

Class test on Assignment 1

30 January 2012, Maximum Marks: 20

1 For subsets $A, B \subseteq \mathbb{R}$ such that B is measurable, check if the following statement is true:

$$m^*(A \cup B) + m^*(A \cap B) = m^*(A) + m^*(B).$$

If yes, then prove the statement. If not, then provide a counterexample. **[5 marks]**

2 For a subset E of \mathbb{R} , show that the following statements are equivalent [**15 marks**]:

(a) E is measurable.

(b) Given $\epsilon > 0$, we can find a closed subset F of \mathbb{R} such that $F \subseteq E$ and $m^*(E - F) < \epsilon$.

(c) There exists a set G such that G is a countable union of closed sets, $G \subseteq E$ and

$$m^*(E - G) = 0.$$

2 (Continue on this page, if required)