Thermal Holography and Charged Black Holes

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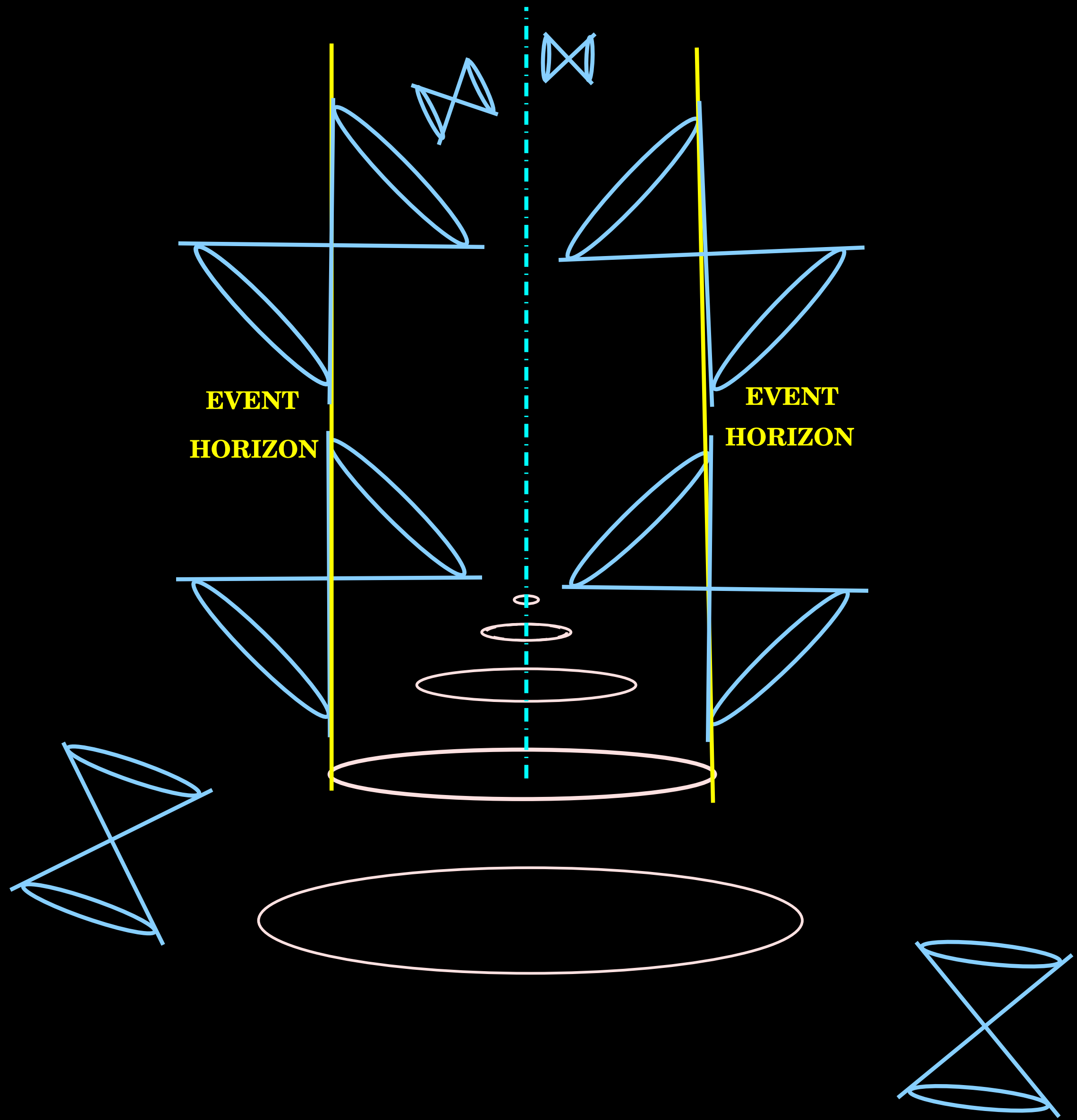
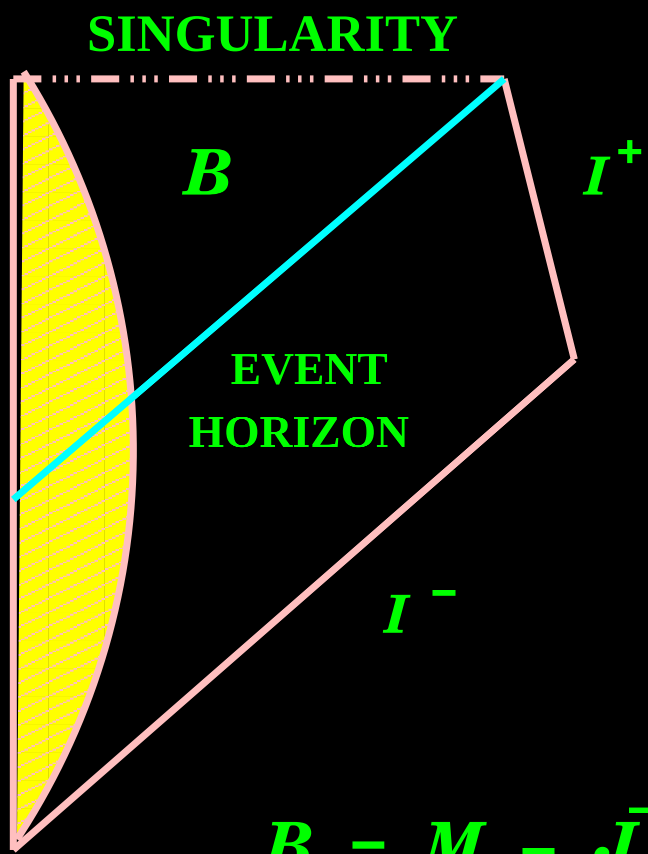
Black hole spacetime Eddington-Finkelstein

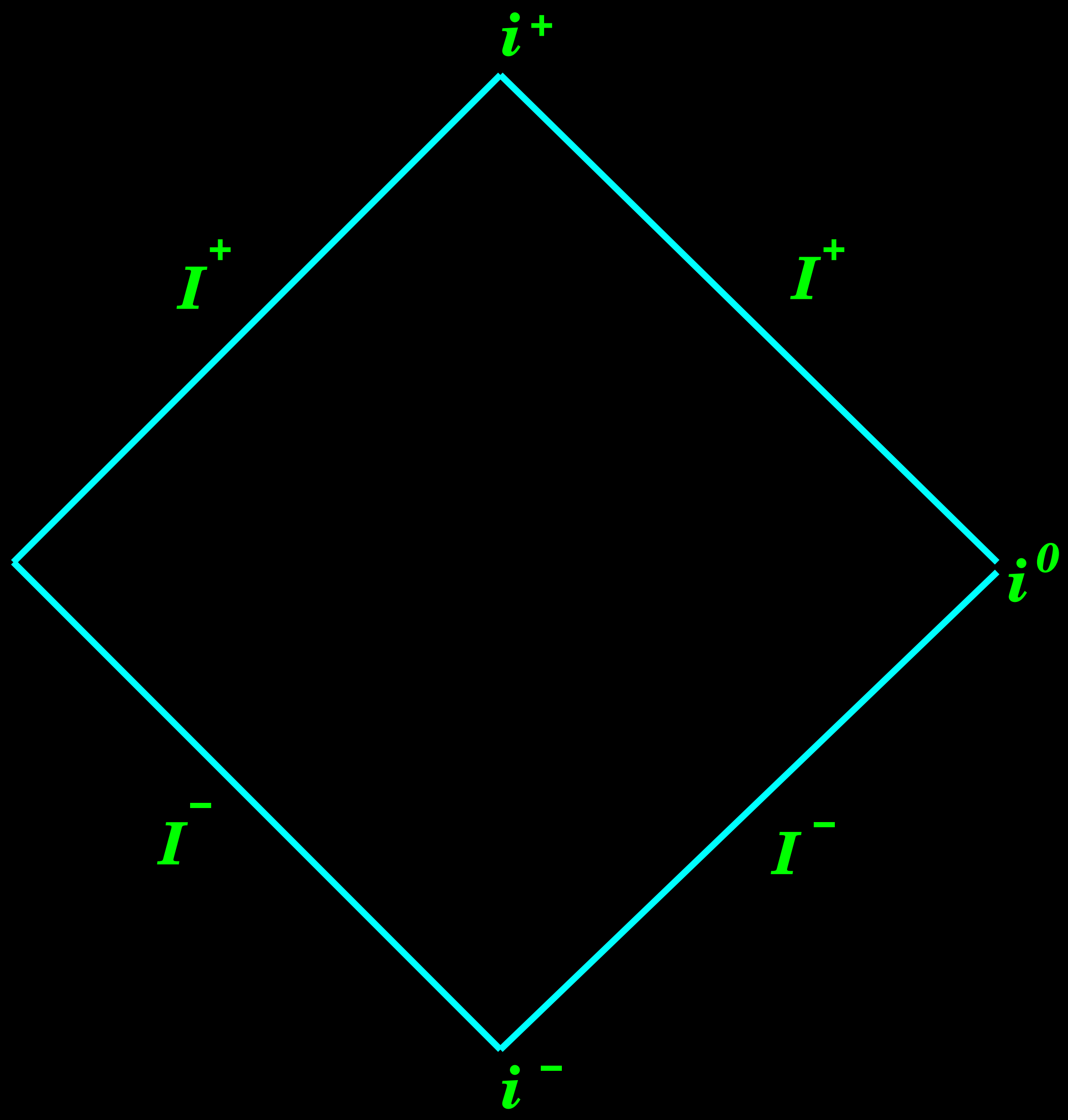


Black hole spacetime : another view

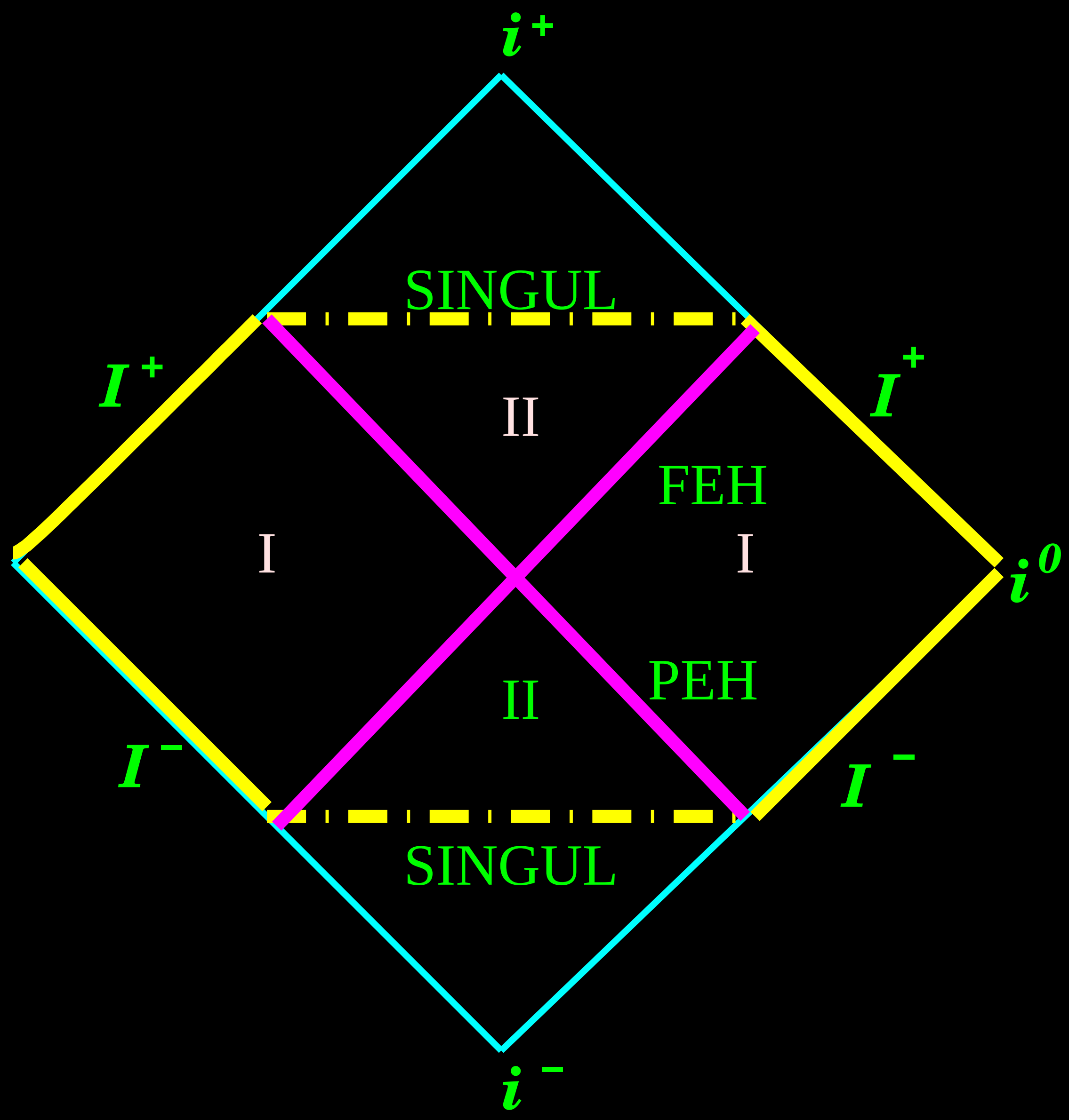


$$B = M - \bar{J}(I^{+})$$

Special relativistic sptm (Minkowski)



Spherical (static) spacetime (Schwarzschild)



Black holes ... are the most perfect macroscopic objects there are in the universe. The only elements in their construction are our notions of space and time ... and because they appear as ... family of exact solutions of Einstein's equation, they are the simplest objects as well. - Subramanian Chandrasekhar

Yet Black hole sptms have

- **Singularities, where all known laws of physics break down**
- **Event horizon : boundary of sptm accessible to asympt. obs.**

Laws of bh mech Bardeen, Carter, Hawking 1972

$$\begin{aligned} \delta A_{hor} &\geq 0 \\ \kappa_{hor} &= const \\ \delta M &= \kappa_{hor}\, \delta A_{hor} + \Phi \delta Q_{hor} + \cdots \end{aligned}$$

Gen. Sec. Law of thermo. **Bekenstein, 1973 :** $\delta(S_{out} + S_{bh}) \geq 0.$

$$S_{bh} = \frac{\mathcal{A}_{hor}}{4l_P^2} \, (k_B = 1)$$

$l_P \equiv (G\hbar/c^3)^{1/2} \sim 10^{-33} cm \longrightarrow$ quantum gravity

Need to go beyond classical GR - compulsion, not aesthetics

$S_{bh} \sim l_P^- 2 \longrightarrow$ **nonperturbative QG**

Physics at 10^{-33} cm determines entropy of bh of size 10^{11} cm – Extreme Macro QM!

Two issues to be addressed:

- **How is it that** $S_{bh} = S_{bh}(\mathcal{A}_{hor})$ **while** $S_{thermo} = S_{thermo}(vol)$ **?**
- **What degrees of freedom contribute to** S_{bh} **?**

Vac EM in Minkowski sptm: $\nabla \cdot \vec{E} = 0$ everywhere in $V \Rightarrow Q(V) = 0$

Can define total charge globally

$$Q_{tot} \equiv \int_{S_\infty} \vec{E} \cdot \hat{n} d^2 a$$

$\longrightarrow$ holographic

But, $\mathcal{H}_v = (1/8\pi)(\vec{E}^2 + \vec{B}^2) \rightarrow$ photons

Vac GR : no $\mathcal{T}^{ab}$ s.t. $\nabla_a \mathcal{T}^{ab} = 0$ in bulk

$$H_v = \int_{\mathcal{S}} [N\mathcal{H} + \mathbf{N} \cdot \mathbf{P}]$$
$$\approx 0 \text{ when } \mathcal{H} \approx 0, \ \mathbf{P} \approx 0$$

$\Rightarrow$ **no analogue of** $\mathrm{E}^2 + \mathrm{B}^2$ **in vac GR! Excitations 'polymeric'**

Grav energy globally defined

$$H_{Komar} = \frac{1}{8\pi}\int_{\mathcal{S}_\infty} d^2\sigma^{ab}\nabla_a K_b$$

Classically, bulk $\Rightarrow$ boundary entirely

Holography: 3 dim bulk info encoded on 2 dim bdy

Gravitons ?

Weak field approx $g_{ab} = \underbrace{\bar{g}_{ab}}_{bkgd} + \underbrace{h_{ab}}_{graviton}$

$$\mathcal{H}_v = (1/8\pi)[({}^3h)^2 + ({}^3\pi)^2]$$

As $|h| \nearrow$, $bkreactn \nearrow$, approx. invalid nonperturbatively

QGR: $\exists$ indep qu fluct on bdy : $\mathcal{H} = \mathcal{H}_v \otimes \mathcal{H}_b$

$$|\Psi\rangle \;=\; \sum_{v,b} c_{vb} \underbrace{|\psi_v\rangle}_{blk} \underbrace{|\chi_b\rangle}_{bdy} \in \mathcal{H}_v \otimes \mathcal{H}_b$$

$$\hat{H} \;=\; \underbrace{\hat{H}_v}_{blk} \otimes \mathbf{1} \;+\; \mathbf{1} \otimes \underbrace{\hat{H}_b}_{bdy}$$

Hamiltonian constraint (bulk)

$$\hat{H}_v \,|\psi_v\rangle \;=\; [\hat{H}_{g,v} + \hat{H}_{m,v}]|\psi_v\rangle = 0$$

$$\hat{Q} = \hat{Q}_v \otimes \hat{I}_b + \hat{I}_v \otimes \hat{Q}_b$$

$$\hat{Q}_v|\psi_v\rangle = 0$$

New Hamiltonian constraint

$$\hat{H}'_v|\psi_v\rangle = 0$$
$$\hat{H}'_v \equiv \hat{H}_v - \Phi\hat{Q}_v$$

Grand Partition Function Majhi, PM 2011

$$Z_G = Tr \, \exp -\beta\hat{H}_T + \beta\Phi\hat{Q}$$
$$= \sum_{v,b} |c_{vb}|^2 \langle\chi_b| \otimes \langle\psi_v| \exp -\beta\hat{H}' |\psi_v\rangle \otimes |\chi_b\rangle$$
$$\hat{H}' = \hat{H}_T - \Phi\hat{Q}$$

Observe

$$\hat{H}' = (\hat{H}'_v \otimes \hat{I}_b + \hat{I}_v \otimes \hat{H}'_b)$$
$$\hat{H}'_v|\psi_v\rangle = 0$$

$$Z_G = Z_{Gb}$$
$$Z_{Gb} = Tr_b \exp -\beta(\hat{H}_b - \Phi\hat{Q}_b)$$

Bulk states decouple! Boundary states determine bh thermodynamics completely $\longrightarrow$ **Thermal holography !** (PM 2001, 2007; Majhi, PM 2011)

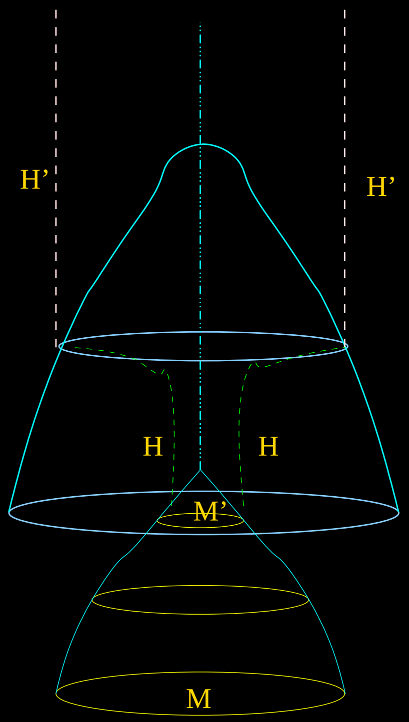
Different from strong holography ('t Hooft 1992; Susskind 1993; Bousso 2002)

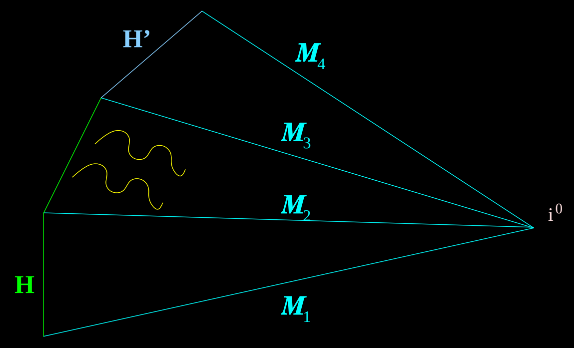
Holographic Hypothesis (HH)

... Given any closed surface, we can represent all that happens (gravitationally) inside it by degrees of freedom on this surface itself. This ... suggests that quantum gravity should be described by a **topological** *quantum field theory in which all (gravitational) degrees of freedom are projected onto the boundary.*

What sort of boundary ? Not asymptotic bdy; not *inner* bdy of accessible sptm $\longrightarrow$ EH (teleological, stationary, ...)

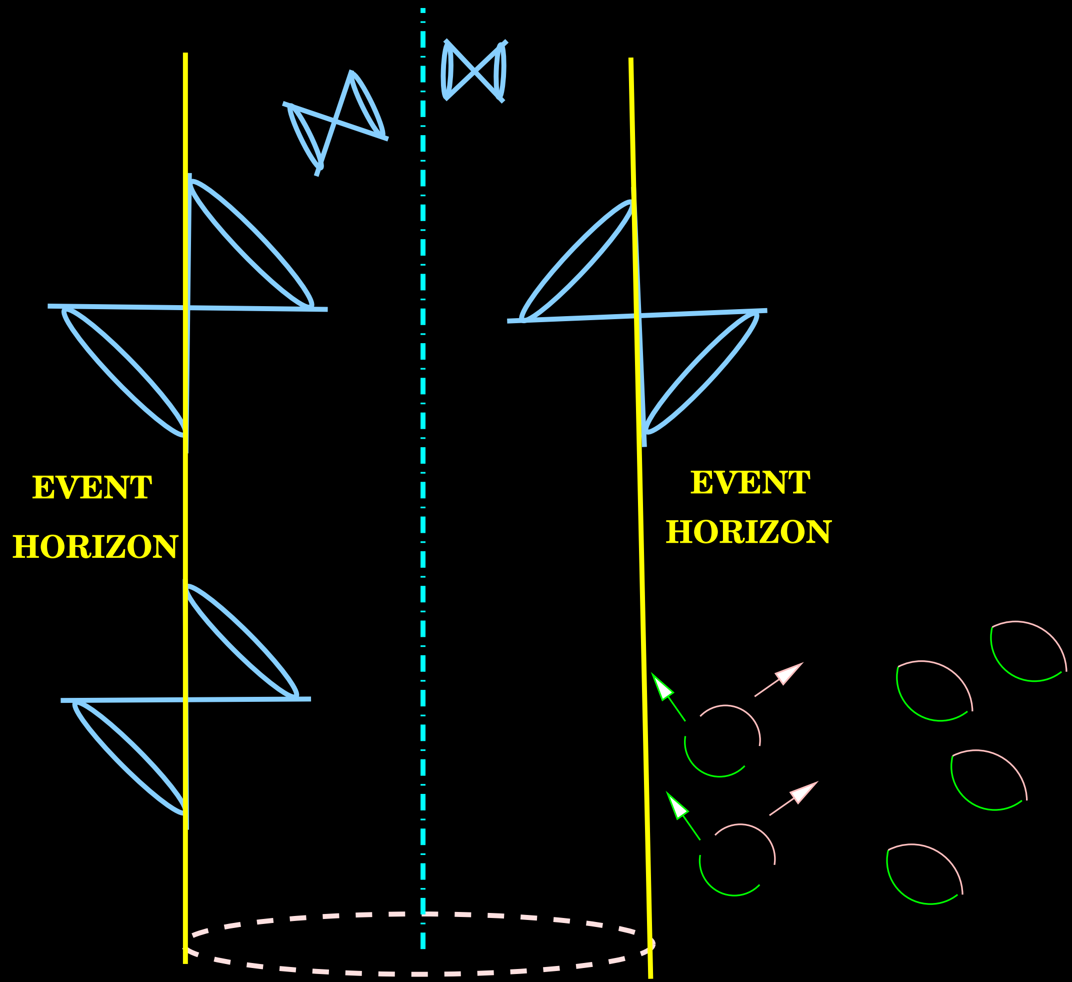
Work with Isolated Horizons (IH) as local, non-stationary equilibrium generalization of EHs (Ashtekar et. al. 1997-2001)





- **Nonstationary**
- Null (lightlike) inner boundary of sptm with topol $R \otimes S^2$
- $\mathcal{A}(S^2) = const \rightarrow$ isolation
- *Zeroth law of IHM* surface grav $\kappa_{IH} = const$
- $M_{IH} \equiv M_{ADM} - \mathcal{E}^{\infty}_{rad}$ s.t. $\delta M_{IH} = \kappa \delta \mathcal{A}_{hor} + \Phi \delta Q_{hor}$ (Ist law of IHM)
- IH microcanonical ensemble with fixed $\mathcal{A}_{hor}, Q_{hor}$
- Hawking radiation requires IH $\longrightarrow$ Dynamical Hor

Black hole radiance



Grand Canonical Ensemble of IHs in rad bath : compute $Z_b \to S_{can}$

- Assume equil. IH with fixed $\mathcal{A}_{IH}, Q_{IH}$ and $M_{IH} = M(\mathcal{A}_{IH}, Q_{IH})$.
- Keep Gaussian fluct. (Das, Bhaduri, PM 2001; Chatterjee, PM 2003)
- $\mathcal{A}_n \sim n l_P^2,\ n >> 1$ (justify later)

$$S_{can}(\mathcal{A}_{IH}) \ = \ S_{IH}(\mathcal{A}_{IH}) \ + \ \underbrace{\frac{1}{2} \log \Delta(\mathcal{A}_{IH})}_{th\ fluc\ corr}$$

Two issues arise :

- Expect S_{can} $+ve\ real\ \Rightarrow\ C\ >\ 0$ (th stab). How/when violated (e.g. Schwarzschild, RN)?
- How to compute S_{IH} ? Need microscopic QG theory of IH

Condition for thermal stability (Majhi, PM 2011)

$$Z_G = \int \frac{dA\, dQ}{A_x\, Q_y}\, g(A,Q)\ \exp -\beta[E(A,Q) - \Phi Q]$$

$$= \int dA\, dQ\, e^{S(A)-\beta E(A,Q)+\beta\Phi Q}$$

$S(A)$ is the microcanonical entropy

Thermal stability criterion

$$\beta \equiv \frac{S_A(\overline{A})}{E_A(\overline{A},\overline{Q})} > 0$$

$$\beta M_{AA}(\overline{A},\overline{Q}) - S_{AA}(\overline{A}) > 0$$

$$\{\beta M_{AA}(\overline{A},\overline{Q}) - S_{AA}(\overline{A})\}\, \beta M_{QQ}(\overline{A},\overline{Q}) - \beta^2 M_{AQ}^2(\overline{A},\overline{Q}) > 0$$

$$\frac{M_{AA}}{M_A} - \frac{M_{QA}^2}{M_{QQ}M_A} > \frac{S_{AA}}{S_A}$$

Partial differential inequality

Ansatz:

$$M(A,Q) = \mu(A) \cdot \chi(Q) \quad , \quad \chi(0) = 1$$

$\longrightarrow$

$$\frac{\mu}{\mu_A}\left[\frac{\mu_{AA}}{\mu_A} - \frac{S_{AA}}{S_A}\right] > \frac{\chi_Q}{\chi}\frac{\chi_Q}{\chi_{QQ}}$$

Solution :

$$\chi(Q) \;=\; (1 + CQ)^{\frac{1}{\kappa-1}}$$

$$\mu(A) \;>\; (\alpha S + \gamma)^{\frac{\kappa}{\kappa-1}}$$

$\kappa > 1\ ,\ \gamma = 0\ ,\ (k_B\alpha)^{\frac{\kappa}{\kappa-1}} = M_P \Rightarrow$

$$\frac{M}{M_P} > \frac{S}{k_B}\left[\frac{S}{k_B(1+CQ)}\right]^{\frac{1}{\kappa-1}}$$
$$= \frac{S}{k_B}\left[1 + \frac{1}{\kappa-1}\ln\left(\frac{S}{k_B(1+CQ)}\right) + \cdots\right]$$

- Checks out with $Q = 0$ case PM 2007
- **Necessary and sufficient condition for black hole tobe stable** : checks out with classical RN and AdS-RN metrics
- No classical metric used in derivation

Bulk dof $g_{ab} \rightarrow e_a^I \rightarrow \omega_a^{IJ} \rightarrow SL(2,C)$ gauge potential $\rightarrow$ Self-dual connection formulation Sen 1982, Ashtekar 1985

IH **null** bdy $\Rightarrow {}^3g_{ab}dx^a dx^b = 0 = {}^3g$

3 dim gravity : $\mathcal{S}_{IH} = \int_{IH} \sqrt{-{}^3g}\, {}^3R$ impossible!

On IH $\omega(bulk) \rightarrow \mathbf{A}(IH) \rightarrow SL(2,C)$ gauge pot of TGT

$$\mathcal{S}_{IH}[\mathbf{A}] = tr \int_{IH} \epsilon^{abc} \left[\left(\frac{k}{2\pi} \right) (\mathbf{A}_a \partial_b \mathbf{A}_c + \mathbf{A}_a \mathbf{A}_b \mathbf{A}_c) + \mathbf{A}_a \mathbf{\Sigma}_{bc} \right]$$
$$\equiv \mathcal{S}_{CS+sources}$$

$\mathcal{S}_{GR} + \mathcal{S}_{IH} \rightarrow$ variational principle OK, provided

$$\left(\frac{k}{2\pi} F_{CS} + E \times E \right)_{S^2} = 0 \,, \; k \equiv (\mathcal{A}_{IH}/4\pi l_P^2) >> 1$$

Loop Quantum Gravity/Canonical QGR (bkgd-indep, nonpert)

$SL(2,C)$ inv self-dual gravity $\longrightarrow$ complex config. space $\longrightarrow$ gauge fix to Barbero-Immirzi $SU(2)$ inv formlation

For $\mathbf{A}$, E canonical quantization $\Rightarrow$

$$\left[\hat{A}^a_I\ ,\ \hat{E}_{b,J}\right]\ =\ i\,\delta^a_b\,\eta_{IJ}\,\delta^{(3)}(...)$$

Global variables

$$\text{holonomies } h_l \equiv \mathcal{P}\exp\int_l \mathbf{A}\ ,\ \text{Fluxes } E_{f,S} \equiv \int_S f_a E^a$$

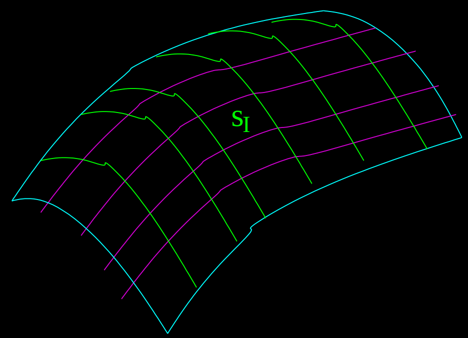
LQG : promote these to operators $\hat{h}_l(\hat{\mathcal{A}})\ ,\ \hat{E}_{f,S}$

Wave functionals in 'position' basis $\Psi = \Psi[\mathcal{A}]$ can be expressed as functions of holonomies $\psi(h_{l_1}, ...h_{l_n}, ...)$.

Holonomies completely specified by spin j_l associated with link l

Spin network : Quantum Space

Area operator (also volume, length) have bded, discrete spectrum



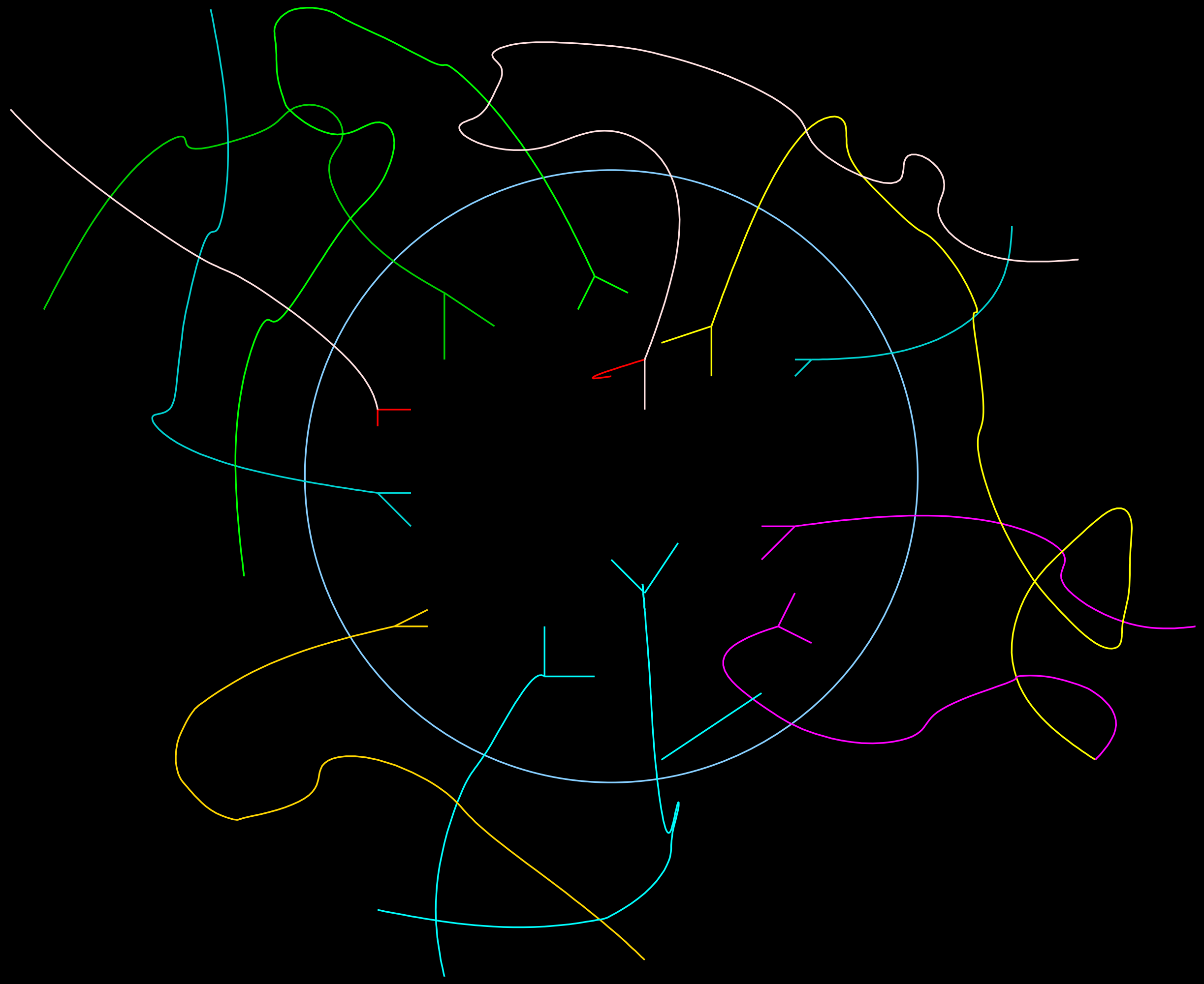
$$\hat{A}_S \equiv \sum_{I=1}^{N} \int_{S_I} \det{}^{1/2}[{}^2g(\hat{E})]$$

$$a(j_1, \ldots, j_N) = \frac{1}{4}\gamma l_P^2 \sum_{p=1}^{N} \sqrt{j_p(j_p+1)}$$

$$\lim_{N\to\infty} a(j_1, \ldots. j_N) \leq \mathcal{A}_{cl} + O(l_P^2)$$

Equispaced $\forall j_p = 1/2$

‘Quantum’ Isolated Horizon → effective description (Ashtekar, Baez, Corichi, Krasnov 1997)



Need to compute $S_{IH} = \log \dim \mathcal{H}_{CS+ptsources(j_1,...j_n)}$ for fixed $\mathcal{A}_{IH} \pm O(l_P^2)$

Witten (1986) : $\dim \mathcal{H}_{CS} = \#conf\ blocks\ of\ SU(2)_k\ WZW\ (CFT_2)$ on punctured S^2

4 dim gravity $\rightarrow$ 2 dim CFT link

$\Rightarrow$ (Kaul, PM 1998)

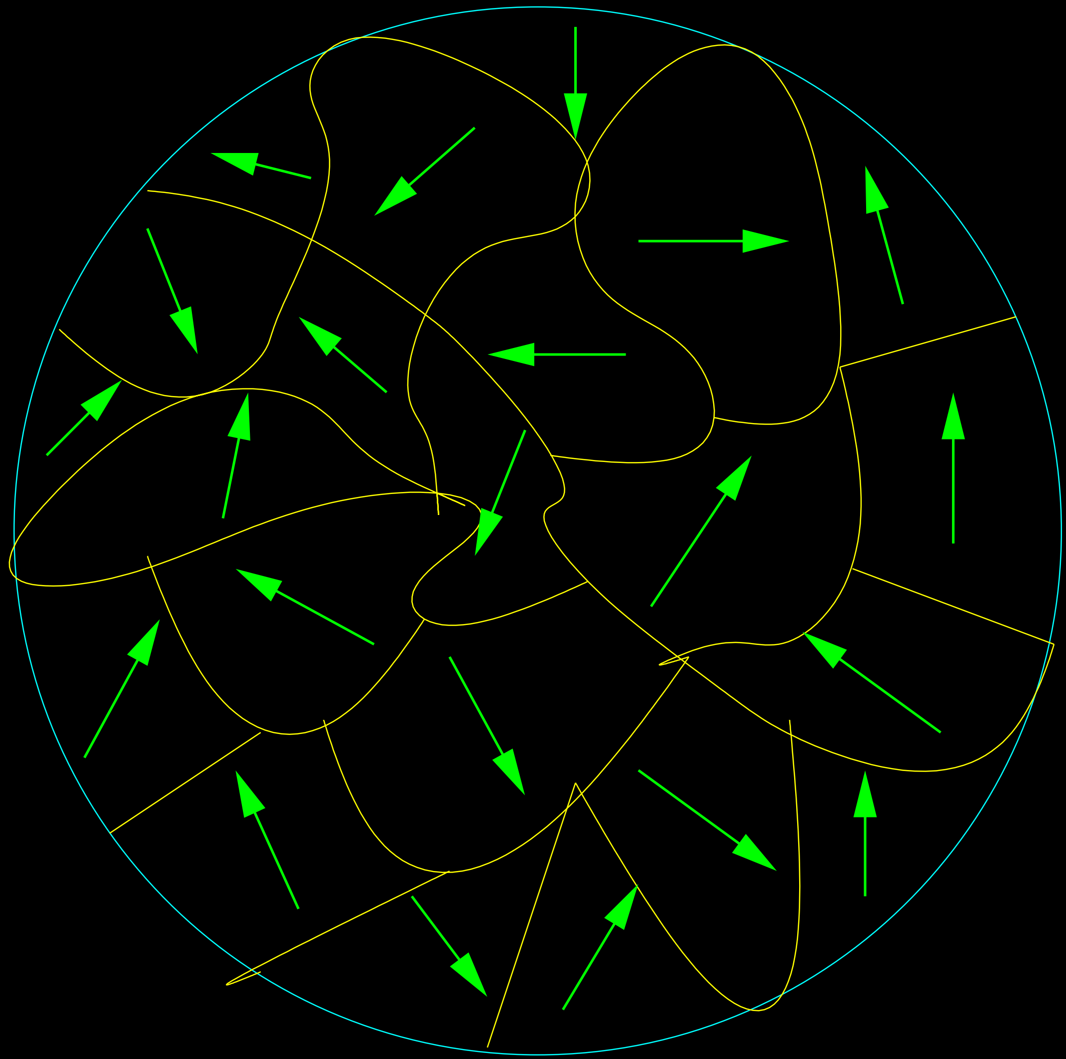
$$\begin{aligned}
\dim\ \mathcal{H}_{CS+(j_1,...,j_n)} &= \prod_{p=1}^{n} \sum_{m_p=-j_p}^{j_p} [\delta_{m_1+\cdots+m_n,0} \\
&- \frac{1}{2}\delta_{m_1+\cdots+m_n,-1} \\
&- \frac{1}{2}\delta_{m_1+\cdots+m_n,1}]
\end{aligned}$$

If $j_p = \frac{1}{2} \, \forall \, p = 1, \ldots, n$

$$S_{mc} = S_{IH} = \underbrace{\frac{A_{IH}}{4l_P^2}}_{\text{(Ashtekar et. al. 1997)}} \underbrace{- \frac{3}{2} \log \left(\frac{A_{IH}}{4l_P^2} \right) + const. + O(A_{IH}^{-1})}_{\text{(Kaul,PM 2000)}}$$

Infinite series of corrections to semicl BHAL : characteristic signature of LQG

IT from BIT



Plaquettes have $A_{pl} \sim l_{Pl}^2 : A_{Ibh}/A_{pl} \equiv N_{Ibh} >> 1$

Each Plaq has a binary BIT (e.g., spin 1/2 state) $\Rightarrow$ count total $\dim\{net\ spin = 0\ states\} \equiv \mathcal{N}$

$$\mathcal{N} = \frac{N_{Ibh}!}{((N_{Ibh}/2)!)^2} - \frac{N_{Ibh}!}{(N_{Ibh}/2+1)!(N_{Ibh}/2-1)!}$$

Use Stirling approximation for $N_{Ibh} >> 1$ and $S_{Ibh} \equiv \log \mathcal{N}$ with units chosen such that $k_B = 1$

For macroscopic isolated black holes ($N_{Ibh} >> 1$) Das, Kaul, PM 2001

$$S_{Ibh} = \frac{A_{Ibh}}{4l_P^2} \underbrace{- \frac{3}{2}\log\left(\frac{A_{Ibh}}{4l_P^2}\right) + const. + O\left(\frac{4l_P^2}{A_{Ibh}}\right)}_{qu.sptm.corr.}$$

Summary

- Weaker version of holography derived from QGR, albeit heuristic
- Can bh entropy receives positive log (area) corrections due to thermal fluct
- Thermal stability: prelim non-semicl understanding why some black holes decay and others may not
- Microcan bh entropy understood for macro bhs; BH area law receives infinite series of finite corrections – signature of LQG
- Bekenstein entropy bound tightened due to LQG corrections

Pending Issues

- IH $\longrightarrow$ Dynamical Hor unclear: Hawking radiation ?
- Info Loss Puzzle: can lowest area quantum be a remnant ? Even so, how do we get back lost info ?
- How does LQG resolve black hole singularities ?
- Gauge-gravity connection : relation between Chern Simons dynamics ?