Dynamical Mass Generation in 2+1 Gauge Field Theories

IISER Workshop on Field Theory

Motivation

- Mapping the conformal window.
- QCD with 16 mass zero quarks is asymptotically free but chirally symmetric and non confining. Now decrease the number of flavors. At some flavor number chiral symmetry starts to break and confinement is restored. What is this flavor number?
- This is termed the zero temperature chiral transition.

Motivation

- Close to the conformal window but on the broken side the theory has an approximate scale symmetry. Does this give rise to a light dilaton when the chiral symmetries are spontaneously broken.
- Analogous behavior in QED3 and QCD3. There is a critical number of flavors below which a set of continuous global symmetries are spontaneously broken and above which the symmetries are exact.

2+1 Gauge theories

- Discussion started by R.Pisarski in 1984.
- He wanted to analytically study chiral symmetry breaking. Only non trivial analytic studies were NJL type models.

Review of QED3

- Map the phase structure of QED3 as a function of N-f, the number of 4-component complex fermion flavors.
- Studied for the past 30 years.
- Nice model field theory to study non-perturbative phenomena.

QED3

- Exhibits a transition similar to the zero T chiral transition in SU(M) vector like gauge theories.
- Dynamical mass generation below a critical number of flavors.
- Of interest to CM?

QED3

$$L = \sum_{i=1}^{N\_f} \overline{\Psi}_i \, i \not{D} \Psi_i - \frac{1}{4} F_{\mu\nu}^2$$

$$D_\mu = \partial_\mu - ie A_\mu$$

$$d(e^2) = 1$$

Theory is super-renormalizable

Large N_f Limit

$$\frac{e^2 N_f}{8} = \alpha$$

Large N Gauge Propagator

- Gauge propagator to the leading order in $\frac{1}{N_f}$. Each vertex is of order $\frac{1}{\sqrt{N_f}}$. Each loop carries N_f fermion flavors.

Large N_f resummed gauge propagator

- Summing a geometric series of bubble graphs gauge propagator becomes

$$\frac{g_{\mu\nu} - k_\mu k_\nu / k^2}{k^2(1+\Pi(k))}$$

and

For k << α, $\Pi(k) = \frac{\alpha}{8\mathrm{k}}$ and the propagator

$$= \frac{P_{\mu\nu}}{k^2(1+\frac{\alpha}{8k})} = \frac{\mathbf{P}_{\mu\nu}}{k\alpha}$$

IR power counting is like that of a 3+1 theory.

Symmetries

- Two component fermion mass term $m\overline{\Psi}\Psi$

violates P and T.

With two two component fermions the

combined mass term $m\left(\overline{\Psi}_1\Psi_1 - \overline{\Psi}_2\Psi_2\right)$

is invariant under P and T.

Combine the two flavors to form a four component fermion.

4 Component Fermions

- The two gamma matrices γ_3 and γ_5 anti commute with the matrices $\gamma_{0,1,2}$. Consider the mass term $i\overline{\Psi}(m+m'\gamma_{3,5})\Psi$ where $\gamma_{3,5}=-i\gamma_3\gamma_5$. The first term is the parity conserving mass while the second violates parity.

Global Symmetries

- $\sum_{i=1}^{2N\_f} \overline{\Psi}_i \, i \not{D} \Psi_i$ has a U(2 N_f) global symmetry where the fermions are two component.
- If a parity invariant mass is generated this breaks down to U(N_f)xU(N_f)
- Yields $2N_f^2$ NGB's.

Effective Running Coupling

$$\alpha(k) = \frac{e^2}{k(1+\Pi(k))} = \frac{\alpha}{N_f k\left(1+\frac{\alpha}{k}\right)} = \frac{\alpha}{N_f(k+\alpha)}$$

Beta Function

$$k\frac{\partial}{\partial k}\alpha(k)=\beta(\alpha(k))=-N_f\left(\frac{1}{N_f}-\alpha(k)\right)$$

Banks-Zaks Fixed Point

- This is like the Banks Zaks F.P. of QCD.

$$\beta(\alpha) = -b\alpha^2 - c\alpha^3$$

- For N_f close to 11N_c/2 b>0 and c<0 and the fixed point coupling $\alpha_* = \frac{-b}{c}$
- Above a critical number of flavors there is no chiral symmetry breaking. Gap equation analysis implies that N_cr=4N_c.

Dynamical Mass generation

- R.Pisarski P.R.D.29,2428(1984) first drew attention to the possibility of spontaneous mass generation in QED3.
- By solving a gap equation he concluded that a P invariant mass is spontaneously generated for all N_f.

Dynamical Mass Generation

- A study of the gap equation done in 88 concluded that dynamical mass generation occurs iff N_f is less than a critical number of flavors N_cr about 4.
- Appelquist, Nash, L.C.R.W, PRL(60)2572(88) and Nash P.R.L.62,3024(89).
- Most studies agree that there is a critical N. No agreement about its magnitude.

Gap Equation

$$\Sigma(p) = \frac{2\alpha}{3N\pi^2 p}\int^{\alpha} \frac{k\,dk\,\Sigma(k)}{k^2+\Sigma^2}\ln\left(\frac{k+p+\alpha}{|k-p|+\alpha}\right)$$

$$\Sigma(p) = \frac{2\alpha}{3N\pi^2 p}$$

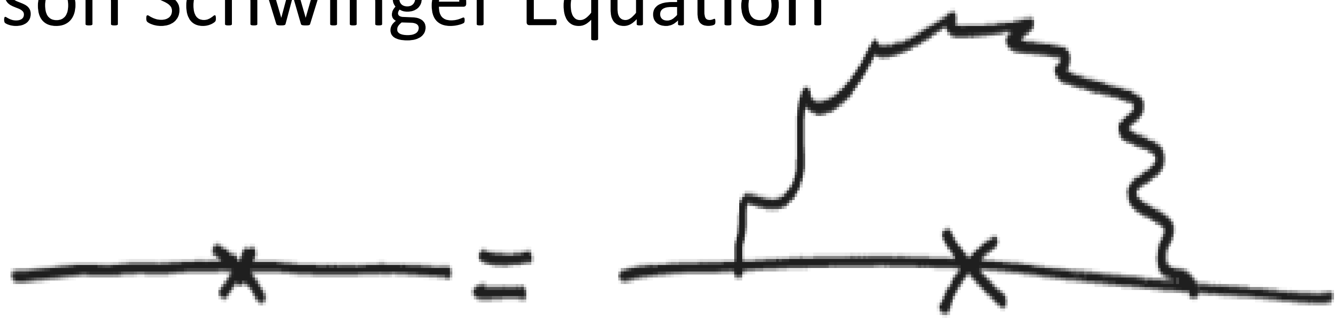
$$\frac{\Sigma(0)}{\alpha} = \exp\left[-\frac{2\pi}{\sqrt{\frac{N_{cr}}{N}-1}}\right]$$

$$\Sigma(p) = \frac{\Sigma(0)^{\frac{3}{2}}}{\sqrt{p}}\sin\left[\frac{1}{2}\sqrt{\frac{N_c}{N}-1}\,\ln\left(\frac{p}{\Sigma(0)}+\delta\right)\right]$$

$$N_{cr} = \frac{128}{3\pi^2}$$

Gap Equation

- Dyson Schwinger Equation



$$\Sigma(p) = \frac{2\alpha}{3Np\pi^2}\int_0^\infty \frac{kdk\Sigma(k)}{k^2+\Sigma(k)^2}\ln\frac{k+p+\alpha}{|k-p|+\alpha}$$

$$\frac{\Sigma(0)}{\alpha} \approx \exp\left(-\frac{2\pi}{\sqrt{\frac{N_{cr}}{N}-1}}\right)$$

$$\Sigma(p) = \frac{\Sigma(0)^{\frac{3}{2}}}{\sqrt{p}} Sin\left(\frac{1}{2}\sqrt{\frac{N_{cr}}{N}-1}\right)\ln\left(\frac{p}{\Sigma(0)}+\delta\right)$$

RG Argument By Pisarski

- R.Pisarski P.R.D44,1866(1991) analyzed the issue of dynamical mass generation in QED3 using the effective Lagrangian method and the epsilon expansion. The is a fluctuation induced first order transition for more than two flavors. He concluded that there is dynamical mass generation in this case.

Counter Argument

- In response to this Appelquist, Terning and L.C.R.W P.R.L.75,2081(95) investigated the nature of the transition.
- No light scalar degrees of freedom

as N ->N_cr from above. Therefore the phase transition is not second order. No light scalar degrees of freedom on both sides of the transition.

Counter Argument

- Effective field theory argument by Pisarski is not valid.
- However the order parameter vanishes continuously in the broken phase as the transition point is approached.
- Therefore this does not look like a conventional first order transition either.
- This unusual behavior is attributed to the presence of long range gauge forces.

Conformal Transition

- S.Chivukula,P.R.D.55(97)5238 and Miransky,Yamawaki, P.R.D.55(97)5051 discussed this transition.
- Transition is continuous but unconventional looking from the symmetric side. All the masses go to zero as the transition point is approached.
- This is attributed to the presence of long range forces. No universality.

QCD3

- Similar arguments suggests that there a critical flavor number in QCD3.
- Above this number of flavors no dynamical mass generation or confinement.
- For a SU(M) gauge theory with fermions in the fundamental representation the critical flavor number according to gap equation analysis is $$N_{cr} = \frac{256}{3\pi^2}\frac{M^2-1}{2M}$$

ACS Thermal Inequality

- Conjectured constraint on the Infra red Structure of an asymptotically free gauge theory.

$$F_{IR} \leq F_{UV}$$

$$F_{IR} = -\lim_{T \to 0} \frac{F(T)}{T^d} f(d)$$

$$F_{UV} = -\lim_{T \to \infty} \frac{F(T)}{T^d} f(d)$$

- f(d) is a function of the number of space-time dimensions d defined such that the contribution from a free Bose field is one.

Thermal Inequality

- In QED3 $F_{UV} = \frac{3}{4}(4N) + 1$. 4N the fermion degrees of freedom. ¾ is the Boltzmann weighting of fermions in 3-d. 1 the number of gauge degrees of freedom.
- F_{IR} depends on whether the global symmetry is spontaneously broken. For large N no symmetry breaking. IR spectrum consists of 1 mass less photon and N mass zero fermions. Theory is described by a weak IR fixed point.

Thermal Inequality with no symmetry breaking

- To the leading order in 1/N $F_{IR} = \frac{3}{4}(4N) + 1 = F_{UV}$.
- Next to leading order contribution to the IR is negative (Fazio Hep-Th/0512307).
- Therefore the inequality is satisfied.

With Symmetry Breaking

- Global symmetry is broken for some finite N.
- Fermions become massive and $2N^2$ NG Bosons are formed.

$$F_{IR} = 2N^2 + 1 \leq F_{UV} = 3N + 1 \Rightarrow N \leq \frac{3}{2}$$

Lattice Studies

- Hands and Kogut Nuc.Phys.B335(1990)455.Quenched approximation indicated that the symmetry is spontaneously broken.
- Simulation of 2 Flavor QED3 by Hands,Kogut and Strouthos on 50^3 lattice indicated that the condensate is two orders of magnitude smaller than the quenched value.
- The dimensionless condensate $\sigma = \frac{<\overline{\Psi}\Psi>}{e^4}$ is smaller than $5x10^{-5}$. Consistent with the thermal inequality.

Lattice Studies

- Kogut and Strouthos **arXiv:0808.2714 [cond-mat.supr-con] and arXiv:0804.0300 [hep-lat].**

Critical N is less than 2.

Critical N depends on the IR cutoff(Gusynin and Reenders,P.R.D.68,025017(2003)Hep-ph/0304302). Finite lattice sizes would affect the measured critical value. They use an IR cutoff

in the gap equation.

Abelian Higgs Model

- Consider the Lagrangian of the Abelian Higgs Model.

$$L = L_{QED3} + \frac{1}{2} D_\mu \Phi^* D_\mu \Phi - \lambda \left(\Phi^* \Phi - N v \right)^2$$

- Remove the Higgs field by taking the four scalar coupling to infinity with v fixed.

$$\mathrm{M} = \mathrm{e}\sqrt{\mathrm{Nv}}$$

$$\text{Keep } \alpha = \frac{\mathrm{e}^2 \mathrm{N}}{8} \text{ and v fixed as } \mathrm{N} \rightarrow \infty.$$

Running Coupling

- Dimensionless running coupling

$$\overline{\alpha}(k) = \frac{e^2 k}{k^2 + M^2 + \frac{e^2 Nk}{8}} = \frac{8}{N} \frac{k}{\frac{k^2}{\alpha} + G^{-1} + k}$$

$$G = \frac{\alpha}{M^2}$$

- This coupling vanishes in the UV and the IR. It is bounded by 8/N. Its maximum value approaches 8/N only when $M \leq \alpha$.

Abelian Higgs Model

- For any $\frac{M}{\alpha}$ the model is weakly coupled in the large N limit at all k.
- Reasonable to expect that as in QED3 a Parity invariant mass is generated below a critical N.
- Forces are strongly damped at scales above α .
- Therefore if a critical N exists it would be a function of $\frac{M}{\alpha}$.

Connection between QED3 and the Thirring Model

- Consider the limit $\frac{M}{\alpha} \to 0.$
- If

$$\alpha \to \infty \text{ with } G = \frac{\alpha}{M^2} \text{ fixed then } \frac{M}{\alpha} \approx \frac{1}{\sqrt{\alpha}} \to 0.$$

This leads to the Thirring Model.

- If

$$M \to 0 \text{ with } \alpha \text{ fixed we get QED3.}$$

- This suggests that the critical N's for these two models should be the same provided the Thirring model is cut off at a scale greater than 1/G.

Comments

- In the QED limit IR Fixed Point at coupling strength 8/N.
- In the Thirring Limit there is a UV fixed point with the same strength.
- As increses from zero low momentum components are damped out. If the force driving symmetry breaking is attractive at all scales like in the large N approximation a stronget IR coupling is needed fro Chiral symmetry breaking. Therefore critical N would decrease.

Critical Curve

- At some point critical N would drop below zero and there will not be any symmetry breaking.
- The critical curve should be similar to the one found by Gusynin and Reenders but with the gauge bosn mass replacing their explicit IR cutoff.
- For M=0 the transition is of infinite order. For finite M the force is of finite range and it is of second order.

Hidden Local Symmetry

- Ito,Kim,Suiura and Yamawaki Prog.Theo.Phys.417(95) used the gap equation method invoking hidden local gauge symmetry to study the dynamical mass generation in the Thirring model.
- Concluded that the two models have the same critical N.

Simulations of the Thirring Model

- Hands and Lucini,Phys.LettB461;263-269(99),hep-lat/9906008. They find symmetry breaking for N=3. Critical value is greater than 3.

Concluding Remarks

- What is the critical N? No definite answer.
- What is the Nature of the Transition?
- Need to do simulations in QCD3. Improved IR convergence?

- Effect of 4-fermion coupling.