a singlet.

Quantum dot models



1CK

$$H = H_{leads} + H_{dot} + H_{leads-dot}$$

For small system sizes, e-e interactions in the dot need to be taken into account – because of the small size, charge tends to be localized in the nanostructure - the flow of charge become correlated due to Coulomb interactions/ charging effects.

Anderson type model:

$$H_d = \sum_{\sigma=\uparrow,\downarrow} \varepsilon_d d_\sigma^\dagger d_\sigma + U n_{d\uparrow} n_{d\downarrow}$$

Anderson type dots

- Non interacting leads

$$H_{And} = \sum_{k,\sigma} \varepsilon_k c^\dagger_{k\sigma} c_{k\sigma} + \varepsilon_d \sum_{\sigma} c^\dagger_{d,\sigma} c_{d,\sigma} + U \cdot n_{d,\uparrow} n_{d,\downarrow}$$
$$+ \sum_{k,\sigma} \left(V_k \, c^\dagger_{d,\sigma} c_{k\sigma} + V^\dagger_k \, c^\dagger_{k\sigma} c_{d,\sigma} \right)$$

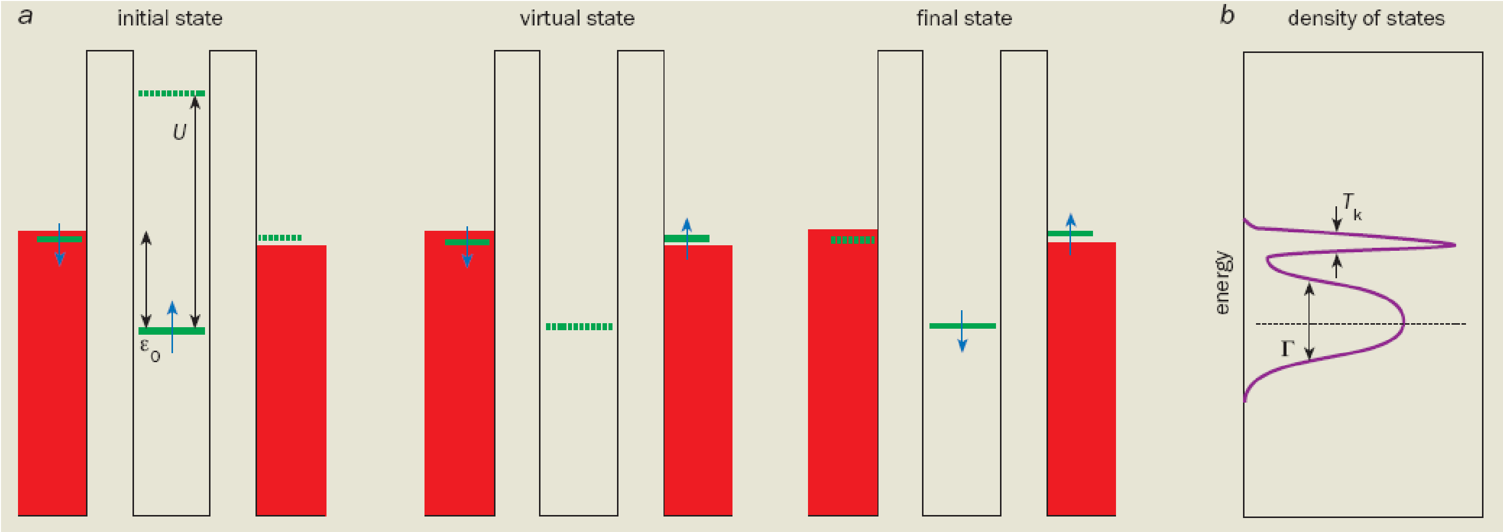
$\varepsilon_d \ll \varepsilon_F \ll 2\varepsilon_d + U$ $\rightarrow$ **Kondo regime**

Dot behaves effectively as a local spin impurity

Effective interaction of the local spin with the lead electrons - Kondo exchange interaction

$$H = J \sum_{k,k'} \vec{S} \, . (c^\dagger_{L,k} + c^\dagger_{R,k}) \frac{\vec{\sigma}}{2} (c_{L,k'} + c_{R,k'}) \qquad J \approx \frac{4t^2}{U}$$

The Kondo or Abrikosov-Suhl Resonance:



Low temperature transport

Linear Conductance

$$G(T) = \frac{2e^2}{h} \int d\epsilon \left(\frac{-d\, n_F}{d\,\epsilon}\right)[-\pi\nu\, Im\, T(\epsilon, T)]$$

$$G(T) = \frac{2e^2}{h} \frac{3\pi^2}{16 \ln^2 T/T_K} \qquad T >> T_K$$

$$G(T) = \frac{2e^2}{h}\left(1 - \left(\frac{\pi T}{4 T_K}\right)^2\right) \qquad T << T_K$$

Strong coupling fixed point : Fermi liquid behaviour $T << T_K$

Nozieres – phenomenological Fermi liquid theory, Wilson

Effective model around the strong coupling fixed point:

Single impurity Anderson model (SIAM)

$$H_{And} = \sum_{k,\sigma} \varepsilon_k c^\dagger_{k\sigma} c_{k\sigma} + \varepsilon_d \sum_{\sigma} c^\dagger_{d,\sigma} c_{d,\sigma} + U \cdot n_{d,\uparrow} n_{d,\downarrow} + \sum_{k,\sigma} \left(V_k \, c^\dagger_{d,\sigma} c_{k\sigma} + V_k^\dagger \, c^\dagger_{k\sigma} c_{d,\sigma} \right)$$

U~ T_K

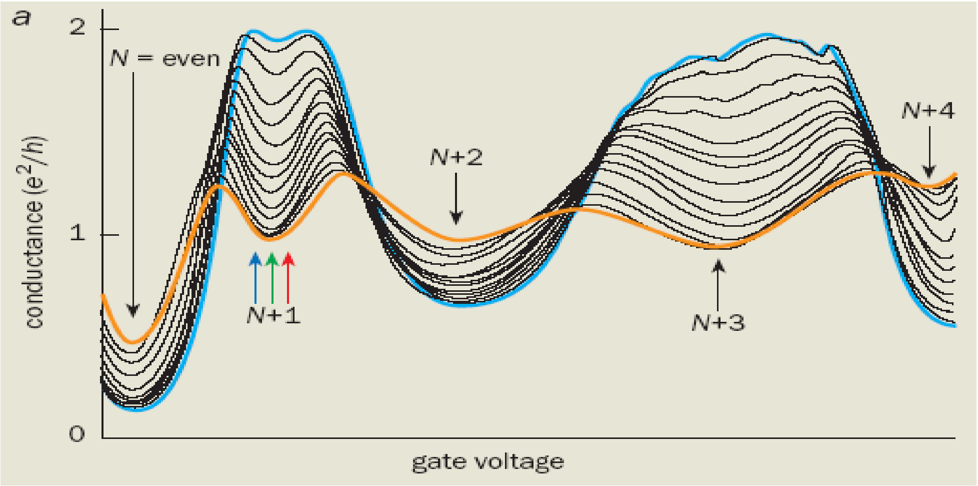
Perturbative study (in U) of the SIAM model Yosida, Yamada ……

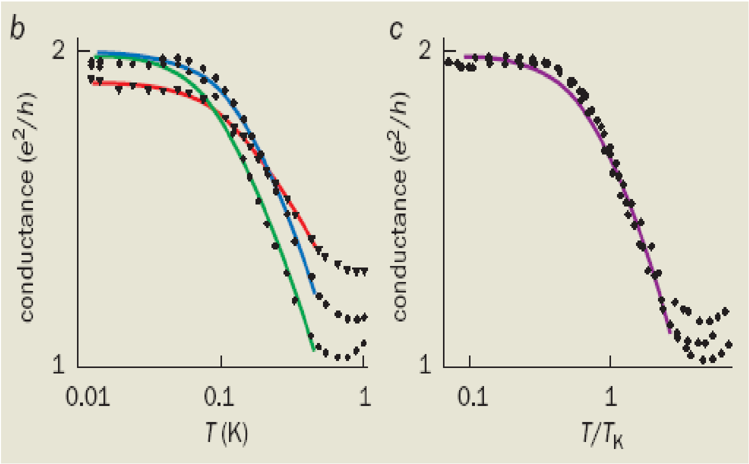
- Convergent expansion for various physical quantities
- Match with the exact solutions from Bethe Ansatz

Renormalized Perturbation theory Hewson 1993,2006

Kondo effect in quantum dot systems

van der Wiel et al , 2000, Science 289, 2105



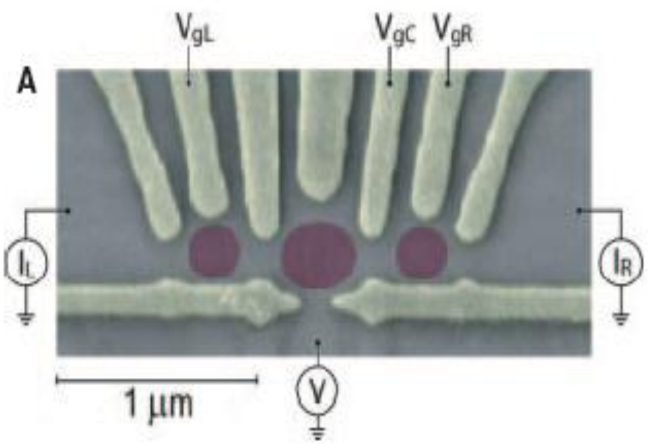


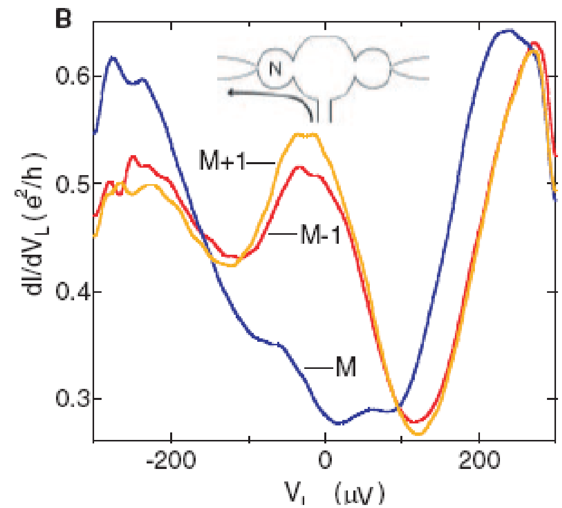
- Unitary value for the Conductance.
- Universal Scaling.

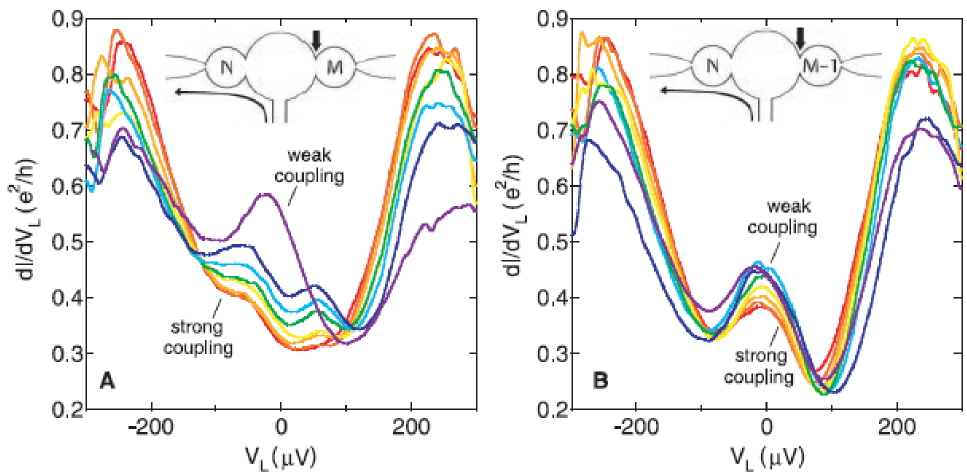
Ng and Lee, 1988

Double dots

N. C. Craig etal , 2004, Science 304, 565



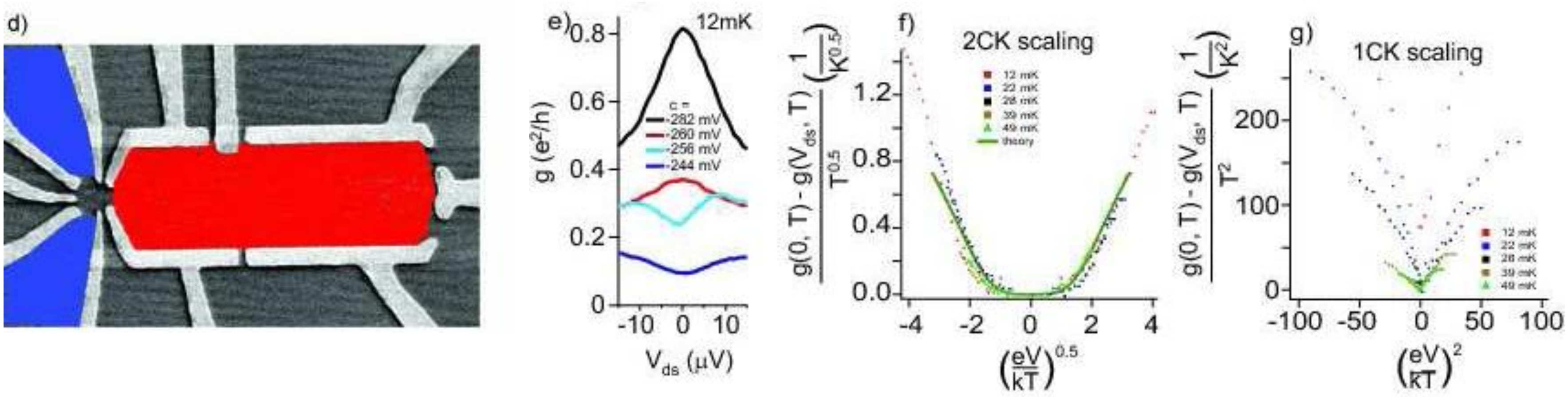




Golovach and Loss,
Simon and Affleck.,
Lopez,Simon…..

Two channel Kondo

2CK R. M. Potok, etal, Nature, 2007



Two impurity NFL FP , Zarand et al, PRL 2006

Tamura and Glazman, PRB 72, 121308(R) 2005

Two impurity Kondo model with ferromagnetic RKKY interaction

→ Effective model - Single channel Kondo model with a S=1 impurity spin

→ Underscreened Kondo physics

→ Singular Fermi liquid behaviour

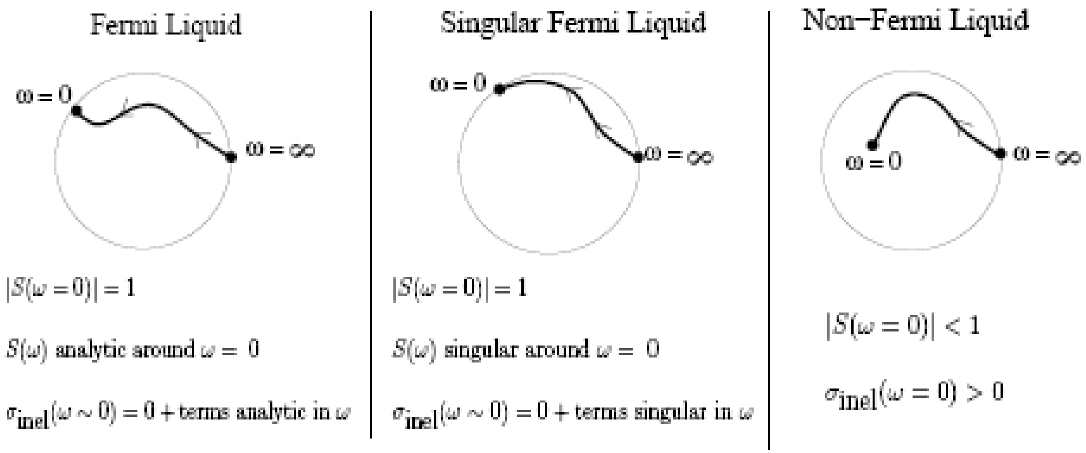
Coleman and Pepin, 2003,
P. Mehta etal, 2005
A. Posazhennikova and P. Coleman, 2005
W. Koller etal , 2006
Rosch et al, Nature, 2008
Rosch et al, PRL, 2009
Logan et al, PRB, 2009

Relevance in Heavy fermion materials at the quantum critical point

Regular and Singular Fermi liquids

P. Mehta, etal, 2005

RG flows of the eigen values of the single particle matrix elements of the many body S matrix



1 CK is a Fermi liquid

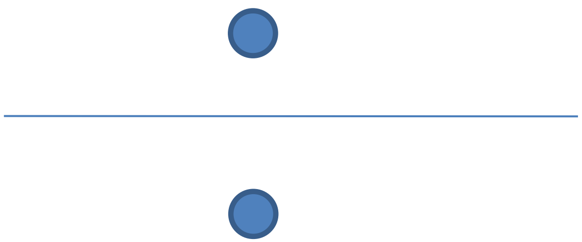
Bethe Ansatz and NRG

Underscreened Kondo is a singular Fermi liquid

Two impurity Anderson model

PD, EPL, 2011

Hamiltonian



$$H = H_{wire} + H_{dots} + H_{wire\text{-}dots},$$

$$\begin{aligned} H_{wire} &= -t \sum_{i} \sum_{\sigma} (c^{\dagger}_{i,\sigma} c_{i+1,\sigma} + + \mathrm{h.c.}), \\ H_{dots} &= \epsilon_0 \sum_{1}^{2} \sum_{\sigma} d^{\dagger}_{i,\sigma} d_{i,\sigma} + \sum_{i=1}^{2} U n_{i,\uparrow} n_{i,\downarrow}, \\ H_{wire\text{-}dots} &= -v_0 \sum_{\sigma} (c^{\dagger}_{0,\sigma} d_{1,\sigma} + \mathrm{h.c.}) \\ & \quad -v_0 \sum_{\sigma} (c^{\dagger}_{0,\sigma} d_{2,\sigma} + \mathrm{h.c.}). \end{aligned}$$

Perturbation (in U) study of spectral and transport properties

Restrict to Half filling

Non-interacting case: U =0

$$H_{wire-dots} = -v_0 \sum_\sigma (c^\dagger_{0,\sigma} d_{1,\sigma} + h.c) - v_0 \sum_\sigma (c^\dagger_{0,\sigma} d_{2,\sigma} + h.c)$$
$$= -\sqrt{2} v_0 \sum_\sigma (c^\dagger_{0,\sigma} d_{+,\sigma} + h.c)$$

$$d_\pm = \frac{1}{\sqrt{2}}(d_1 \pm d_2)$$

d_+ impurity mode gets hybridized; d_- impurity mode remains free

Green functions: $G_{+,\sigma}(\omega^+) = [\omega - 2v_0^2 g_{00}(\omega^+)]^{-1}$, $G_{-,\sigma}(\omega^+) = [\omega^+]^{-1}$;

$$\omega^+ = \omega + i\delta,$$

Impurity density of states: $A_{+,\sigma}(\omega) = \frac{1}{\pi}\frac{\Delta}{\omega^2 + \Delta^2}$ $A_{-,\sigma}(\omega) = \delta(\omega).$

Impurity contribution to free energy:

$$F^0_{imp} = \frac{1}{\pi}\int_{-1}^{1} f(\omega)\tan^{-1}\left(\frac{\Delta}{\omega}\right) \mathrm{d}\omega - \mathrm{T}\ln 2.$$

Residual entropy – ln 2
Two fold degeneracy

Impurity charge and spin densities

$$N_i = \frac{1}{2}(d^\dagger_{i,\uparrow} d_{i,\uparrow} + d^\dagger_{i,\downarrow} d_{i,\downarrow} - 1) = \frac{1}{2}(n_i - 1);$$

$$S^z_i = \frac{1}{2}(d^\dagger_{i,\uparrow} d_{i,\uparrow} - d^\dagger_{i,\downarrow} d_{i,\downarrow}),$$

Charge and spin Susceptibilities

$$\chi^{ch}_{ij}(\tau) = -\langle T_\tau N_i(\tau) N_j(0)\rangle \qquad \chi^{sp}_{ij}(\tau) = -\langle T_\tau S^z_i(\tau) S^z_j(0)\rangle$$

$$\chi^{ch}_{ij}(\tau) = \chi^{sp}_{ij}(\tau) = \chi_{ij}(\tau) = \frac{1}{4}(G_{ij,\uparrow}(\tau)^2 + G_{ij,\downarrow}(\tau)^2).$$

Matsuba-Fourier space

$$\chi_{\alpha\beta}(i\omega_n) = \frac{1}{2\beta}\sum_{\omega_m} G_\alpha(i\omega_m) G_\beta(i\omega_m + i\omega_n); \;\; \alpha, \beta = +, -$$

Zero frequency behaviour :

Diagonal components

$$\lim_{\omega\to 0} \chi_{++}(\omega) \propto \frac{1}{\Delta}; \quad \lim_{\omega\to 0} \chi_{--}(\omega) \propto \frac{1}{T}.$$

Off-diagonal component of the susceptibility

$$\begin{aligned}\mathrm{Im}\,\chi_{+-}(\omega,T) &= -\frac{1}{2}\frac{\Delta}{\omega^2+\Delta^2}\tanh\frac{\omega}{2T}\\ &\sim -\frac{\omega}{T}, \quad \omega<T\\ &\sim -\mathrm{const}, \quad \Delta\gg\omega>T\end{aligned}$$

$$\mathrm{Re}\,\chi_{+-}(\omega,T)=\frac{1}{2\pi\Delta}\ln\frac{x}{\Delta}; \quad x=\max(|\omega|,T).$$

(Bosonic) fluctuation spectrum of the off-diagonal component of the susceptibility coincides with that postulated for high T_c superconductors.

C. M. Varma, 1989

Non interacting theory shows non-Fermi liquid behaviour

Interacting case:

$$\begin{aligned} H_U &= U n_{1\uparrow} n_{1\downarrow} + U n_{2\uparrow} n_{2\downarrow} \\ &= \frac{U}{2} n_{+\uparrow} n_{+\downarrow} + \frac{U}{2} n_{-\uparrow} n_{-\downarrow} - U \vec{S}_{+}.\vec{S}_{-} + U \vec{T}_{+}.\vec{T}_{-} \end{aligned}$$

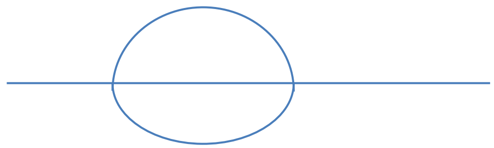
Impurity Green functions

$$G = [\delta_{ij}(\epsilon - \epsilon_j) - \Sigma]^{-1},$$

Self energy

$$\Sigma^U_{d_i,d_j,\sigma} = \delta_{i,j} U_j \langle n_{d_j,-\sigma} \rangle + \tilde{\Sigma}^U_{d_i,d_j,\sigma}$$

Perturbation (to second order in U):



$$\Sigma^{(2)}_{\pm}(\omega) = \Sigma^{u}_{\pm}(\omega) + \Sigma^{s}_{\pm}(\omega)$$

Low frequency and low temperature behaviour:

$$\tilde{\Sigma}^u_+(\omega^+) \approx -\left(\frac{U}{2\pi\Delta}\right)^2 \omega - i\left(\frac{U}{2\pi\Delta}\right)^2 \frac{\Delta}{2}\left[\left(\frac{\omega}{\Delta}\right)^2 + \left(\frac{\pi T}{\Delta}\right)^2\right].$$

Hybridized mode:

$$\tilde{\Sigma}^s_+(\omega^+) = \frac{3U^2}{16}\frac{1}{\omega + i\Delta}.$$

$$\omega^+ = \omega + i\delta.$$

Self energy for the unhybridized impurity mode

Coulombic contribution

$$\hat{\Sigma}^{u}_{-}(\omega^{+}) = \frac{U^{2}}{16}\frac{1}{\omega^{+}}.$$

$$\begin{aligned}\mathrm{Im}\,\hat{\Sigma}^{s}_{-}(\omega^{+}) &= -\frac{3U^{2}}{(2\pi\Delta)^{2}}\frac{\pi}{2}|\omega|\coth\left(\frac{|\omega|}{2T}\right)\\ &\sim -\frac{3U^{2}}{(2\pi\Delta)^{2}}(\pi T), \quad |\omega| < T\\ &\sim -\frac{3U^{2}}{(2\pi\Delta)^{2}}\frac{(\pi|\omega|)}{2}, \quad |\omega| > T\end{aligned}$$

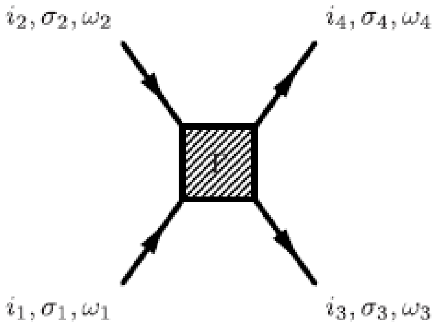
Marginal Fermi liquid behaviour

C. M. Varma, 1989

$$\mathrm{Re}\,\hat{\Sigma}^{s}_{-}(\omega^{+}) \sim \frac{3U^{2}}{(2\pi\Delta)^{2}}\left(\omega\ln\frac{x}{\Delta}\right), \quad x = \max(|\omega|, T).$$

Such a fermionic self –energy here is due to the exchange of the off-diagonal component of the impurity charge and spin fluctuations.

Two particle Green functions and vertex functions



(Dimensionless)Vertex function: $\tilde{\Gamma} = \quad 2\rho_0 \delta_{\sigma_1+\sigma_2, \sigma_3+\sigma_4} \Gamma_{+,\sigma_1,-\sigma_2;+,\sigma_3,-\sigma_4}(0,0;0,0)$

To first order in U $\quad \tilde{\Gamma} = -2\rho_0 U$

To second order in U

$$\tilde{\Gamma} = \tilde{\Gamma} + \tilde{\Gamma}^2 \left[\frac{\pi\Delta}{\beta} \sum_{i\omega_n} G_+(i\omega_n) G_-(-i\omega_n) \right]$$

$$\approx \tilde{\Gamma} + \tilde{\Gamma}^2 \ln \frac{T}{\Delta}.$$

Such Logarithmic terms in higher order diagrams also.

Scaling equation: $\quad \frac{\mathrm{d}(\tilde{\Gamma})}{\mathrm{d}\ln\Delta} = -\tilde{\Gamma}^2$

$\Rightarrow \quad \tilde{\Gamma} \approx \frac{1}{\ln(T/\Delta)}$ Marginally irrelevant

Impurity density of states

Hybridized mode:

$$\rho_+(\omega) \approx \frac{1}{\pi\Delta}\left(1 - \frac{3}{16}\frac{\pi^2}{4}\frac{1}{\ln^2(\omega/\Delta)}\right)$$

T =0

Unhybridized mode

$$\rho_-(\omega) \approx \delta(\omega - \tilde{J}/4) + \delta(\omega + \tilde{J}/4), \qquad \tilde{J} = \Delta/\ln(T/\Delta)$$

Logarithmically vanishing gap in the density of states

At zero temperature and zero frequency, the impurity spectral functions are of the same form as the non-interacting case.

---- Marginal fermi liquid behaviour preserved!

Conduction electron density of states

$$\rho_{00}(\omega) \approx \frac{1}{\pi\Delta}\frac{3}{16}\frac{\pi^2}{4}\frac{1}{\ln^2(\omega/\Delta)}.$$

Transport properties

Linear response - Kubo approach

Linear conductance

$$G = \frac{-e^2}{\hbar} \lim_{\omega \to 0} Im \frac{\Pi_{\alpha\alpha'}(\omega + \iota\eta) - \Pi_{\alpha\alpha'}(\iota\eta)}{\omega}, \quad \alpha, \alpha' = l, r.$$

$$\Pi_{\alpha\alpha'}(\iota\omega_m) = \int_0^\beta d\tau \langle T_\tau J_\alpha(\tau) J_{\alpha'}(0)\rangle e^{\iota\omega_m\tau}, \quad \alpha, \alpha' = l, r.$$

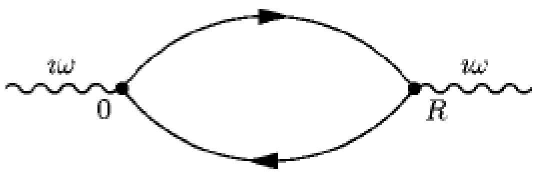
$$J_l = -\iota \sum_\sigma t_l (c^\dagger_{0,\sigma} c_{-1,\sigma} - \mathrm{H.c.}).$$

Current flowing from the left into the sample

Conductance

$$G = \frac{e^2}{h} \int d\epsilon \left(- \frac{\partial f}{\partial \epsilon} \right) T(\epsilon),$$

Generalized transmission probability



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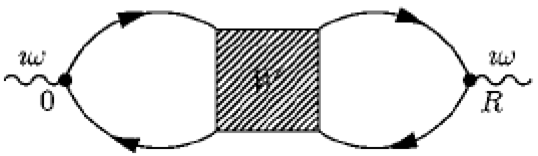


$T^0(\varepsilon)$

$T^{vert}(\varepsilon)$

Landauer result -
Elastic Scattering

A. Oguri, J. Phys. Soc. Jpn, 2001
PD, PRB 2006

Vertex corrections –
Inelastic scattering

Low energy behaviour for the transmission probability

Elastic scattering

$$T^0(\epsilon) \approx \left(\frac{3}{16}\right)^2 \frac{1}{\ln^4(\epsilon/\Delta)};$$

Inelastic scattering

$$T^{corr}(\epsilon) \approx \frac{3}{16}\frac{1}{\ln^2(\epsilon/\Delta)}.$$

$$T(\epsilon) = T^0(\epsilon) + T^{corr}(\epsilon) \approx \frac{3}{16}\frac{1}{\ln^2(\epsilon/\Delta)}.$$

Singular energy (and temperature) dependence

Fermi liquid

$$T(\omega) \propto (\frac{\omega}{\rho_0})^2 + (\frac{\pi T}{\rho_0})^2$$

Conductance

$$G(T) \propto \frac{1}{\ln^2 T_0/T}$$

$$\longrightarrow \quad \frac{dG(T)}{dT} \propto \frac{1}{T}$$

Underscreened Kondo physics

Tamura and Glazman

Zero frequency Johnson-Nyquist noise also shows such logarithmic corrections

Scattering cross section

Current conservation and Ward identities

$$T(\omega) = Im\ t(\omega)$$

T-matrix

$$T^0(\epsilon) = \frac{k_F^2}{2\pi}\sigma_{el}(\epsilon)$$

$$T^{corr}(\epsilon) = \frac{k_F^2}{2\pi}\sigma_{inel}(\epsilon),$$

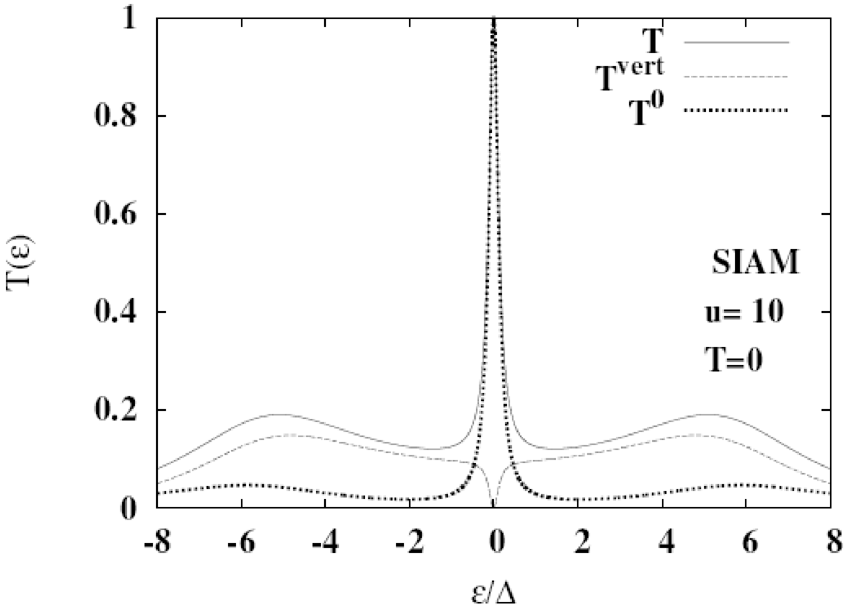
Inelastic scattering rate

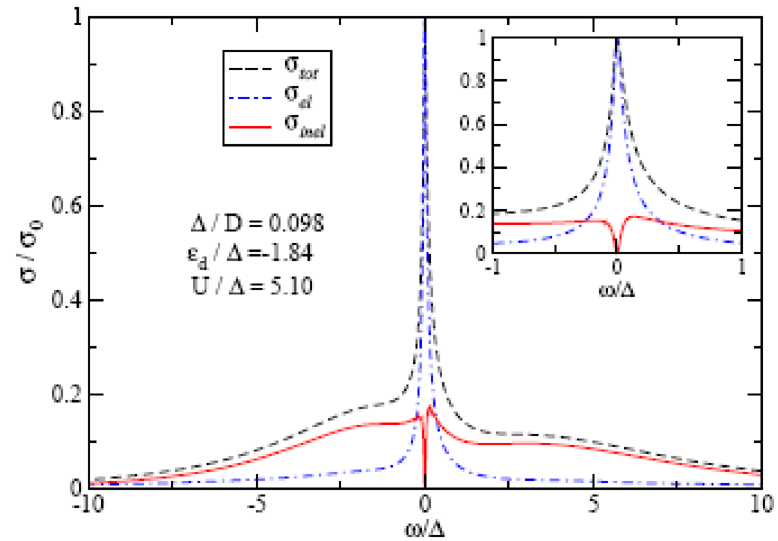
$$\frac{1}{\tau_{inel}} \approx \frac{1}{\rho_0}\frac{3}{16}\frac{1}{\ln^2(\epsilon/\Delta)}.$$

Measurement of dephasing time

Mohanty and Webb, 1997, 2003

Inelastic Scattering from magnetic impurities





Borda etal, PRB (2007)

Saturation of the dephasing time or the inelastic scattering time at low temperatures

Mohanty and Webb, 1997, 2003

Bauerle etal, Alzoubi and Birge, 2006

Robustness of the results

Marginal Fermi liquid is due to the presence of the unhybridized mode .

Which is due to the specific form of the hybridization Hamiltonian with zero inter-dot distance.

Finite inter-dot distance

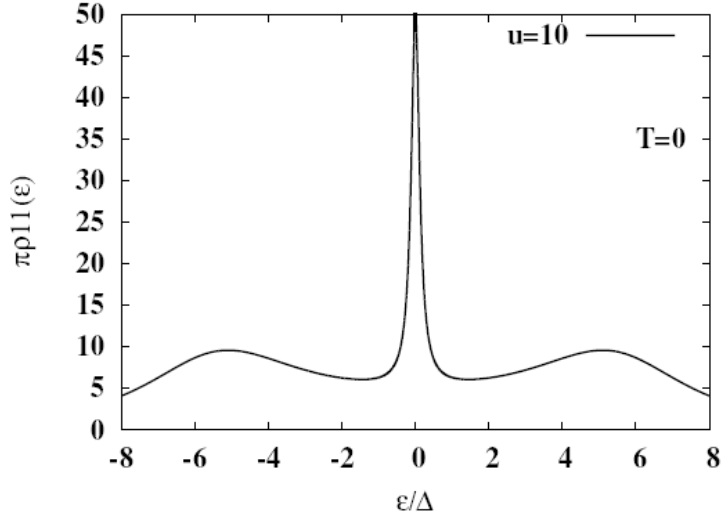
Both even and odd impurity modes get hybridized.

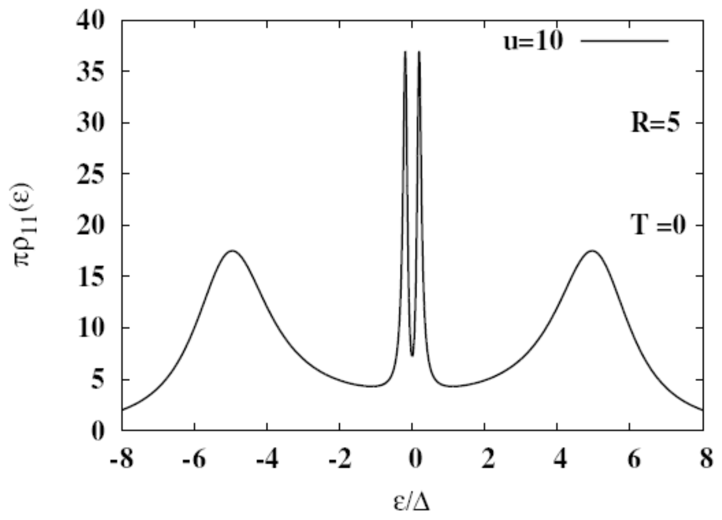
Depending on the distance between the impurities, there can be either ferromagnetic (FM) or antiferromagnetic correlations(AFM) between the impurtities.

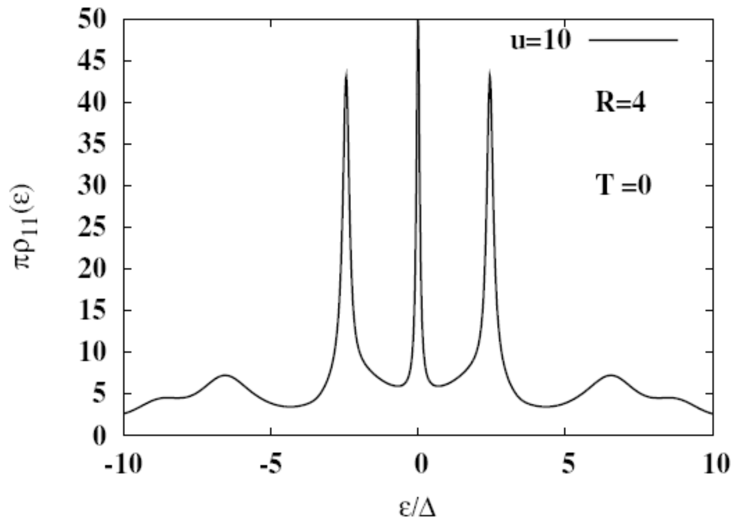
Impurity density of states at T=0

R finite

PD, PRB 74, 2006



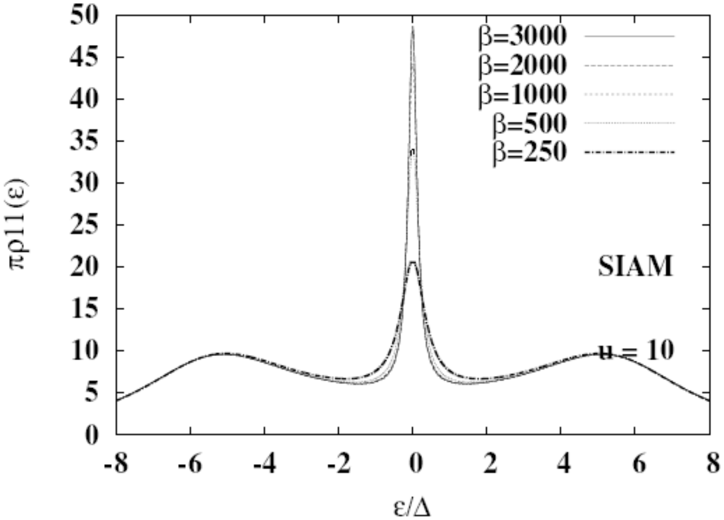


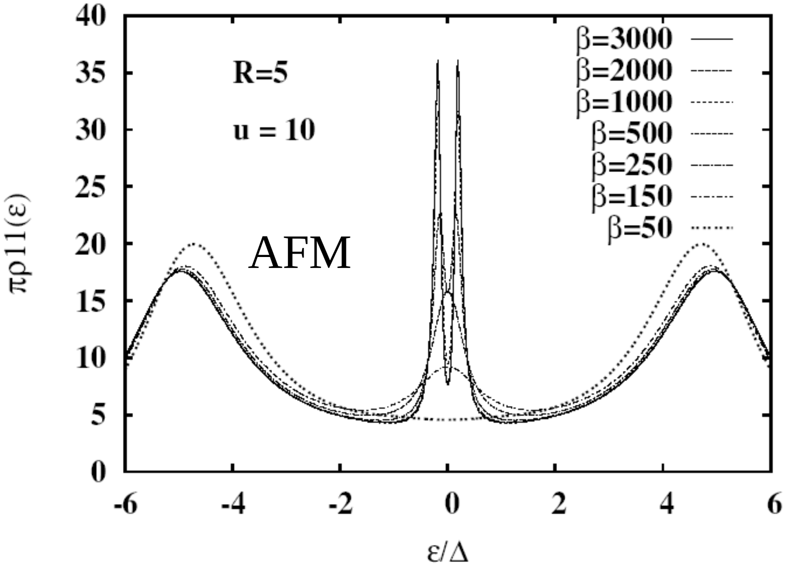


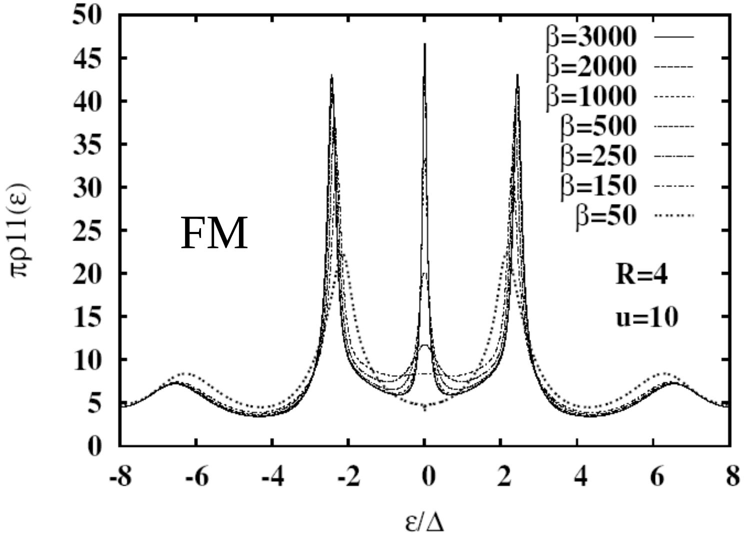
SIAM, AFM TIAM and FM TIAM

Impurity density of states – Finite T

Finite inter-dot distance

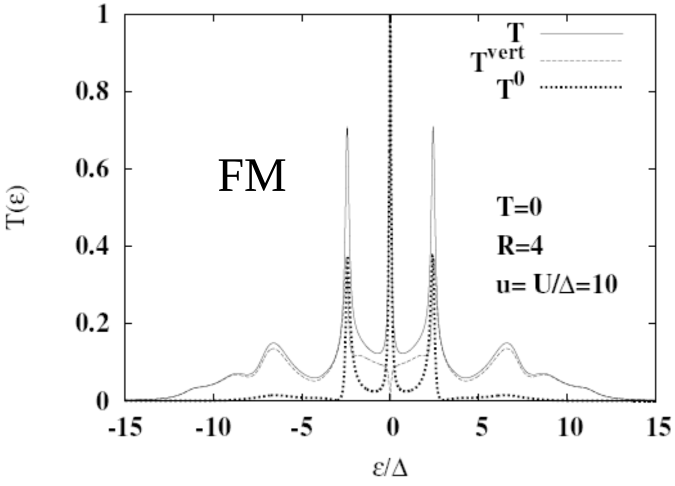


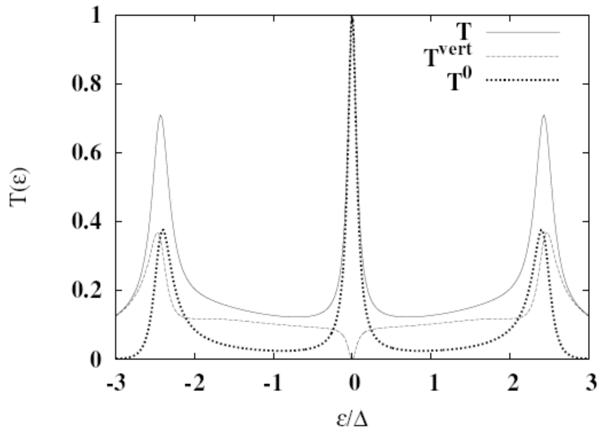




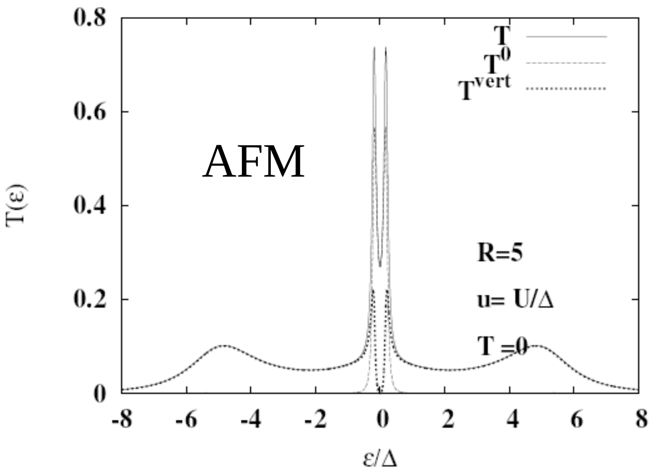
At low energies and low temperatures,
Fermi liquid behaviour

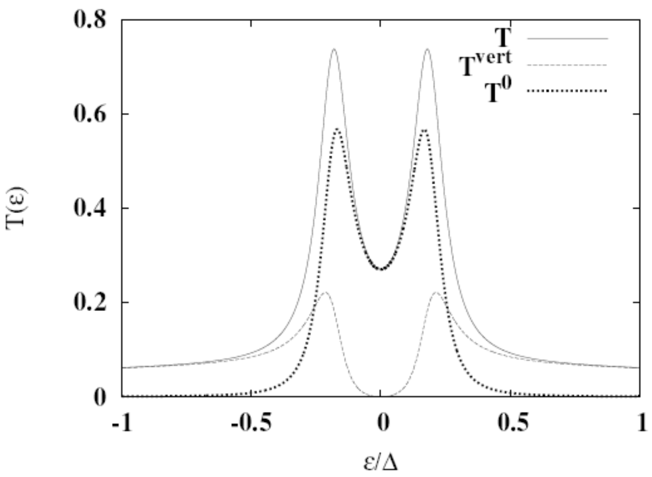
Transmission probability – R finite





T=0



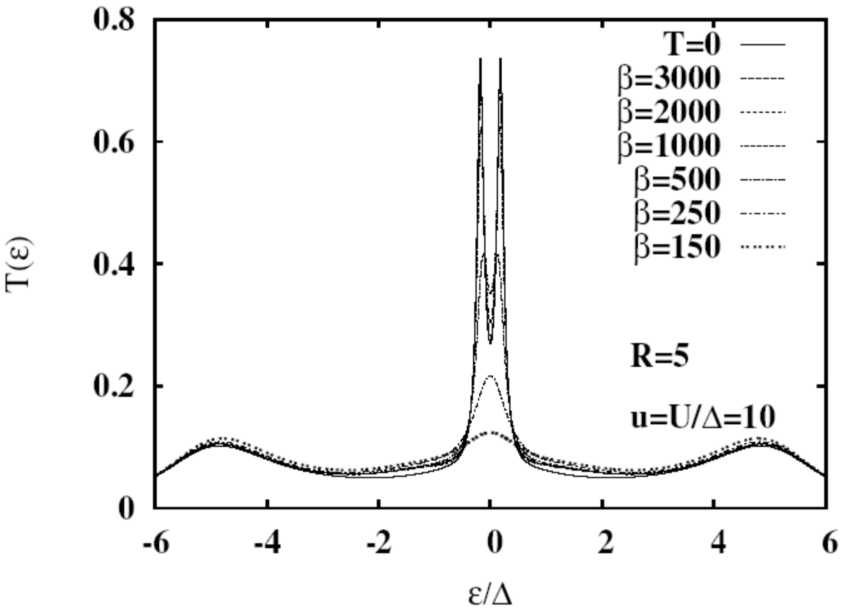


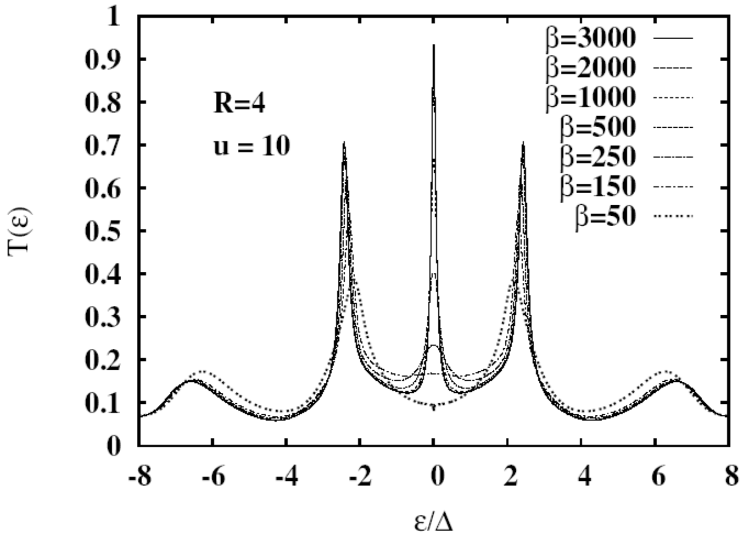
PD, 2006
2007

Elastic scattering; Inelastic scattering

Transmission probability

Finite T





Formation of the Kondo resonance as temperature is lowered.

Splitting of the Kondo peak as temperature is further lowered.

Formation of the Kondo peak as temperature is lowered

Conductance

R=0

$$G(T) \propto \frac{1}{\ln^2 T_0/T}$$

$$\rightarrow \quad \frac{dG(T)}{dT} \propto \frac{1}{T}$$

Crossover between MFL and FL behaviour as the inter-dot distance is tuned.

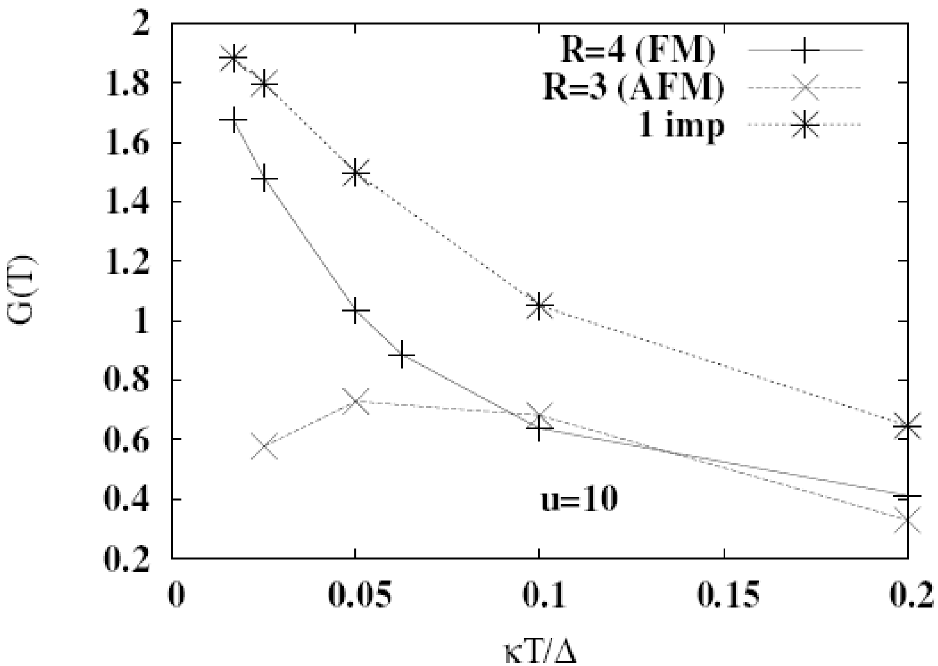
Experiments
Sasaki etal, PRB 2006

(Non) monotonic temperature dependence of the conductance for (odd) even R.

A new temperature scale in the problem – coherence temperature.

(Anti) ferromagnetic correlations develop between the impurities below this scale

R finite PD, PRB, 2006



Conclusions

- Model shows local marginal Fermi liquid behaviour for the spectral properties and logarithmically singular low temperature behaviour characteristic of the underscreened Kondo model for the transport properties.

- Inelastic scattering rate also shows a similar singular behaviour – in principle can be observed by measuring the dephasing rate

- . Negative U - Underscreened 'charge' Kondo physics

- Magnetic field, Non-equilibrium

- Renormalized perturbation theory around the strong coupling fixed point of the underscreened Kondo model ?

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