A theory of non-BCS type superconductivity in gapped Graphene*

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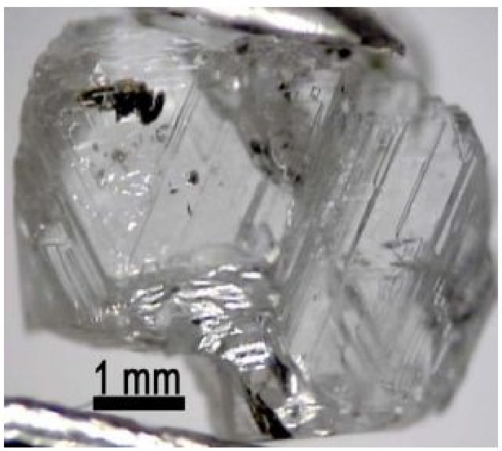
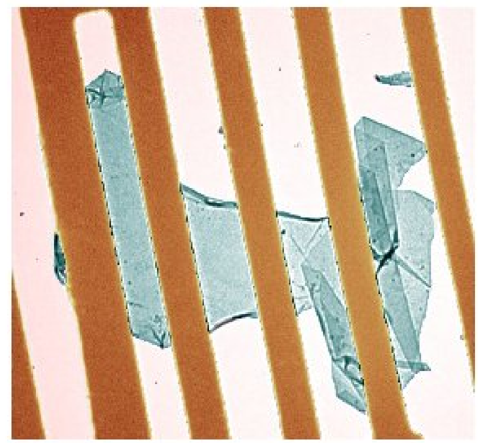
*Based on arXiv: 0901.1034 & 1107.5521

Carbon's Family

Diamond

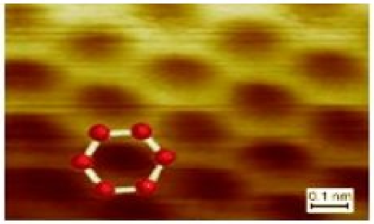
Graphene & derivatives



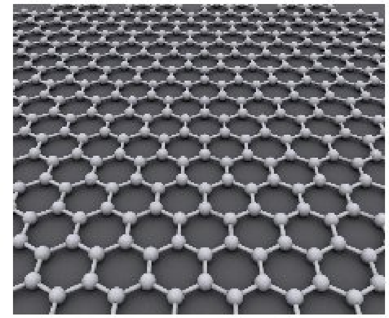


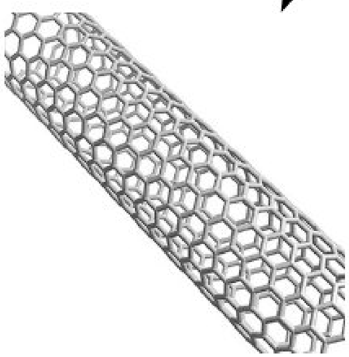
sp^3 hybrid Carbon

sp^2 hybrid Carbon

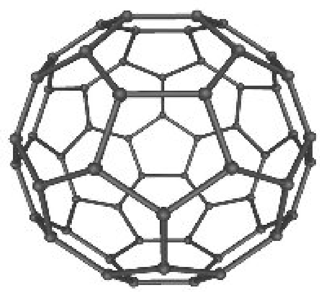


Graphene

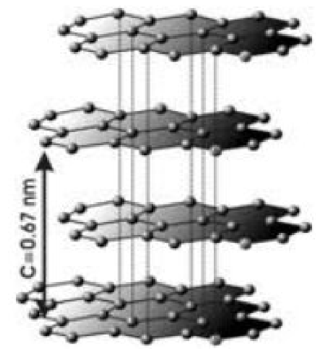




Nanotube

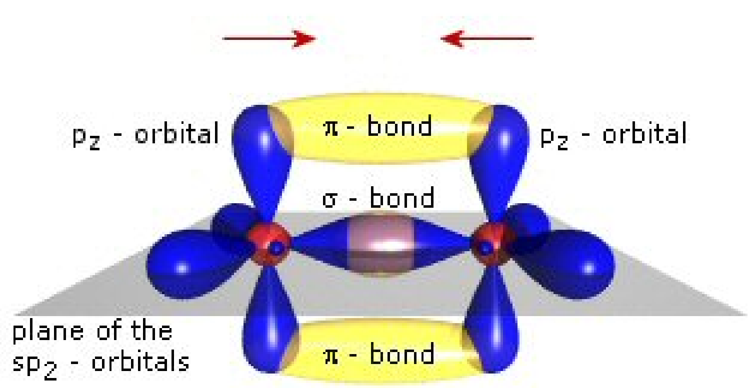


Fullerenes



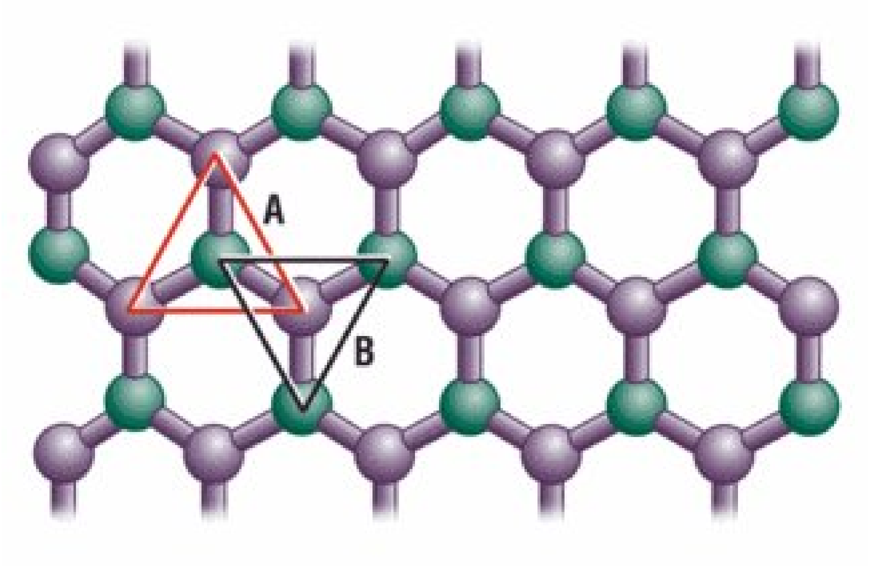
Bulk Graphite

Graphene: Electronic Structure



- Carbon atoms are in sp^2 hybrid state
- Hybrid orbitals form strong & directional σ bonds
- Out-of-plane p_z orbitals merge and form π bonds

Graphene: Electronic Structure



- Hexagonal tiling can be thought of as two interpenetrating triangular lattices (A,B)
- Unit Cell consists of two sites, one from each lattice

Graphene: Electronic Structure

- Delocalised π electrons can be modeled by simple hopping Hamiltonian:

$$H = -t \sum_{\vec{r},\vec{s}_i} \left(a^\dagger(\vec{r}) b(\vec{r} + \vec{s}_i) + h.c. \right)$$

$\vec{r}$ point A lattice points and $\vec{s}_i (i = 1, 2, 3)$ point B lattice points from any A site

- In Fourier space Hamiltonian is:

$$H = -t \sum_{\vec{k}} \left(a^\dagger(\vec{k}), \;\; b^\dagger(\vec{k}) \right) \begin{pmatrix} 0 & \sum_i e^{i\vec{k}\cdot\vec{s}_i} \\ \sum_i e^{-i\vec{k}\cdot\vec{s}_i} & 0 \end{pmatrix} \begin{pmatrix} a(\vec{k}) \\ b(\vec{k}) \end{pmatrix}$$

Graphene: Electronic Structure

- Single particle Energy (Band structure)

$$E(\vec{k}) = \pm t|\sum_i e^{i\vec{k}\cdot\vec{s}_i}|$$

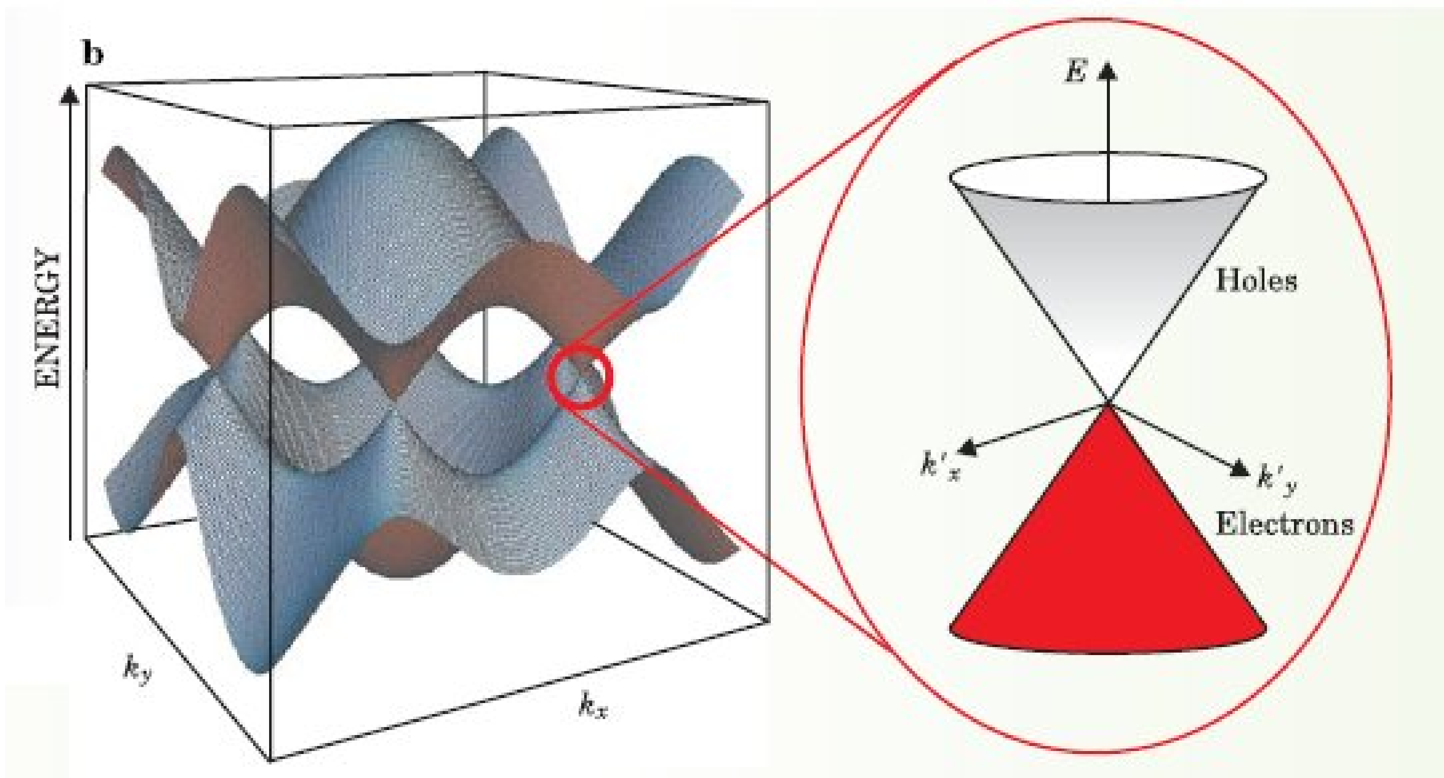
where $\pm$ stand for Conduction/Valence Band

- Wallace* observed that for six points in Fourier space, above energy vanishes

- Only two points, called Dirac points $K_\pm$, in k-space are independent, rest can be reached via symmetry operations

*Wallace, P. R., 1947, Phys. Rev. 71, 622.

Graphene: Band Structure



Graphene: Electronic Structure

- Semenoff* showed that linearised Hamiltonian around two Dirac points (also commonly referred to as valleys) has Dirac structure

$$\mathsf{H}_D = \int d^3x\, \psi_+^\dagger \left(v_f \vec{\sigma} \cdot \vec{p} \right) \psi_+ + \psi_-^\dagger \left(v_f \vec{\sigma^*} \cdot \vec{p} \right) \psi_- .$$

v_f is Fermi velocity ($\sim 10^6 m/s$), $\psi_\pm(\vec{k}) = (a_\pm(\vec{k}), b_\pm(\vec{k}))^T$

- Low energy π electron dynamics ($<$ 1 eV) is captured by two species of massless Dirac electrons each living at $K_\pm$ valley

*Semenoff, G. W., 1984, Phys. Rev. Lett. 53, 2449.

- Long wavelength modes see an emergent relativity, albeit c is replaced by v_f

- $\vec{\sigma}$ represents NOT spin but pseudo-spin

- Understanding of this pseudo-spin is still not clear* (namely whether it is a genuine 'spin' ?)

- Mass-Gap can be induced by say onsite(local) interaction that breaks sublattice symmetry, essential for any semiconducting application

$$\mathrm{H}_{onsite} = \beta \sum_i \left(a_i^\dagger a_i - b_i^\dagger b_i \right).$$

*Mecklenburg M. and Regan B. C., 2011,Phys. Rev. Lett. 106, 116803.

- This can be done selectively functionalising or doping one sublattice

- Gap can also be induced by placing Graphene on carefully choosen substrate (lattice mismatch)

- Boron Nitride has Dirac fermions and is naturally gapped

Integer Quantum Hall Effect @ Room Temperature

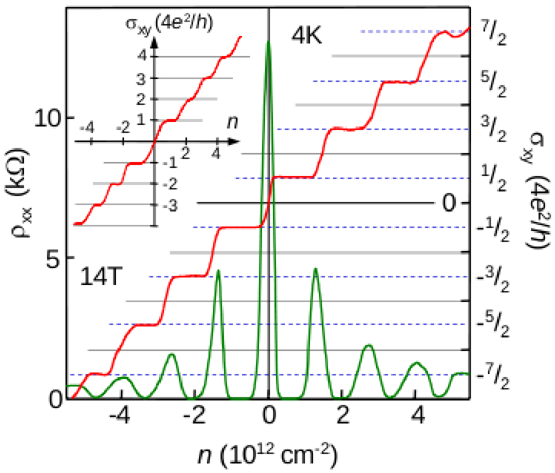


Figure 4. Quantum Hall effect for massless Dirac fermions. Hall conductivity σ_{xy} and longitudinal resistivity ρ_{xx} of graphene as a function of their concentration at B =14T. σ_{xy} =$(4e^2/h)\nu$ is calculated from the measured dependences of $\rho_{xy}(V_g)$ and $\rho_{xx}(V_g)$ as $\sigma_{xy} = \rho_{xy}/(\rho_{xy} + \rho_{xx})^2$. The behaviour of $1/\rho_{xy}$ is similar but exhibits a discontinuity at $V_g \approx 0$, which is avoided by plotting σ_{xy}. Inset: σ_{xy} in "two-layer graphene" where the quantization sequence is normal and occurs at integer ν. The latter shows that the half-integer QHE is exclusive to "ideal" graphene.

(K. S. Novoselov *et. al.*, Nature **438**, 197-200 (10 November 2005))

Graphene

- Klein Paradox was predicted and observed
- Universal Conductance, Ballistic transport
- Proposals:
  - Quantum Spin Hall Effect
  - Fermion fractionalisation
- Fractional Quantum Hall effect is observed in freely standing Graphene

- In Graphene natural units $\hbar = v_f = 1$, electronic Lagrangian in manifestly covariant form reads:

$$\mathscr{L}_D = \bar{\psi}_+(i\gamma_+^\mu \partial_\mu - m)\psi_+ + \bar{\psi}_-(i\gamma_-^\mu \partial_\mu - m)\psi_-. \tag{1}$$

- Above Lagrangian is invariant under two types of independent global transformations:

$$\psi_+(r) \to e^{-i\theta}\psi_+(r),\ \psi_-(r) \to e^{-i\theta}\psi_-(r), \tag{2}$$

$$\psi_+(r) \to e^{-i\lambda}\psi_+(r),\ \psi_-(r) \to e^{i\lambda}\psi_-(r). \tag{3}$$

- Independence of these two transformations can be seen easily by working in a reducible representation:

$$\mathscr{L}_D = \bar{\Psi}(i\gamma^\mu \partial_\mu - m)\Psi$$

where $\Psi = (b_+, -b_-, a_-, a_+)^T$, and

$$\gamma^0 = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1} \end{pmatrix},$$

$$\gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix} \; (i = 1, 2, 3),$$

$$\gamma^5 = \begin{pmatrix} 0 & \mathbf{1} \\ \mathbf{1} & 0 \end{pmatrix}.$$

- Above two transformations now read:

$$\Psi(r) \rightarrow e^{-i\theta}\,\Psi(r), \text{and} \tag{4}$$

$$\Psi(r) \rightarrow e^{-i\gamma^3\gamma^5\lambda}\,\Psi(r) \tag{5}$$

which clearly shows their independence.

- Since these are continuous symmetry operations, Noether theorem holds, and as a result one finds two independently conserved currents:

$$\partial_\mu (j_+^\mu + j_-^\mu) = 0 \text{ and } \partial_\mu (j_+^\mu - j_-^\mu) = 0,$$

where $j^\mu(r) = \bar{\psi}(r)\gamma^\mu \psi(r)$.

- Above relations imply conservation of both the valley currents separately, which means that no intervalley scattering takes place.

- Transformations of first type can be gauged using external electromagnetic field A_μ

$$\begin{aligned}\mathscr{L} =&\bar{\psi}_+(i\gamma_+^\mu \partial_\mu - m + \gamma_+^\mu A_\mu)\psi_+ \\ &+ \bar{\psi}_-(i\gamma_-^\mu \partial_\mu - m + \gamma_-^\mu A_\mu)\psi_- .\end{aligned}$$

- This means that above remains invariant under local gauge transformations:

$$\psi_+(r) \to e^{-i\Lambda(r)}\psi_+(r), \psi_-(r) \to e^{-i\Lambda(r)}\psi_-(r),$$
$$\text{and} \quad A_\mu \to A_\mu + \partial_\mu \Lambda(r).$$

- Above action is responsible for observed electromagnetic response of Graphene

- What happens when one gauges the transformations of second type ?

- Two routes to gauge invariance

  - Route No. 1: By introducing a gauge field explicitly (arxiv: 0901.1034)

  - Route No. 2: By introducing a gauge field implicitly (*i.e.*, via a constraint) (arxiv: 1107.5521)

- In this talk only Route No. 1 will be discussed

Route No. 1

- We shall assume that there exist a gauge field a_μ such that,

$$
\begin{aligned}
\mathscr{L} =& \bar{\psi}_+(i\gamma_+^\mu\partial_\mu - m + \gamma_+^\mu a_\mu)\psi_+ \\
&+ \bar{\psi}_-(i\gamma_-^\mu\partial_\mu - m - \gamma_-^\mu a_\mu)\psi_- - \frac{1}{4\tilde{g}^2} f_{\mu\nu} f^{\mu\nu},
\end{aligned}
$$

and remains invariant under local gauge transformations:

$$
\psi_+(r) \rightarrow e^{-i\chi(r)}\psi_+(r), \psi_-(r) \rightarrow e^{i\chi(r)}\psi_-(r),
$$

and $a_\mu \rightarrow a_\mu + \partial_\mu\chi(r)$.

Electromagnetic response

- In functional integral formulation, vacuum functional is defined as:

$$Z = N \int \mathscr{D}[\bar{\psi}, \psi, a_\mu] \, e^{i\mathcal{S}[\bar{\psi},\psi,a_\mu,A_\mu]},$$

  where action is given by,

$$\begin{aligned}\mathcal{S} = \int d^3x \, \bar{\psi}_+ (i\gamma_+^\mu \partial_\mu - m + \gamma_+^\mu a_\mu + \gamma_+^\mu A_\mu)\psi_+ \\ + \bar{\psi}_- (i\gamma_-^\mu \partial_\mu - m - \gamma_-^\mu a_\mu + \gamma_-^\mu A_\mu)\psi_- - \frac{1}{4\tilde{g}^2} f_{\mu\nu} f^{\mu\nu}.\end{aligned}$$

- Fermion spectrum is gapped and hence at low energy they are only virtually excited

- Hence they can be integrated out from above action, and using the method of derivative expansion*, it yields an effective action in terms of a and A to the lowest order:

$$Z = N \int \mathscr{D}[a_\mu]\, e^{i \int d^3x\, \mathscr{L}_{eff}}, \text{ where,}$$

$$\mathscr{L}_{eff}[a, A] = -\frac{1}{4\tilde{g}^2} f_{\mu\nu} f^{\mu\nu} - \frac{m}{\pi|m|} \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu a_\lambda + \mathcal{O}(\frac{1}{m}).$$

- An additional factor of 2 have multiplied in above action to take into account spin degeneracy

- Virtual fermion loops have generated a mixed Chern-Simons term in above action

*Babu K.S., Das A. and Panigrahi P.K., 1988, Phys. Rev. D 36, 3725

- This term is topological, but unlike pure Chern-Simons term, this does not violate P and T symmetry

- Inorder to study electromagnetic response, one needs to find the effective action for external A_μ field by integrating out a field from above action, which is given by

$$\mathscr{L}_{eff}[A] = \frac{\tilde{g}^2}{2\pi}\left(A_\mu A^\mu - A_\mu \frac{\partial^\mu \partial^\nu}{\partial^2} A_\nu\right).$$

- Amazingly, one finds that photon has become massive

- In regime of linear response, all the electromagnetic response functions can be obtained from above action

- Electric current induced in the system due to presence of an external electromagnetic field is given by:

$$\langle j^\mu(x)\rangle_{ind} = \frac{\delta}{\delta A_\mu(x)}\mathcal{S}_{eff}$$

- When the system is subjected to a constant external magnetic field, in Coulomb gauge $\nabla \cdot \vec{A} = 0$, one finds:

$$\langle \vec{j}(x)\rangle_{ind} = -\frac{\tilde{g}^2}{\pi}\vec{A}(x),$$

  which is the celebrated London equation

- Further, when external electric field is applied such that $\vec{A} = 0$, one obtains:

$$\langle \vec{j}(\omega)\rangle_{ind} = \frac{\tilde{g}^2}{\pi\omega}\vec{E}(\omega)$$

- This means that conductivity $\sigma(\omega) \sim \frac{1}{\omega}$, and blows up at $\omega = 0$

- Hence, we see that our theory exhibits both

  1. Meissner Effect

  2. Infinite DC conductivity

- It is tempting to conclude that our theory describes superconductivity

- But, what about flux quantisation ?

- Note, that transformation $A_\mu \to A_\mu + \partial_\mu \Lambda$, does not leave mixed Chern-Simons term ($\epsilon^{\mu\nu\lambda} A_\mu \partial_\nu a_\lambda$) invariant

- However, the action remains invariant, assuming that fields decay sufficiently quickly as one approaches the boundary

- Consider above theory at finite temperature using imaginary time formulation

- One does a Wick rotation from Minkowskii space-time to Euclidean space-time: $t \to -i\tau$, where $\tau \in [0, \beta]$ is a compact variable and $\beta = \frac{1}{T}$

- In this Euclidean space-time, bosonic (fermionic) fields are required to satisfy periodic (anti-periodic) boundary conditions:

$$F(\vec{x}, 0) = \pm F(\vec{x}, \beta)$$

- These conditions alongwith analyticity, imposes restriction on choice of gauge function $\lambda(x)$: $\lambda(\beta) = \lambda(0) + 2\pi n$

- Vacuum functional now reads:

$$Z^{Euclid} = N \int \mathscr{D}[a_\mu] \, e^{-S_{CS}}, \text{ where}$$

$$S_{CS} = \int_0^\beta d\tau \int d^2x \, \frac{m}{\pi|m|} \epsilon^{\mu\nu\rho} A_\mu \partial_\nu a_\rho$$

- Variation of Chern-Simons action under these restricted gauge transformations (where $\lambda(\tau)$ only depends on τ) is given by:

$$\delta S_{CS} = \frac{2m}{|m|} n\Phi, \text{ where}$$
$$\Phi = \int d^2x \, \epsilon_{ij}\partial_i A_j,$$

is the magnetic flux

- Demanding invariance of vacuum functional requires: $\delta S_{CS} = 2\pi i N$, where N is an integer

- This implies that $\Phi = N\frac{\pi|m|}{gm}$. This, when appropriately scaled to SI units, reads:

$$\Phi = N\left(\frac{m}{|m|}\right)\frac{hc}{2g}$$

Hence we find that, magnetic flux is quantized in this model, with flux unit $\frac{hc}{2g}$, akin to that of a BCS superconductor

- Hence we find that, magnetic flux is quantized in this model, with flux unit $\frac{hc}{2g}$, akin to that of a BCS superconductor.

- So this forces us to conclude that, Graphene minimally coupled to a_μ field behaves like a superconductor.

- Interestingly, there is no BCS type fermion pairing in this theory !

- Further, it is well known in context of BCS theory, that superconductivity does not appear in any finite order of perturbative calculation & it occurs due to the phenomenon of spontaneous symmetry breaking.

- In above calculation, which is done perturbatively at one loop order, we find superconductivity.

- This only indicates that this type of superconductivity has an origin different than the conventional pairing based theories

- Origin of this superconductivity is topological, since it crucially depends on mixed Chern Simons term

- Above reminds one of Anyon Superconductivity, proposed to explain high T_c

- There is no symmetry breaking, then how is this kind of superconductivity is lost ?

- It can be shown that the effective Lagrangian can be written in the London form after a Hubbard-Stratonovich transformation:

$$\mathscr{L}_{eff} = 2\tilde{g}^2 \left(\partial_\mu \theta + \frac{2m}{\pi |m|} A_\mu \right)^2 ,$$

where $\theta(x,t)$ is an auxiliary field

- Note, that θ field transforms under a gauge transformation as:

$$\theta \to \theta - \frac{2m}{\pi |m|} \Lambda,$$

and hence it precisely mimics Nambu-Goldstone mode of BCS theory

- As argued by Weinberg*, existence of a mode that transforms like Nambu-Goldstone mode is sufficient for superconductivity
- Hence, in this light, occurence of superconductivity is not a surprise

*Weinberg S., 1986, Prog. Theor. Phys. Suppl. 86, 43

Vortices

- Above Lagrangian in absence of external electromagnetic field resembles the Lagrangian of 2D XY model in continuum limit

- It is well known that the latter shows a topological phase transition called Berezinskii-Kosterlitz-Thouless phase transition, wherein above a certain finite temperature, the system supports spontaneous occurrence of multivalued field configuration or vortices

- Since our Lagrangian is already in XY form, critical temperature can be readily found to be

$$T_{BKT} = 2\pi\tilde{g}^2.$$

- Inorder to study effect of vortices, one decomposes $\theta = \theta_r + \theta_v$

- Integrating out the regular part to arrive at the following effective action, depicting interaction of vortices and external electromagnetic field:

$$\mathscr{L}_{eff} = -2\tilde{g}^2 J^\mu \frac{1}{\partial^2} J_\mu - \frac{4m\tilde{g}^2}{\pi|m|}\left(\tilde{F}_\mu \frac{1}{\partial^2} J^\mu + J^\mu \frac{1}{\partial^2}\tilde{F}_\mu\right) - \frac{16\tilde{g}^2}{\pi^2} F^{\mu\nu}\frac{1}{\partial^2}F_{\mu\nu},$$

where vortex current $J_\mu = \epsilon_{\mu\nu\lambda}\partial^\nu\partial^\lambda\theta_v$ and dual $\tilde{F}_\mu = \epsilon_{\mu\nu\lambda}F^{\nu\lambda}$

- Integrating out vortex current would give rise to a term exactly the same as the third one but with opposite sign

- Hence, cancellation of the last term take place and so presence of vortices would destroy superconductivity

- So, superconducting-to-normal phase transition is an infinite order topological one

Edge theory

- We have assumed that Graphene sheet is of infinite extent, and fields fall of sufficiently quickly, so that surface terms give negligible contribution

- In reality, one encounters finite Graphene samples with a boundary

- Owing to its hexagonal tiling, Graphene can exhibit boundary of two kinds: Arm chair and Zig-zag

- It is known that, the latter exhibits localised electronic egde states, whereas the former does not

- Hence, in case of arm-chair egdes, fermions present in bulk can freely interact with the ones living on boundary and vice versa

- Bulk effective Lagrangian is

$$\mathscr{L}_{eff}[a, A] = -\frac{1}{4\tilde{g}^2} f_{\mu\nu} f^{\mu\nu} - \frac{m}{\pi|m|} \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu a_\lambda \qquad (6)$$

- As was observed earlier, the last term in above Lagrangian is not invariant under local gauge transformation: $A_\mu \rightarrow A_\mu + \partial_\mu \Lambda$, where Λ is some regular function of x. As a result, the change in action is given by:

$$\delta S_{CS} = \left(\frac{sgn(m)}{2\pi}\right) \int d^3x \; \epsilon^{\mu\nu\rho} \partial_\mu \left(\Lambda \, f_{\nu\rho}\right) .$$

- Above volume integral can be converted to a surface integral, defined on closed Graphene boundary, to give an action:

$$\delta S_{CS} = \left(\frac{sgn(m)}{2\pi}\right) \int_B d^2x \; \epsilon^{\mu\nu} \Lambda \, f_{\mu\nu}$$

- This term, as it stands, is not gauge invariant, and is defined on Graphene boundary, which encloses the bulk

- We demand that the full theory *i.e.*, Bulk + Boundary must be Gauge Invariant

- So, there must exist a corresponding gauge theory living on the boundary, defined such that it contributes a gauge noninvariant term of exactly opposite character and hence cancels the one written above

- The simplest term, living on boundary, that obeys above condition is:

$$S_B = \frac{-sgn(m)}{2\pi} \int_B d^2x \, \theta \epsilon^{\mu\nu} f_{\mu\nu},$$

where $\theta(x, t)$ is Stückelberg field, which transforms like $\theta \rightarrow \theta + \Lambda$.

- Because of peculiar transformation property, a quadratic mass term for θ is not gauge invariant

- So with a gauge invariant kinetic term, the boundary action reads:

$$S_B = \int_B d^2x \left[c\,(\partial_\mu \theta - A_\mu)^2 - \frac{sgn(m)}{2\pi} \theta \epsilon^{\mu\nu} f_{\mu\nu} \right].$$

  and in a gauge theory framework like this, θ field remains massless

- Using the same idea for gauge invariance with respect to a_μ field, one gets net action describing gapless surface modes:

$$S_B = \int_B d^2x \left[c\,(\partial_\mu \theta - a_\mu - A_\mu)^2 - \frac{sgn(m)}{2\pi} \theta \epsilon^{\mu\nu} (f_{\mu\nu} + F_{\mu\nu}) \right] \tag{7}$$

- Firstly, note that the action for θ is in manifest London form, and hence is indicative of non-dissipative transport on the boundary

- Secondly, θ field is electromagnetically charged, and hence boundary supports dissipationless electric current, or in other words boundary is superconducting

- Thirdly, the coupling of θ field, with that of a and A field is anomalous, as a result chiral current in this quantum theory is no longer conserved

- This ultimately results in chirality of these surface modes

Summary

- We show that, an Abelian gauge field which couples to difference of valley fermion currents in gapped Graphene, gives rise to a special type of superconductivity

- This mechanism is fundamentally different then the pairing based ones, where spontaneous symmetry breaking occurs

- Since, the vacuum of our theory is not a condensate, 2Δ or Amplitude collective mode is absent

- It remains to be seen however, whether the gauge field assumed in above discussion is realizable in Graphene or not.

Summary

- It is well known that, optical phonons in Graphene couple to Dirac fermions as vector fields, albeit with different sign for both valley fermions, very much like the a_μ gauge field *

- However, phonon vector field is massive(gapped) and hence differs fundamentally from the one that is required

- Open Questions:

  1. What carries current in this theory ?

  2. Do plasmons exist in this theory ?

*K. Sasaki and R. Saito, Prog. Theor. Phys. Suppl. **176**, 253 (2008)

Thank You for your attention