HALDOR TOPSØE
SEMICONDUCTOR DIVISION

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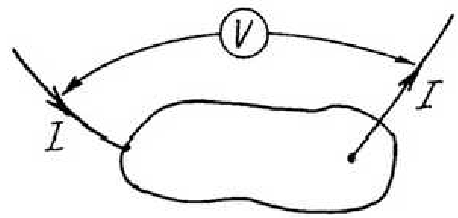
Geometric factors in four point resistivity measurement.

Bulletin No. 472-13.

A.1) INTRODUCTION

The electrical resistance R of an object is defined as the ratio between the voltage V that causes the current I to flow through the object, and the current I itself:

Figure 1:



$$R = \frac{V}{I}$$

Two-point measurement.

So, the direct way to measuring resistance, is to measure these two magnitudes: Voltage and current.

When the resistivity ρ of a sample [1] is wanted, the geometrical dimensions of the sample must also be known. The resistivity is given by:

$$\rho = G \frac{V}{I} \qquad (1)$$

where: I is the current passed through the sample,
V is a measured voltage,
G is a correction factor dependent on sample shape and dimensions, and the arrangement of electrical contacts.

When performing direct current or low frequency resistance measurements, one has to make electrical contacts to the sample. By making two contacts, two-point resistivity measurements can be performed (figure 1).

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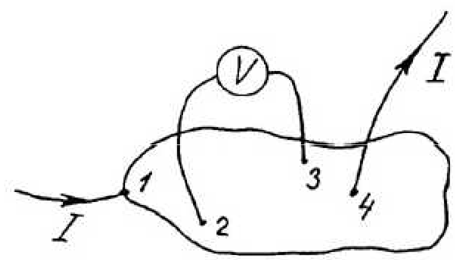
1) Only homogeneous samples are considered.

As the resistance of the contacts becomes comparable to sample resistance, it becomes necessary to separate the contacts conducting the current from the contacts between which the voltage is measured, in order not to measure contact resistance together with sample resistance. This is the rule when measuring resistivity of semiconducting materials, and also when measuring very small resistances of good conductors.

The separation of current and voltage contacts is fulfilled in the four-point method indicated in figure 2.

Figure 2:



Four-point measurement.

The resistivity is given by:

$$\rho = G \frac{V_{2-3}}{I_{1-4}} \qquad (2)$$

where

V_{2-3} is the voltage between contacts 2 and 3,

I_{1-4} is the current through contacts 1 and 4.

From the reciprocal theorem of electromagnetic theory we conclude that if instead we pass the current I through contacts 2 and 3, we shall get between the contacts 1 and 4 the same voltage V as before between 2 and 3.

Thus we can extend equation (2) :

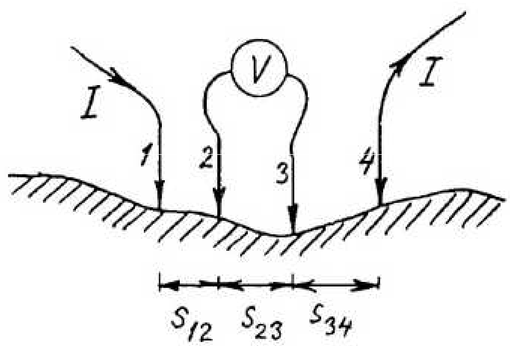
$$\rho = G \frac{V_{2-3}}{I_{1-4}} = G \frac{V_{1-4}}{I_{2-3}} \qquad (3)$$

In words: We can deliberately exchange the current contact pair and the voltage contact pair, using the same correction factor G.

A.2) Four Points in a Line.

In semiconductor resistivity measurements the most common arrangement of the four points is in a line. This arrangement will be dealt with exclusively in the following. Normally, the outer contacts conduct the current and voltage is measured between the inner points as shown in figure 3.

Figure 3:



In practice, the contacts are produced by four probes with parallel movements as indicated. The resistivity is :

$$\rho = G \frac{V}{I},$$

where the correction factor G is a function of sample geometry, the position of the probes on the sample, and the spacings between probes. Normally, equal probe spacing $s_{12} = s_{23} = s_{34} = s$ is aimed at.

The very important task of calculating the correction factor G for various shapes and dimensions has been performed during the last decade for most practical applications. The results of highest interest for our application are compiled on the following pages. Where no other statement is made, it is assumed that the four points are in a line and are equi-distant. As above, we will consider homogeneous samples only.

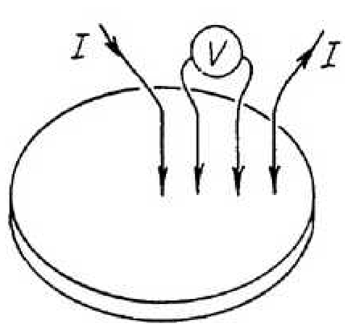
A.3) Symmetry Considerations.

If the sample has a symmetry plane through the four points of measurement , the electric field in the sample will be symmetrical with respect to this plane, and no current will pass the plane. Consequently, we can remove one half of the sample without affecting the electric field in the other half part. If the sample contains more symmetry planes through the four points, any part between two such planes can be removed without effecting the electric field in the remaining part.

This can be utilized in deriving geometric factors for configurations with similar symmetry.

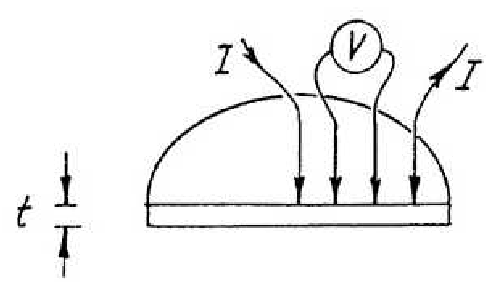
As an example from the way of reasoning we consider a circular slice with the probes on a diameter. The resistivity is given by:

a)



$$\rho = G \frac{V}{I}$$

b)

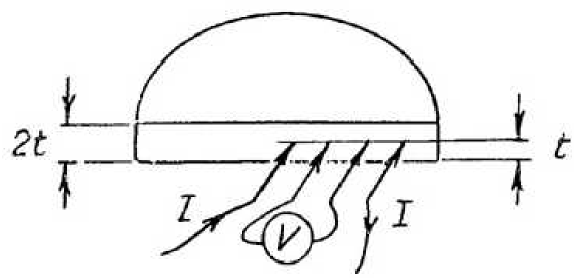


If we bisect the slice as shown, twice the voltage will develop between the inner probes for the same current.

So,

$$\rho = \frac{G}{2} \cdot \frac{V}{I}$$

c)



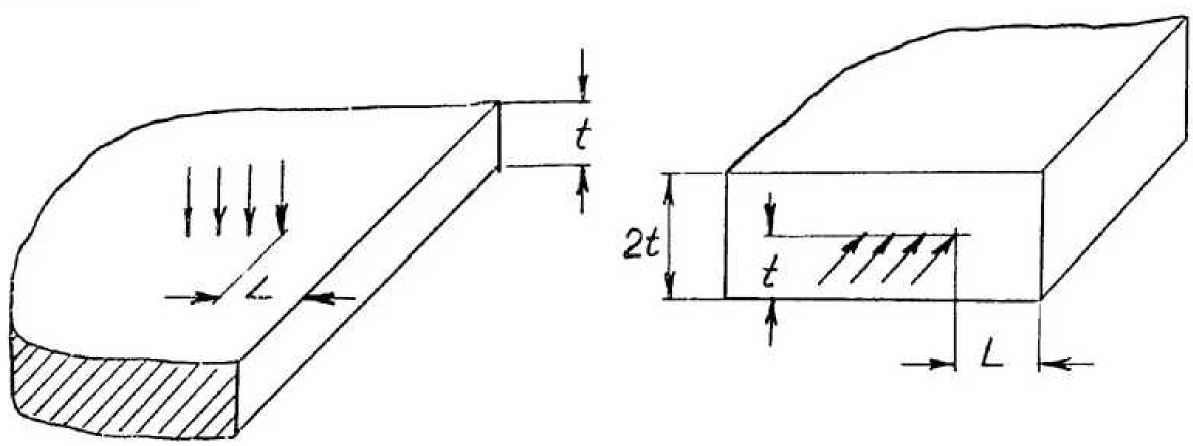
If we now double the slice as shown in figure c), the voltage to current ratio is again like that in fig. a), and:

$$\rho = G \frac{V}{I}$$

So, the same geometric factor applies for the two situations a) and c), both of practical interest.

Another example of configurations having the same geometric factor is shown below (f), (g) :

Example 2:

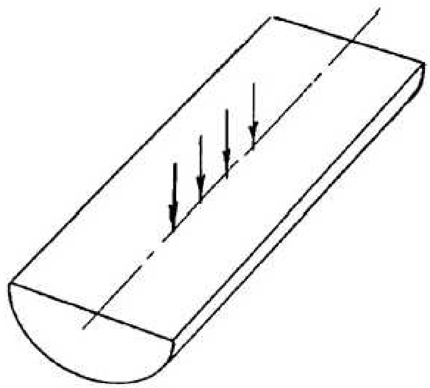


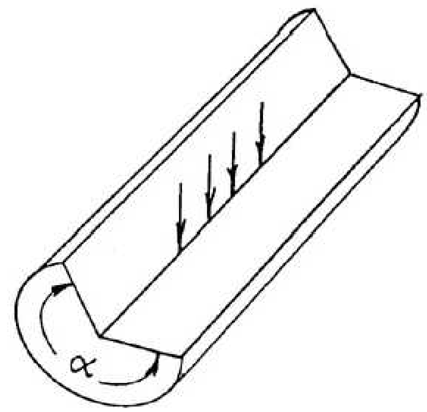
Semi-infinite slice
Thickness t.

Quarter-infinite slice
Thickness 2t

For the measurement along the axis of cylinder segments we get the relation :

Example 3:



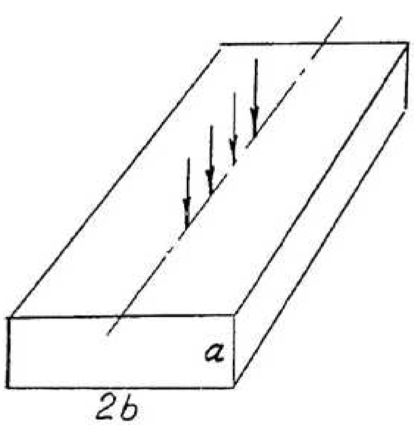


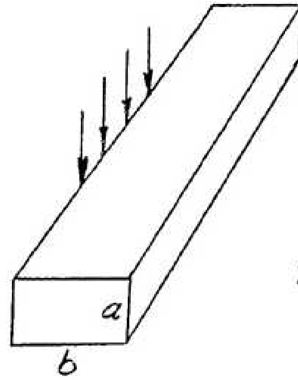
$$\rho = G\,\frac{V}{I}$$

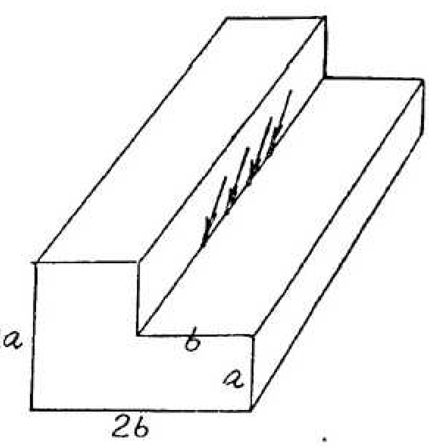
$$\rho = \frac{\alpha^\circ}{180^\circ}\,G\,\frac{V}{I}$$

A fourth example on box-shaped samples:

Example 4:







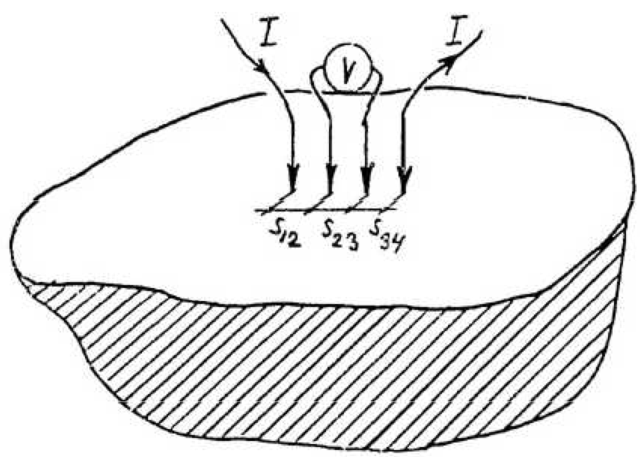
$$\rho = G\,\frac{V}{I}$$

$$\rho = \frac{1}{2}G\frac{V}{I}$$

$$\rho = \frac{3}{2}\,G\,\frac{V}{I}$$

B. SEMI-INFINITE VOLUME.

Figure 4:



Semi-infinite volume of material.

We call a sample semi-infinite, if it extends to infinity in all directions below a plane, which is the plane on which the four probes are located. The resistivity of the sample is given by (a):

$$\rho = G \frac{V}{I},$$

$$\text{where } G = \frac{2\pi}{\frac{1}{s_{12}} + \frac{1}{s_{34}} - \frac{1}{s_{12} + s_{23}} - \frac{1}{s_{23} + s_{34}}}\ ; \qquad (4)$$

B.1) Equidistant Probes.

In this case (4) reduces to:

$$G = 2\pi s, \qquad \rho = 2\pi s \frac{V}{I}, \qquad (5)$$

where s is the probe distance.

A convenient practice is to make

$$s = \frac{1}{2\pi}\text{ cm} = 1{,}59 \text{ mm, whereby } \rho = \frac{V}{I} \text{ cm.}$$

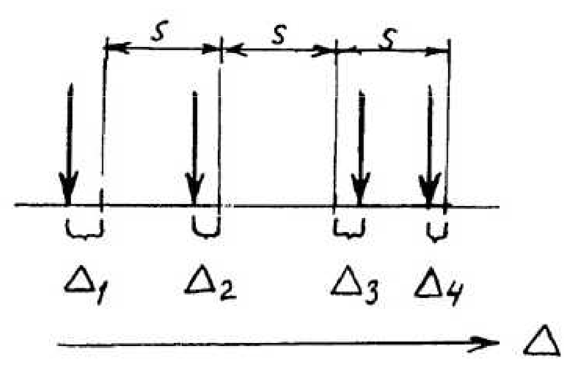
B.2) <u>Different Probe Distances (fig. 4).</u>

$$\rho = G \frac{V}{I}$$

$$\text{where } G = \frac{2\pi}{\frac{1}{s_{12}} + \frac{1}{s_{34}} - \frac{1}{s_{12} + s_{23}} - \frac{1}{s_{23} + s_{34}}} ; \qquad (6)$$

From equation (6) one can calculate the change in G caused by small probe displacements from their nominal locations (b).

<u>Figure 5:</u>



Probe displacements from equal distances.

For equi-distant probes (figure 5), the result is to a first approximation:

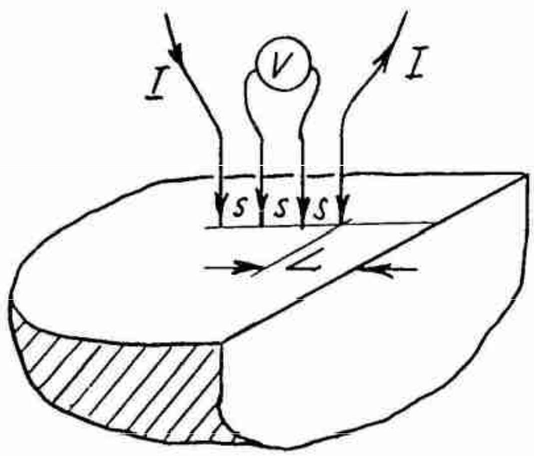
$$G = \frac{2\pi s}{1 + \frac{3.\Delta_1}{4\,s} - \frac{5.\Delta_2}{4\,s} + \frac{5.\Delta_3}{4\,s} - \frac{3.\Delta_4}{4\,s}} , \qquad (7)$$

We notice that to a first approximation, G is independent of probe displacement, if $\Delta_1 = \Delta_4$ and $\Delta_2 = \Delta_3$, which means that the distance between current probes mutually and between voltage probes mutually is fixed. A four-point probe head construction utilizing this fact, has been published. (c).

C. QUARTER-INFINITE VOLUME.

C.1) Probe Array Perpendicular to Edge.

Figure 14:



The resistivity of the sample is given by (a):

$$\rho = G \frac{V}{I},$$

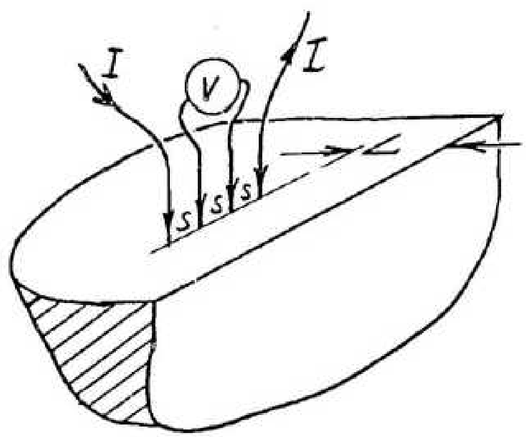
$G = 2\pi s \cdot D_1(\frac{L}{s})$, where

$$D_1 = \frac{1}{1 + \frac{s}{2L+s} - \frac{s}{2L + 2s} - \frac{s}{2L + 4s} + \frac{s}{2L + 5s}} \qquad (8)$$

$D_1(\frac{L}{s})$ is tabulated and plotted on page 12.

C.2) Probe Array Parallel to Edge.

Figure 15:



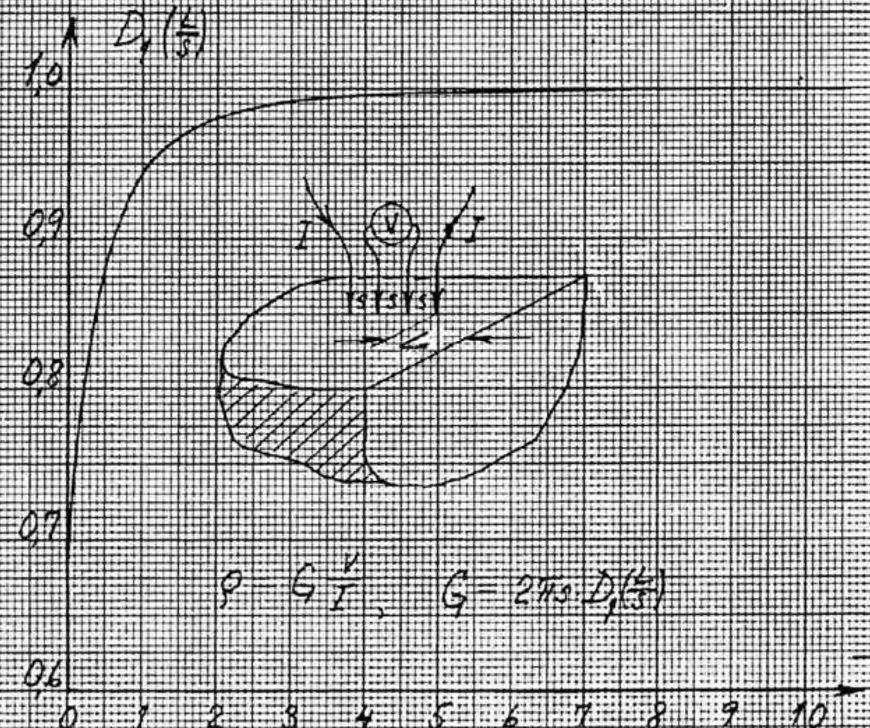
The resistivity is given by (a): $\rho = G\frac{V}{I}$,

$G = 2\pi s \cdot D_2(\frac{L}{s})$ when

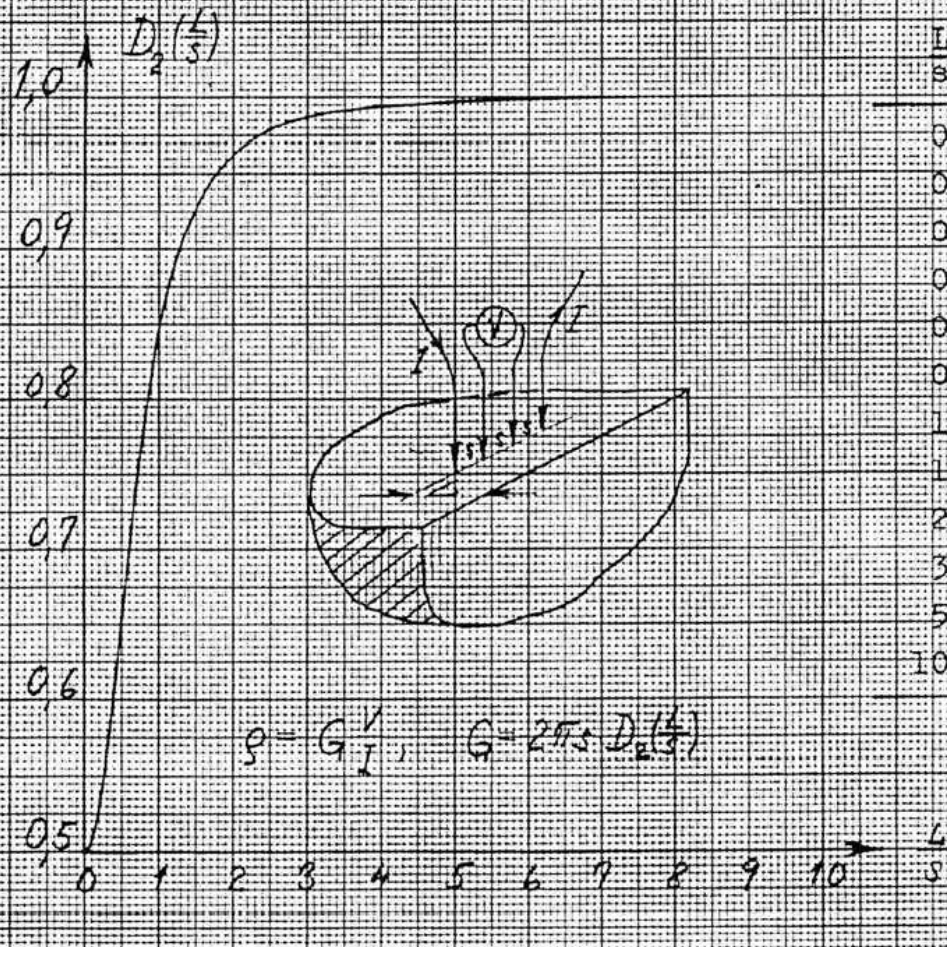
$$D_2 = \frac{1}{1 + \frac{2}{\sqrt{1 + (2L/s)^2}} - \frac{1}{\sqrt{1 + (\frac{L}{s})^2}}}; \qquad (9)$$

$D_2(\frac{L}{s})$ is tabulated and plotted on page 12.

Quarter-infinite volume.



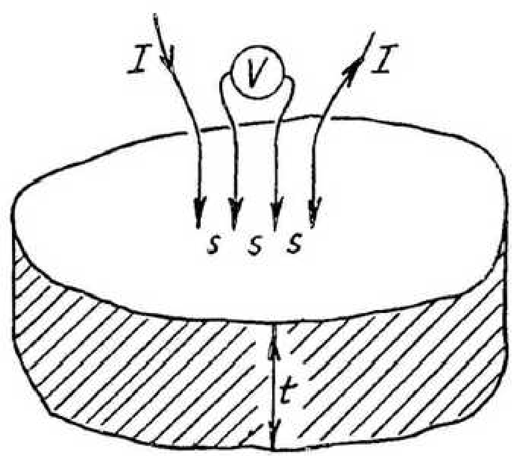
$\frac{L}{s}$	$D_1(\frac{L}{s})$
0	0.6897
0.1	0.7502
0.2	0.7965
0.5	0.8824
1	0.9438
2	0.9809
3	0.9915
5	0.9972
10	0.9995



$\frac{L}{s}$	$D_2(\frac{L}{s})$
0	0.5
0.1	0.5086
0.2	0.5329
0.3	0.5691
0.5	0.6580
0.7	0.7445
1	0.8422
1.4	0.9162
2	0.9635
3	0.9876
5	0.9971
10	0.9996

D. INFINITE PLANE SAMPLE OF FINITE THICKNESS.

D.1) Thick Sample.



The geometric factor was derived by Uhlir (f) (g). The resistivity ρ is given by :

$$\rho = G\frac{V}{I}, \qquad G = 2\pi s \cdot T_1(\frac{t}{s}), \tag{10}$$

where

$2\pi s$ is the geometric factor for a semi-infinite volume, see section A, and

$T_1(\frac{t}{s})$ is an additional correction to apply for the finite thickness t of the sample.

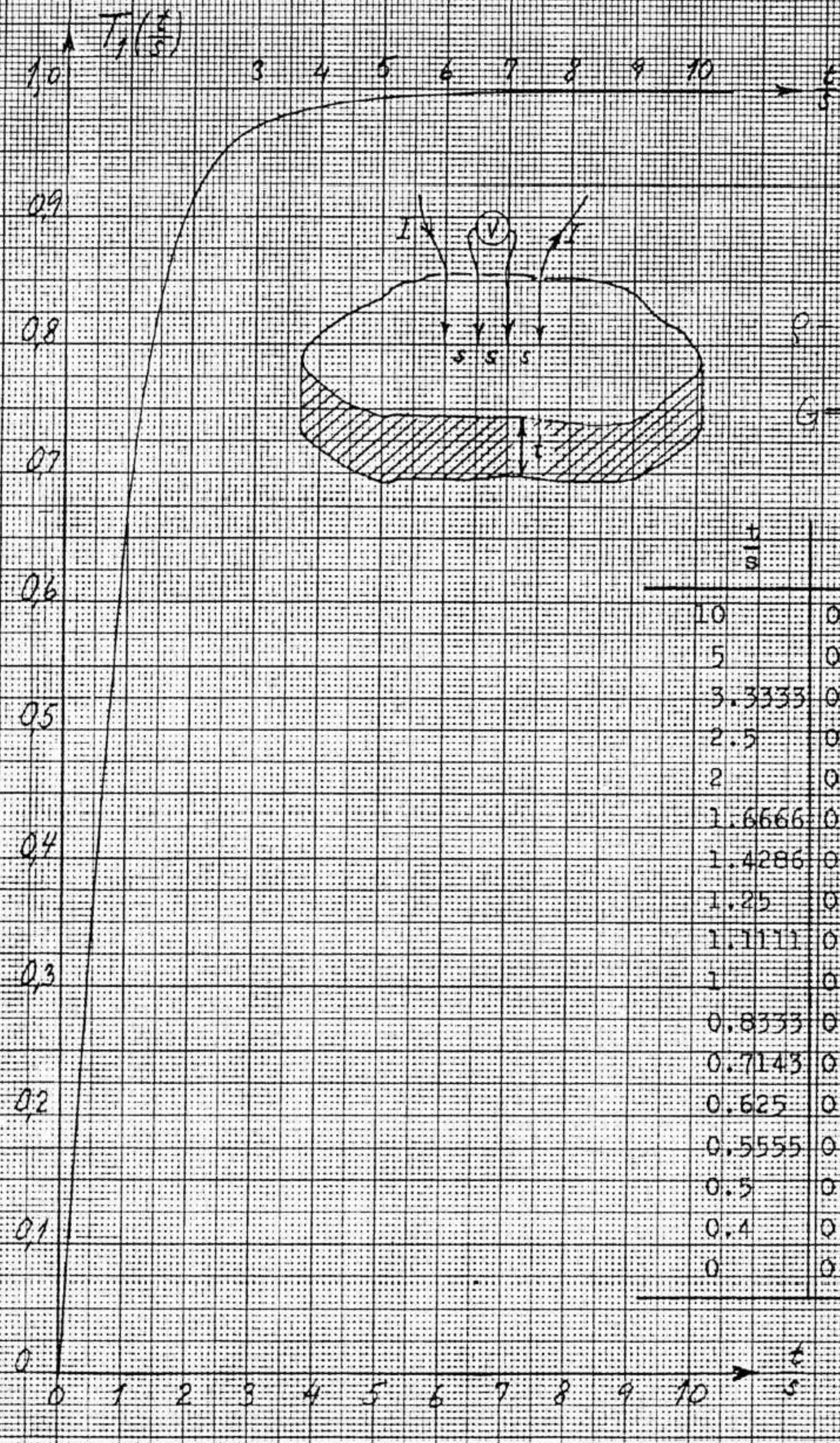
$T_1(\frac{t}{s}) \longrightarrow 1$ as $t \longrightarrow \infty$.

$T_1(\frac{t}{s})$ is tabulated and plotted on page 14.

Notice that when the thickness $t \geq 5s$, the geometric factor $2\pi s$ for a semi-infinite volume is correct within 0.7 %.

Infinite Plane Sample of Finite Thickness.

1) Thick Sample.

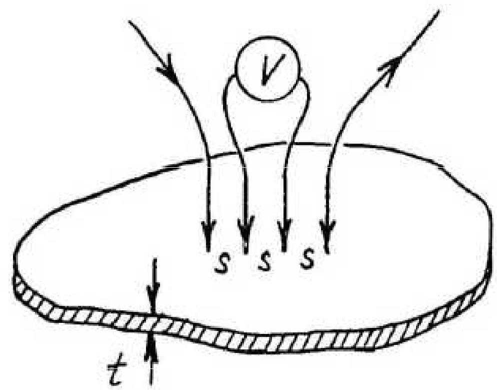


$$\rho = G\,\frac{V}{I}$$

$$G = 2\pi s\,T_1\left(\frac{t}{s}\right)$$

$\frac{t}{s}$	$T_1(\frac{t}{s})$
10	0.999
5	0.993
3.3333	0.978
2.5	0.951
2	0.914
1.6666	0.869
1.4286	0.818
1.25	0.766
1.1111	0.714
1	0.665
0.8333	0.577
0.7143	0.505
0.625	0.446
0.5555	0.399
0.5	0.360
0.4	0.288
0	0

D.2) Thin Sample.



When the sample is thin, it is more convenient to write the resistivity in the form given by Smits (e) :

$$\varrho = G\frac{V}{I}, \text{ where}$$

$$G = \frac{\pi}{\ln 2}\cdot t\cdot T_2(\frac{t}{s}) = 4.5324\cdot t\cdot T_2(\frac{t}{s}); \qquad (11)$$

$\frac{\pi}{\ln 2}\cdot t = 4.5324\cdot t$ is the geometric factor for an infinitely large, thin slice (e). Thin slice means $\frac{t}{s} \ll 1$ (in practice $\frac{t}{s} \leqq 0.5$).

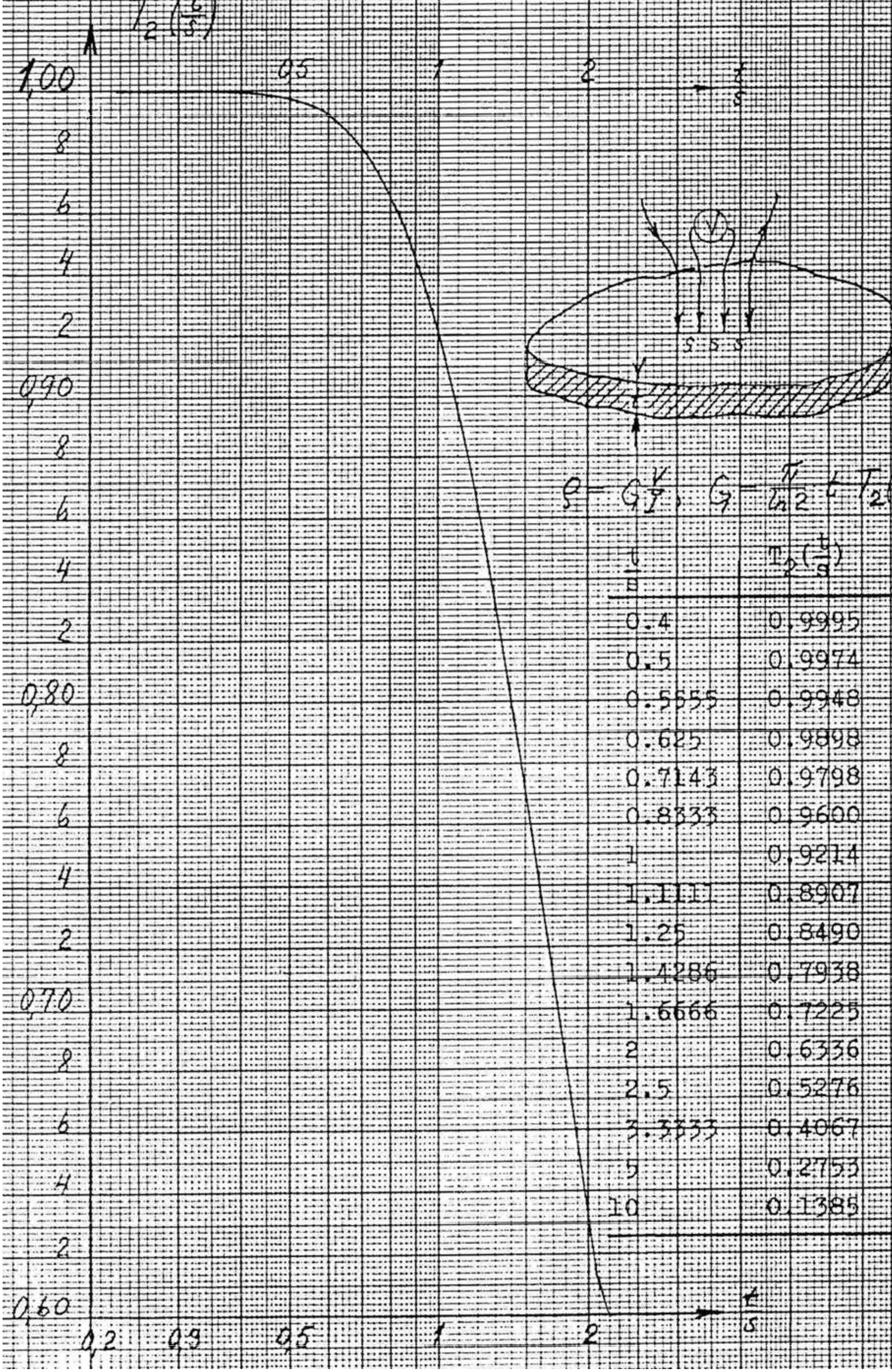
T_2 is an additional correction factor to apply when t is not much less than s.

$$T_2 \rightarrow 1 \text{ as } \frac{t}{s} \rightarrow 0.$$

$T_2(\frac{t}{s})$ is tabulated and plotted on the following page.

Infinite Plane Sample of Finite Thickness.

2) Thin Sample.

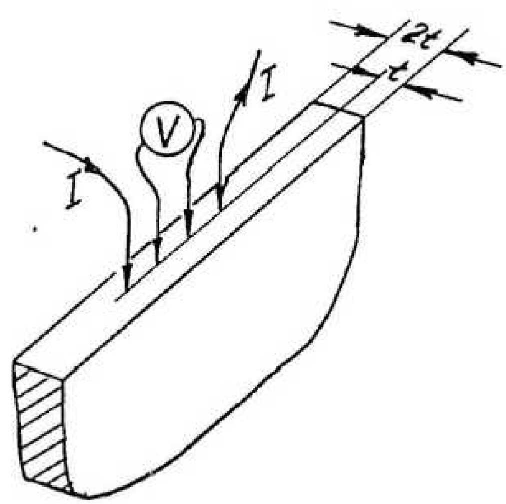


$$\frac{\rho}{s} = G\frac{V}{I}, \quad G = \frac{\pi}{\ln 2}\, t\, T_2\left(\frac{t}{s}\right)$$

$\frac{t}{s}$	$T_2(\frac{t}{s})$
0.4	0.9995
0.5	0.9974
0.5555	0.9948
0.625	0.9898
0.7143	0.9798
0.8333	0.9600
1	0.9214
1.1111	0.8907
1.25	0.8490
1.4286	0.7938
1.6666	0.7225
2	0.6336
2.5	0.5276
3.3333	0.4067
5	0.2753
10	0.1385

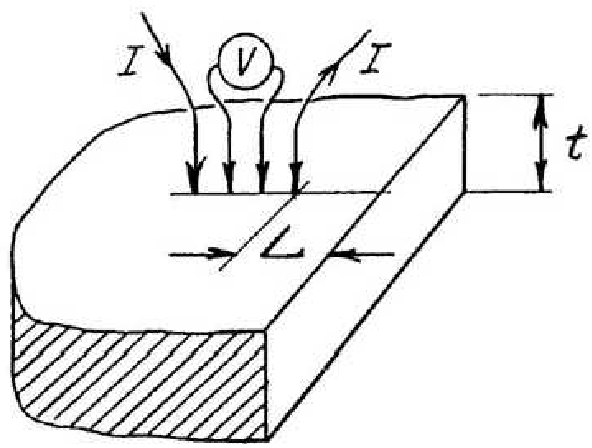
E. SEMI-INFINITE PLANE SAMPLE OF FINITE THICKNESS.

E.1) Probe Array on the Edge.



In example 2, section A.3, we showed that this configuration has the same geometric factor as has an infinite slice of thickness t, which is treated in section D.1 (thick sample) and section D.2 (thin sample).

E.2) Probe Array Perpendicular to Edge, Thick Sample.



The geometric factor was calculated by Uhlir (f)(g) The result is presented here in the following way :

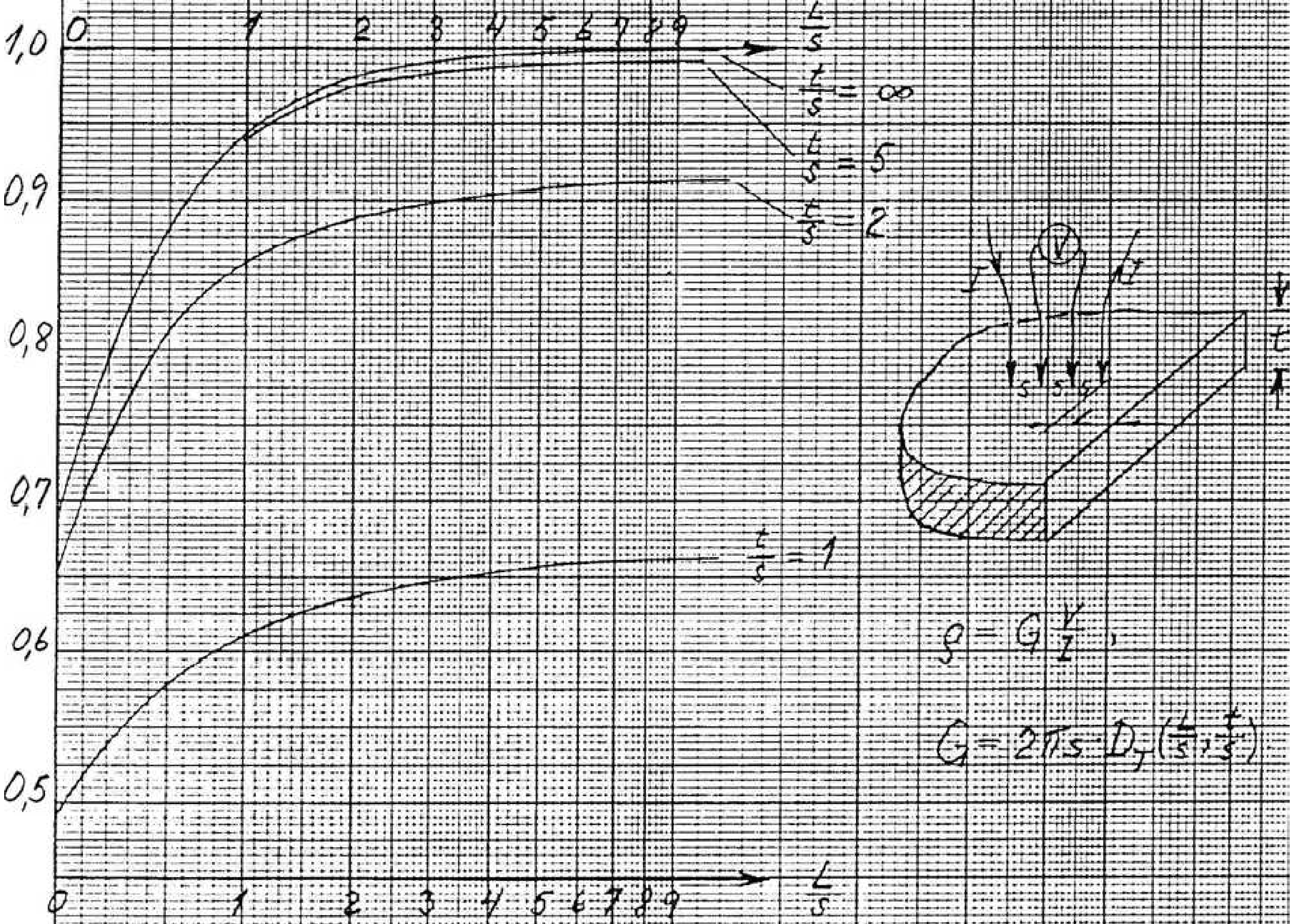
$$\rho = G\frac{V}{I}, \qquad G = 2\pi s \cdot D_T\left(\frac{L}{s},\frac{t}{s}\right) \tag{12}$$

where

$2\pi s$ is the geometric factor for a semi-infinite volume, and

$D_T(\frac{L}{s},\frac{t}{s})$ is tabulated and plotted on page 19 .

$D_T\left(\frac{L}{s}, \frac{t}{s}\right)$ Thick, semi-infinite plane sample.

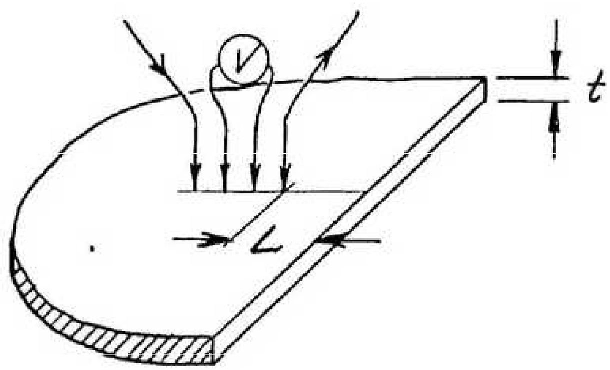


$$\rho = G\frac{V}{I},$$

$$G = 2\pi s\, D_T\left(\frac{L}{s}, \frac{t}{s}\right)$$

$D_T\left(\frac{L}{s}, \frac{t}{s}\right)$	L/s								
t/s	0	0,1	0,2	0,5	1	2	5	10	∞
	0,6897	0,7502	0,7965	0,8824	0,9438	0,9809	0,9972	0,9995	1
10	0,6896	0,7501	0,7964	0,8822	0,9437	0,9806	0,9965	0,9985	0,9991
5	0,6888	0,7490	0,7950	0,8800	0,9401	0,9751	0,9894	0,9917	0,9930
2	0,6542	0,7061	0,7421	0,8125	0,8585	0,8879	0,9067	0,9118	0,9142
1	0,4918	0,5193	0,5398	0,5782	0,6105	0,6373	0,6568	0,6622	0,6647
0,5	0,2686	0,2843	0,2900	0,3100	0,3282	0,3438	0,3551	0,3583	0,3597
0,2	0,1077	0,1124	0,1162	0,1243	0,1316	0,1379	0,1424	0,1437	0,1443
0,1	0,0539	0,0562	0,0581	0,0621	0,0658	0,0689	0,0712	0,0718	0,0721

E.3) Probe Array Perpendicular to Edge, Thin Sample.



When the sample is thin, it is convenient to express the resistivity as follows :

$$\rho = G\frac{V}{I}, \qquad G = \frac{\pi}{\ln 2}\cdot t\cdot D_3(\tfrac{L}{s})\cdot F_2(\tfrac{t}{s},\tfrac{L}{s}) \qquad (13)$$

where

$\frac{\pi}{\ln 2}\cdot t = 4.5324\cdot t$ is the geometric factor for an infinite slice of thickness $t \ll s$,

$$D_3(\tfrac{L}{s}) = \frac{1}{1 + \frac{1}{2\ln 2}\ln\frac{(\frac{L}{s}+2)\,(\frac{L}{s}+1)}{(\frac{L}{s}+\frac{5}{2})\,(\frac{L}{s}+\frac{1}{2})}} \qquad (14)$$

is the additional correction to apply when measuring at a distance L from the edge.

$D_3(\frac{L}{s}) \longrightarrow 1$ as $\frac{L}{s} \rightarrow \infty$

$F_2(\frac{t}{s},\frac{L}{s})$ deviates from unity when the thickness t of the slice becomes comparable to the probe distance s.

The expression for $D_3(\frac{L}{s})$ was obtained from formula (21) in section I.2 for a circular slice when the probes are on a diameter, by letting the diameter go to infinity.

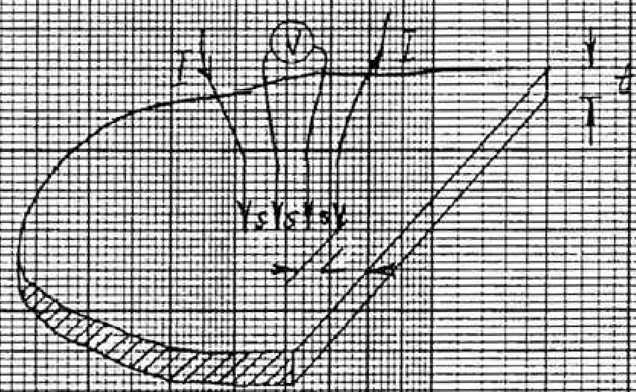
$D_3(\frac{L}{s})$ is tabulated and plotted on page 22.

The values of $F_2(\frac{t}{s},\frac{L}{s})$ were computed on the basis of Uhlir's work (f)(g), and the result presented on page 23.

$F_2(\frac{t}{s},\frac{L}{s})$

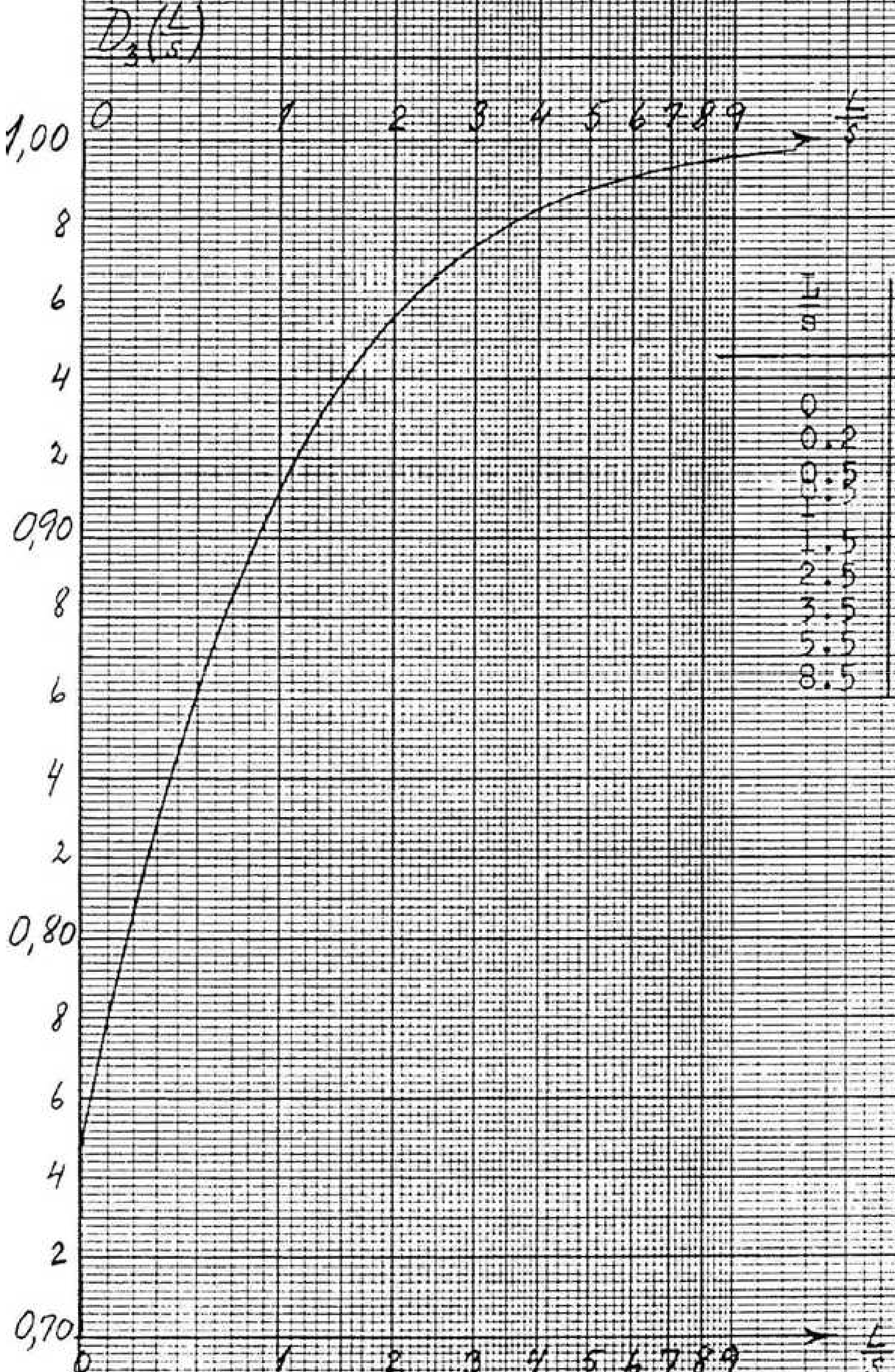
t/s \ $\frac{L}{s}$ =	0	0.1	0.2	0.5	1	2	5	10	∞
0	1	1	1	1	1	1	1	1	1
0.5	0.9970	0.9977	0.9978	0.9977	0.9976	0.9975	0.9974	0.9974	0.9974
0.5555									0.9948
0.6250									0.9898
0.7143									0.9798
0.8333									0.9600
1	0.9129	0.9238	0.9287	0.9306	0.9279	0.9247	0.9224	0.9217	0.9214
1.1111									0.8904
1.25									0.8490
1.4286									0.7938
1.6666									0.7225
2	0.6072	0.6280	0.6384	0.6539	0.6524	0.6441	0.6366	0.6346	0.6336

Thin, semi-infinite plane sample.



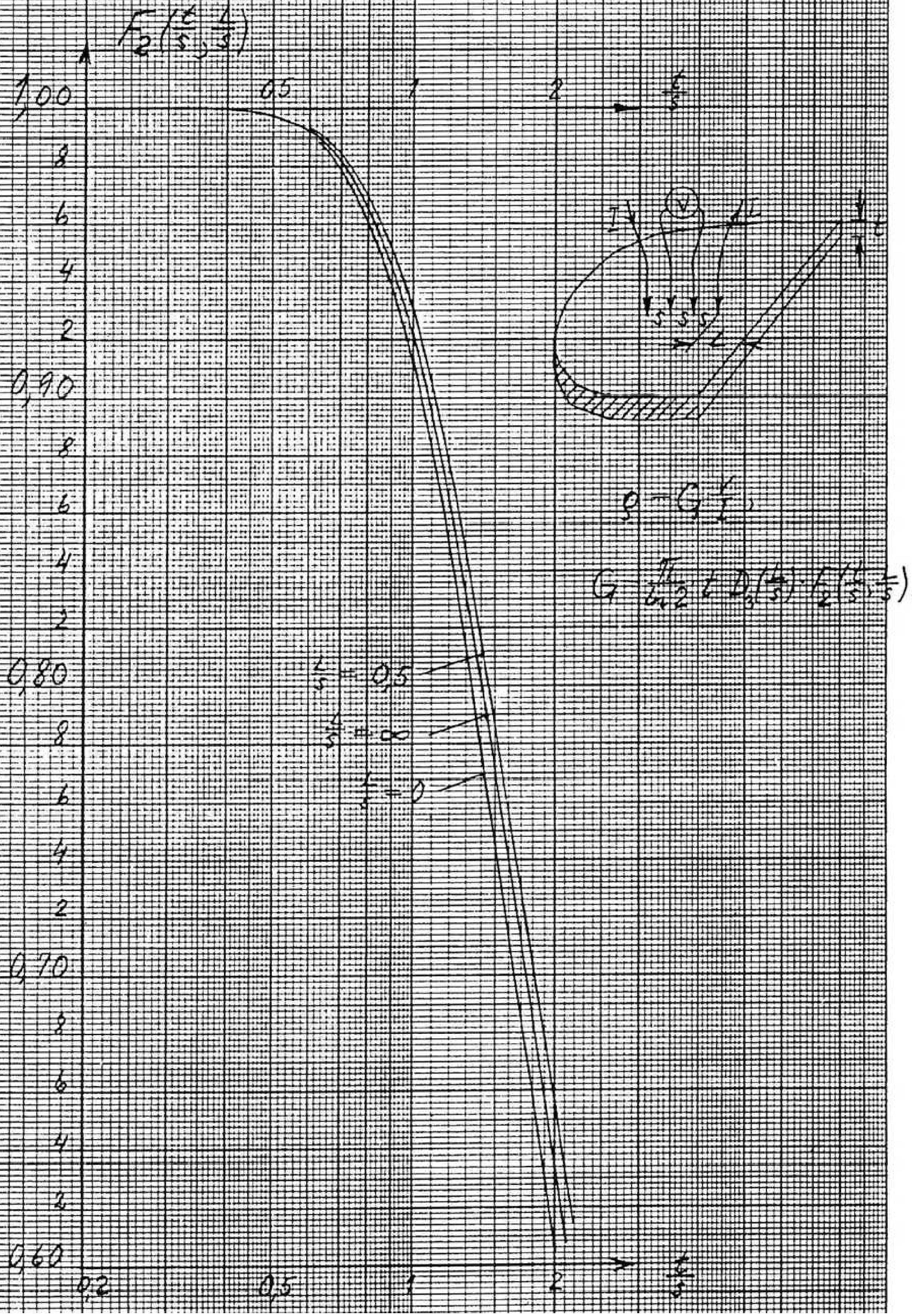
$$\rho = G \frac{V}{I},$$

$$G = \frac{\pi}{\ln 2} \cdot t \cdot D_3\left(\frac{L}{s}\right) \cdot F_2\left(\frac{t}{s}, \frac{L}{s}\right).$$

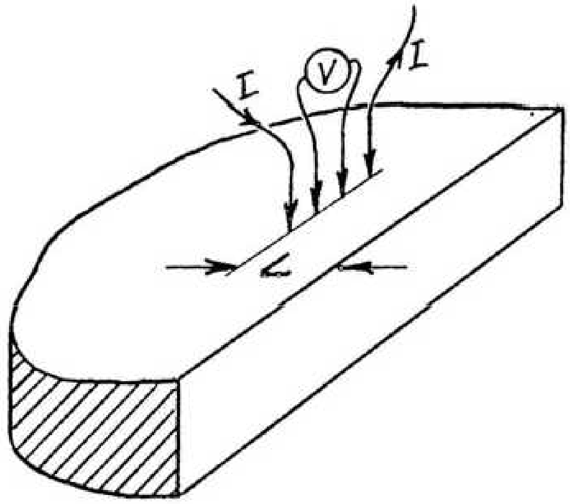


$\frac{L}{s}$	$D_3(\frac{L}{s})$
0	0.7468
0.2	0.8058
0.5	0.8614
1	0.9121
1.5	0.9393
2.5	0.9660
3.5	0.9783
5.5	0.9889
8.5	0.9946

Thin, semi-infinite plane sample.



E.4) Probe Array Parallel to Edge, Thick Sample.



The geometric factor was calculated by Uhlir (f) (g). The results are presented here in a somewhat different form.

The resistivity is given by the equations:

$$\rho = G\,\frac{V}{I}, \qquad G = 2\pi s \cdot D_2\left(\frac{L}{s}\right) \cdot F_3\left(\frac{t}{s},\frac{L}{s}\right), \qquad (15)$$

where

$2\pi s$ is the geometric factor for a semi-infinite volume.

$$2\pi s \cdot D_2\left(\frac{L}{s}\right) = \frac{2\pi s}{1 + \frac{2}{\sqrt{1 + (2L/s)^2}} - \frac{1}{\sqrt{1 + (L/s)^2}}}$$

is the geometric factor to apply when measuring on a quarter-infinite volume with the probe array parrallel to the edge, see section C.2. $D_2\left(\frac{L}{s}\right) \rightarrow 1$ as $L/s \rightarrow \infty$ and $D_2(0) = \frac{1}{2}$.

$F_3\left(\frac{t}{s},\frac{L}{s}\right)$ is the additional correction because of the finite thickness t of the sample.

As $L/s \rightarrow \infty$, F_3 approaches the factor $T_1(t/s)$ for an infinite plane sample, see section D.1. Furthermore, it is seen from symmetry considerations (see section A.3), that

$$F_3\left(\frac{t}{s}, 0\right) = F_3\left(\frac{t}{s},\infty\right) = T_1(t/s).$$

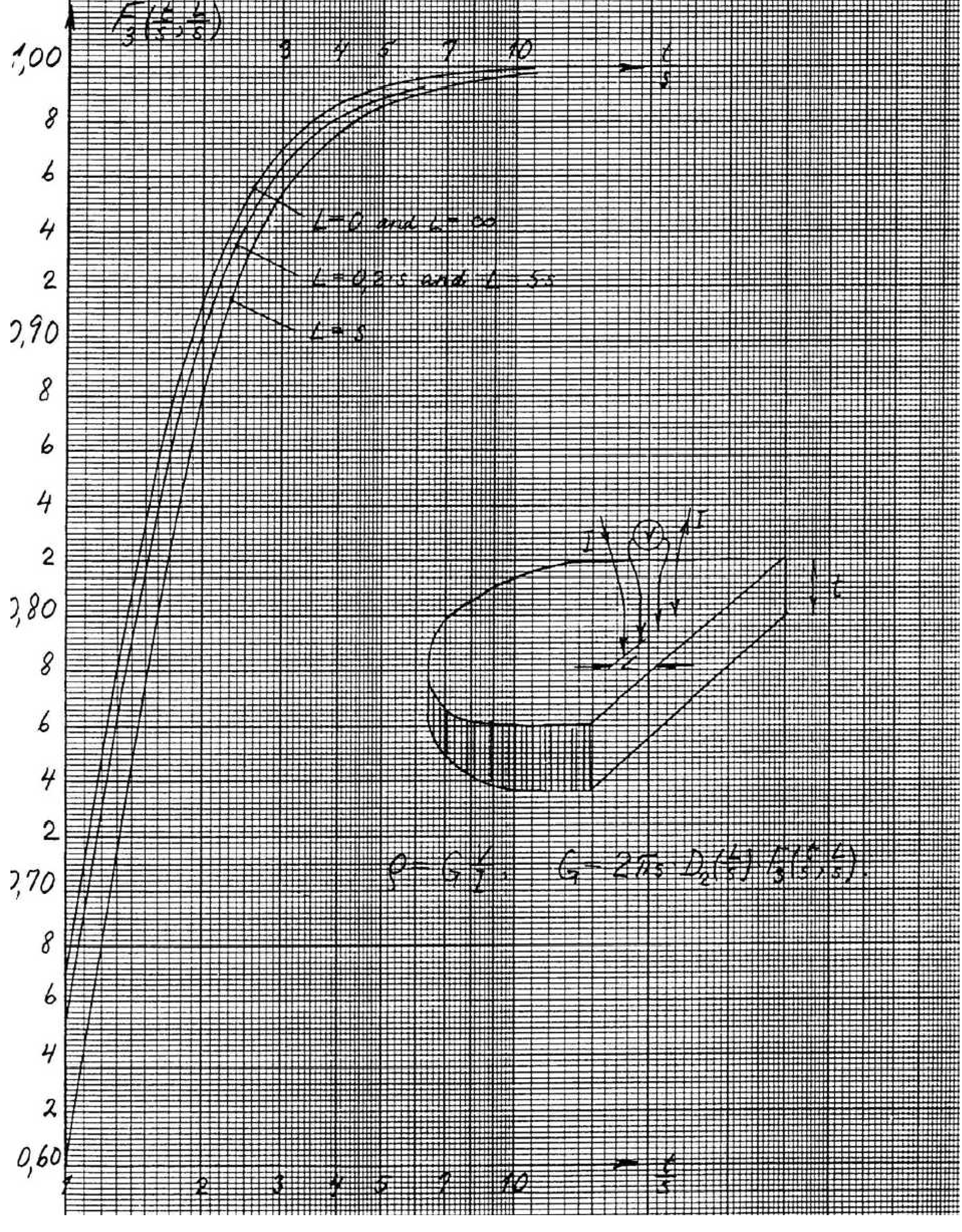
In the interval $0<(\frac{L}{s})<\infty$, $F_3(\frac{t}{s}, \frac{L}{s})$ differs from $T_1(\frac{t}{s})$ for an infinite plane sample.

$F_3(\frac{t}{s}. \frac{L}{s})$ is tabulated below and shown at page 26. The deviation from $T_1(t/s)$ is greatest, when $L \cong s$.

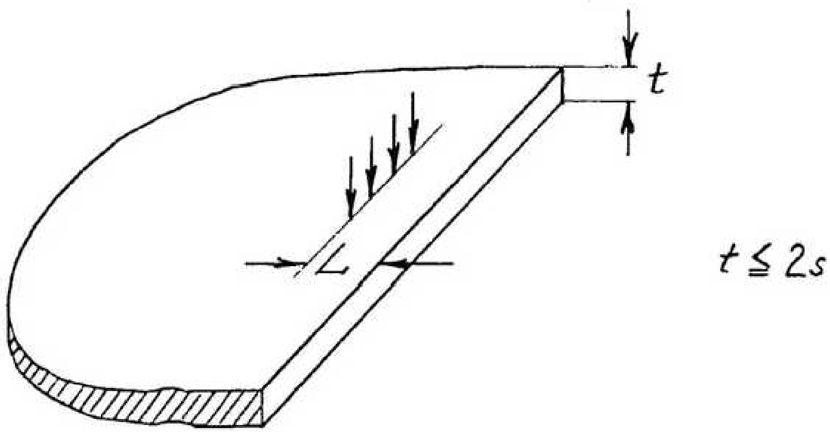
$F_3(\frac{t}{s}, \frac{L}{s})$ / L/s

t/s	0	0,1	0,2	0,5	1	2	5	10	∞
∞	1	1	1	1	1	1	1	1	1
10	0,9991	0,998	0,998	0,998	0,997	0,998	0,999	0,999	0,9991
5	0,9930	0,99	0,99	0,99	0,99	0,987	0,989	0,992	0,9930
3,3333	0,9778								0,9778
2,5	0,9514								0,9514
2	0,9142	0,91	0,91	0,89	0,880	0,883	0,904	0,911	0,9142
1,6666	0,8687								0,8687
1,4286	0,8180								0,8180
1,25	0,7656								0,7656
1,1111	0,7139								0,7139
1	0,6647	0,662	0,654	0,620	0,600	0,623	0,654	0,662	0,6647

Thick, semi-infinite plane sample.



E.5) Probe Array Parallel to Edge, Thin Sample.



When the sample is thin, it is convenient to write the resistivity in the following way:

$$\rho = G\,\frac{V}{I}, \qquad G = \frac{\pi}{\ln 2}\cdot t\cdot D_4\left(\frac{L}{s}\right)\cdot F_4\left(\frac{t}{s},\frac{L}{s}\right) \qquad (16)$$

Where

$\frac{\pi}{\ln 2}\cdot t = 4{,}5324\cdot t$ is the geometric factor for an infinite slice of thickness $t \ll s$,

$$D_4\left(\frac{L}{s}\right) = \frac{1}{1+\frac{1}{2\ln 2}\cdot \ln\left[\frac{(L/s)^2+1}{(L/s)^2+\frac{1}{4}}\right]} \qquad (17)$$

is the additional correction to apply when measuring at a distance L from the straight edge on the semi-infinite slice of thickness $t \ll s$.

$F_4\left(\frac{t}{s},\frac{L}{s}\right)$ deviates from unity, when the thickness t of the slice is not much less than the probe distances s.

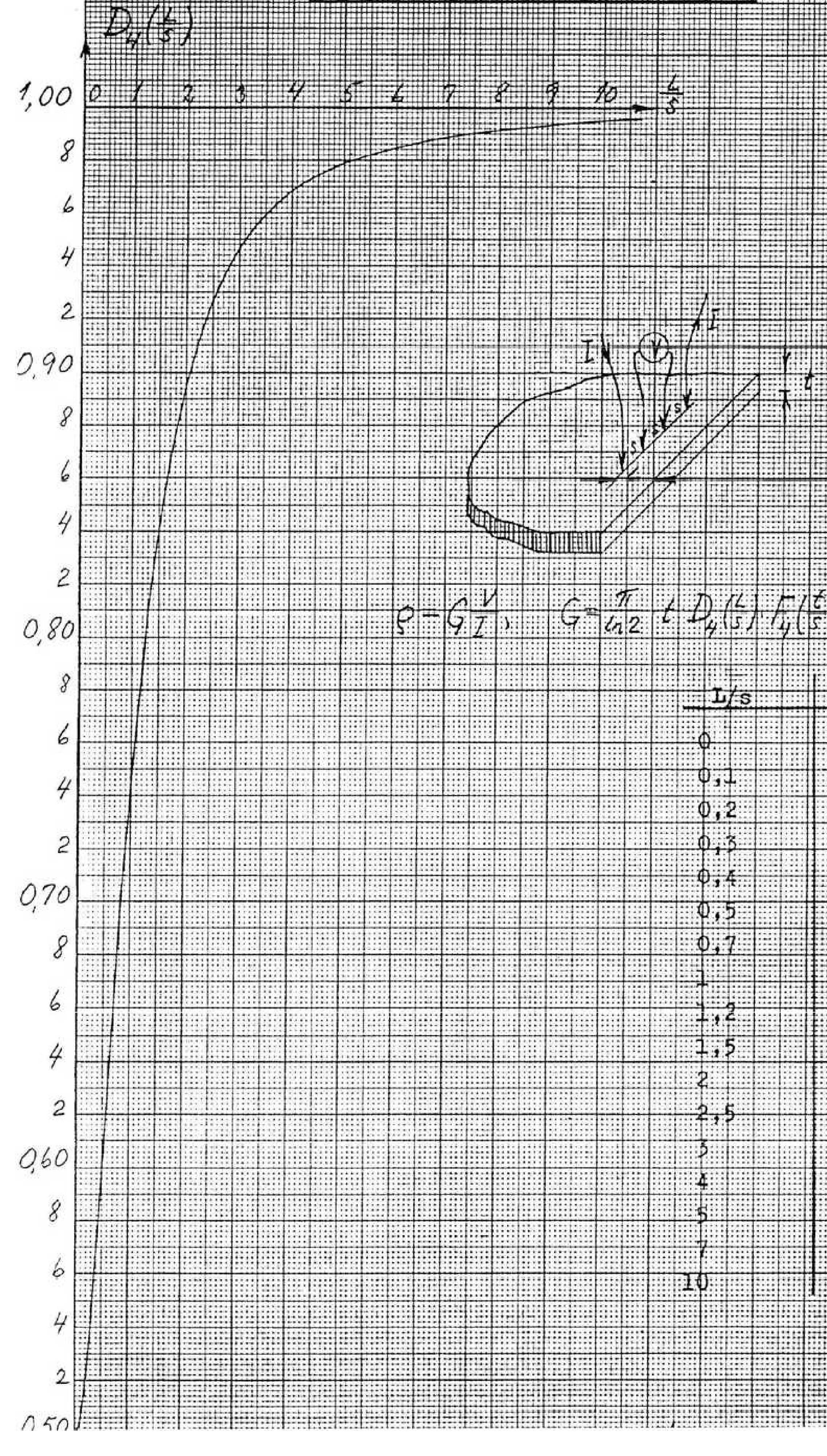
The expression for $D_4\left(\frac{L}{s}\right)$ was obtained from the formula (23) section I.3. for a circular slice when the probes are perpendicular to a diameter, by letting the diameter go to infinity. $D_4\left(\frac{L}{s}\right)$ is tabulated and plotted at page 29.

The additional correction $F_4\left(\frac{t}{s},\frac{L}{s}\right)$ to apply when t is not much less than s, was computed on the basis of Uhlir's paper (f) (g) and tabulated below. Curves for $F_4\left(\frac{t}{s},\frac{L}{s}\right)$ are shown at page 30.

$F_4(\frac{t}{s},\frac{L}{s})$	L/s								
t/s	0	0,1	0,2	0,5	1	2	5	10	∞
1	1	1	1	1	1	1	1	1	1
0,5	0,9974	0,998	0,998	0,998	0,998	0,998	0,998	0,997	0,9974
0,5555	0,9948								0,9948
0,6250	0,9898								0,9898
0,7143	0,9798								0,9798
0,8333	0,9600								0,9600
1	0,9214	0,92	0,93	0,94	0,939	0,929	0,923	0,922	0,9214
1,1111	0,8907								0,8907
1,25	0,8490								0,8490
1,4286	0,7938								0,7938
1,6666	0,7225								0,7225
2	0,6336	0,64	0,65	0,68	0,685	0,658	0,638	0,635	0,6336

It is evident from the curves that when $t \leq s/2$, we can put $F_4(\frac{t}{s},\frac{L}{s}) = 1$ for all practical purposes. Furthermore, $F_4(\frac{t}{s},\frac{L}{s})$ is only slightly dependent on L/s.

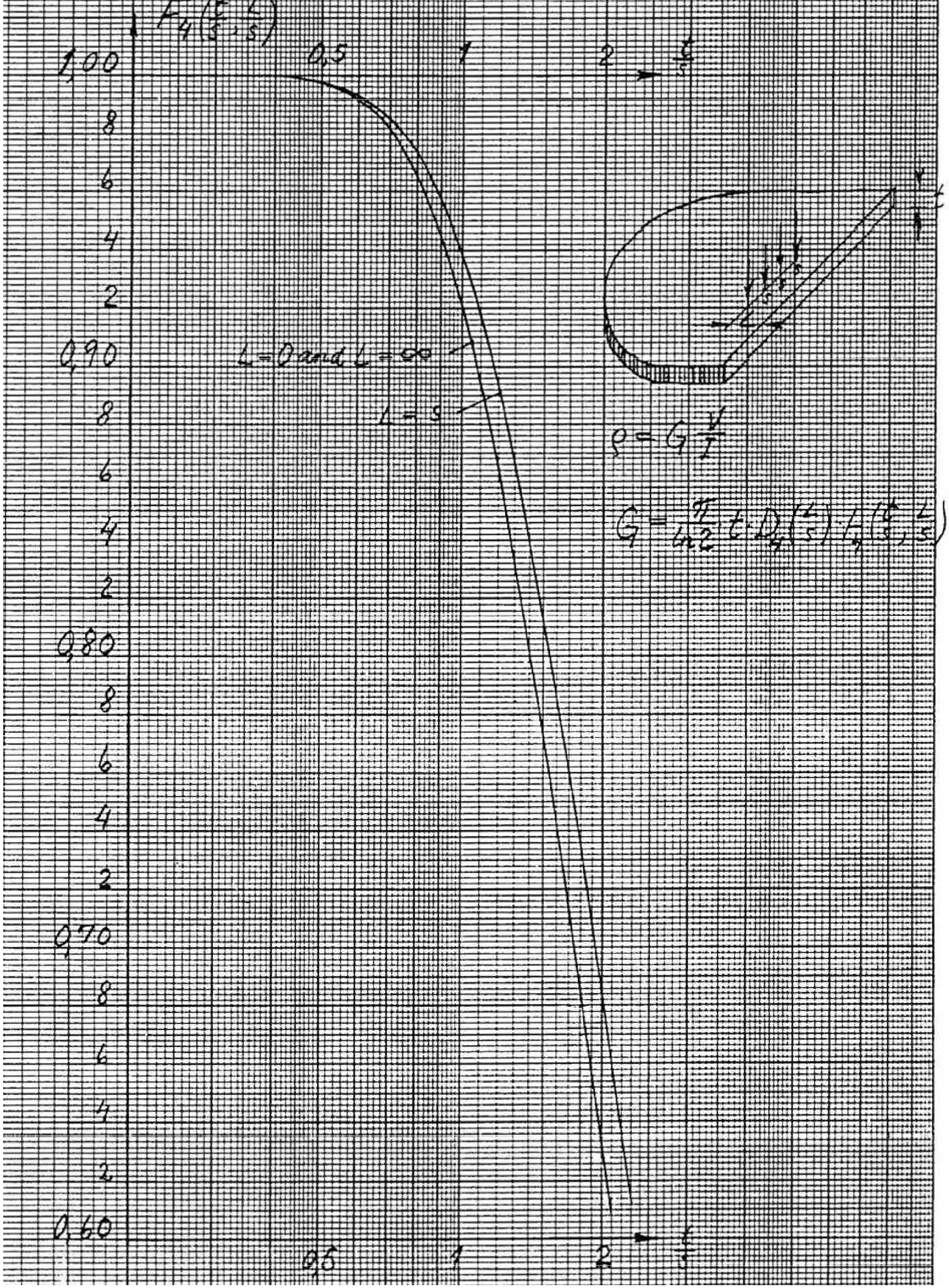
Thin, semi-infinite plane sample.



$$\rho = G\frac{V}{I}, \quad G = \frac{\pi}{\ln 2}\, t\, D_4\left(\frac{L}{s}\right) F_4\left(\frac{t}{s}, \frac{L}{s}\right)$$

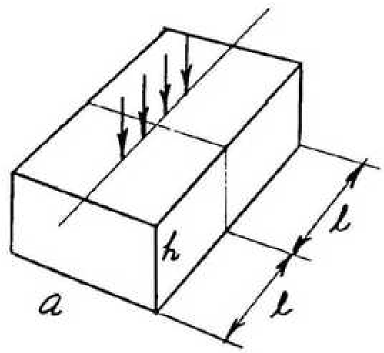
L/s	$D_4(\frac{L}{s})$
0	0,5
0,1	0,5053
0,2	0,5205
0,3	0,5434
0,4	0,5714
0,5	0,6021
0,7	0,6645
1	0,7468
1,2	0,7905
1,5	0,8409
2	0,8951
2,5	0,9270
3	0,9468
4	0,9685
5	0,9793
7	0,9892
10	0,9946

Thin, semi-infinite plane sample.



F.) BAR OF RECTANGULAR CROSS SECTION.

F.1) Rectangular Cross Section.



The geometric factor has been derived by Uhlir (f) (g) and Hansen (d). Hansen's results are given in the form:

$$\rho = G \frac{V}{I}, \qquad G = \frac{2\pi s}{F}, \qquad F = F\left(\frac{a}{s}, \frac{h}{s}, \frac{l}{s}\right) \tag{17}$$

The values of F for an infinitely long bar are shown at page 33.

Bar of finite length $2l$

An examination of the results of Uhlir (f) (g) shows that when measuring on a semi-infinite plane sample at a distance L from the edge, the dependence of G on variation in L is the greater, the thinner the sample. This can also be seen by comparing the factor $D_1(\frac{L}{s})$, section C.1 with the factor $D_3(\frac{L}{s})$ section E.3, and factor $D_2(\frac{L}{s})$ section C.2 with factor $D_4(\frac{L}{s})$ section E.5.

When the bar becomes finite, the deviation of G from $2\pi s/F$ will be greatest in the cases when $a \ll h$ or $h \ll a$ (see also(d) pp. 99-100). In estimating the deviation, an upper limit is obtained from the factor $D_3(\frac{L}{s})$ in section E.3, doubling the deviation from unity, when this is small.

From this we conclude, that when

$2l \geq 13s$, then $0{,}97 \cdot \frac{2\pi s}{F} \leqq G \leqq \frac{2\pi s}{F}$

for all values of a and h. $2l \leqq 13s$ means, that the current

Probes are at a distance 5s from the bar ends.

For a given ratio h/a, the influence of the bar ends is the smaller, the smaller a and h, approaching zero as a and h go to zero, where the sample reduces to a filament, and the current is restricted to the region between the current probes.

The special case, when $h = \frac{1}{2}a$ and $2\ell = 3s + \frac{1}{2}\cdot a$, was treated in detail by Hansen (d). The decrease in G from $2\pi s/F$ was less than 1,6% for all values of a/s.

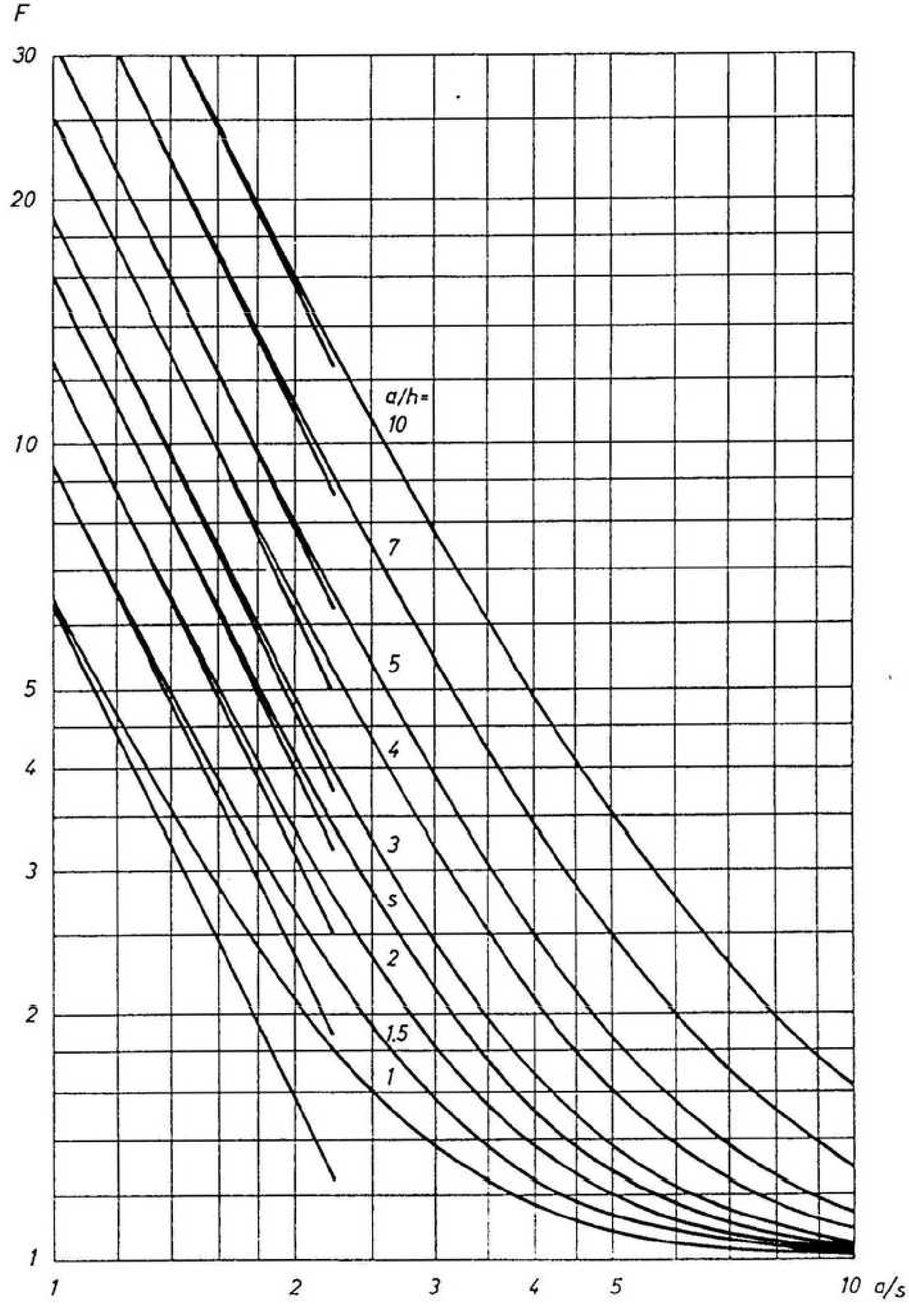
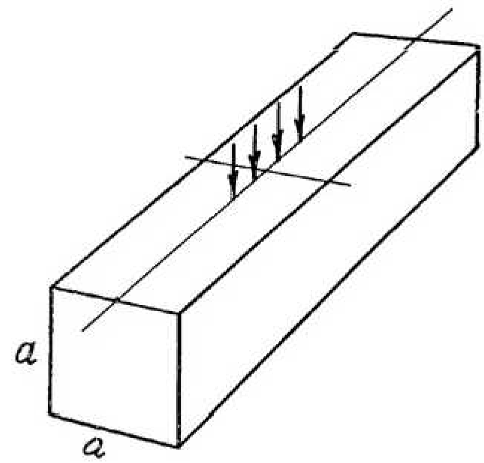


Fig. 2. The correction factor for an infinite bar of rectangular cross-section $a \times h$ and for an infinite bar of semi-circular cross-section (indicated by an s). The straight lines correspond to a homogeneous current distribution. The geometrical arrangements are shown in fig. 1 and fig. 4 respectively.

F.2) <u>Quadratic Cross Section.</u>

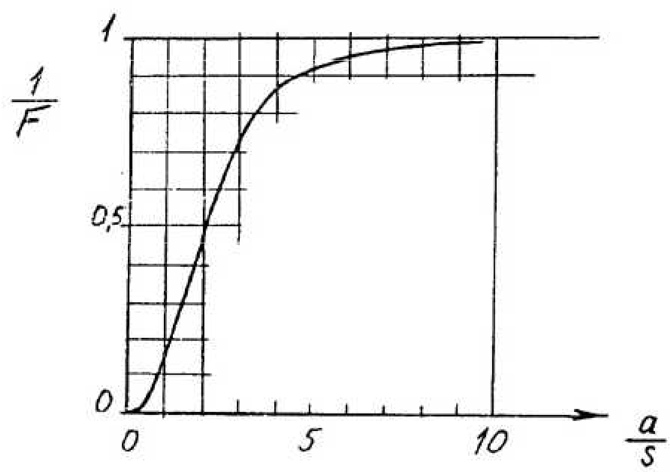


From the curves at page 33 we read F for the special case of quadratic cross section (a = h), and compute the magnitude:

$$\frac{1}{F(\frac{a}{s})} = \frac{G}{2\pi s}, \qquad \text{where} \qquad \rho = G \cdot \frac{V}{I}$$

We get

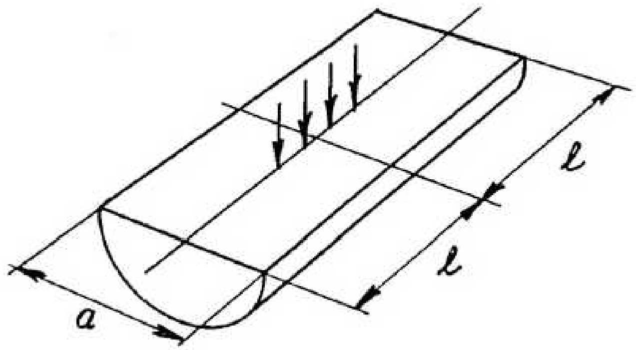
$\frac{a}{s}$	$\frac{1}{F} = \frac{G}{2\pi s}$
1	0,155
2	0,478
3	0,73
4	0,86
5	0,92
6	0,95
7	0,97
8	0,98
9	0,99



Geometric factor for infinitely long bar of quadratic cross section.

The geometric factor approaches that for a semi-infinite volume ($G = 2\pi s$) when $\frac{a}{s} \rightarrow \infty$.

G) BAR OF SEMI-CIRCULAR CROSS SECTION



The geometric factor for an <u>infinitely long bar</u> of semi-circular cross section is included in (d):

$$\rho = G\,\frac{V}{s}, \qquad G = \frac{2\pi s}{F}$$

where $F = F(\frac{a}{s})$ is shown at page 33.

The value of F for $a = 10 \cdot s$ is $F(\frac{a}{s} = 10) = 1{,}038$, or $G(\frac{a}{s} = 10) = 0{,}963 \cdot 2\pi s$.

When $a \geq 10s$, then $0{,}963 \cdot 2\pi s \leqq G \leqq 2\pi s$, where $2\pi s$ is the geometric factor for a semi-infinite volume.

The deviation of G from $G = \frac{2\pi s}{F}$ for a <u>bar of finite length 2l</u> can be estimated from the special case of a box-shaped bar treated by Hansen (d), see p. 31, as the rectangular shape with $h = \frac{1}{2} \cdot a$ is not very different in this context from a semi-circular cross section with diameter a.

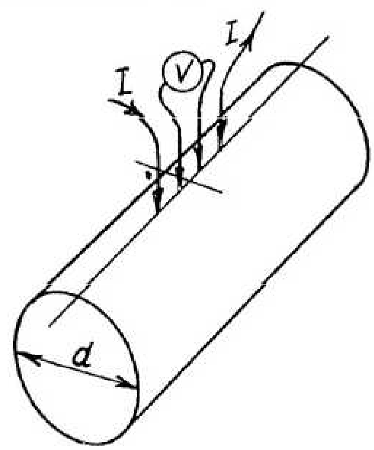
So, on the basis of (d) we estimate that when

$2l \geqq 3s + \frac{1}{2}a$, then $0{,}98\,\frac{2\pi s}{F} \leqq G \leqq \frac{2\pi s}{F}$.

H.) BAR OF CIRCULAR CROSS SECTION:

H.1) Probe Array on a Generator.

$$\rho = G \frac{V}{I}$$

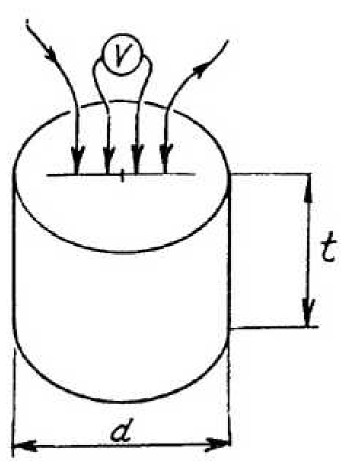


A calculation of the geometric factor G for this configuration has not been published although it is of great practical interest.

Preliminary calculations (k) show that in this case G approaches $2\pi s$ at a considerably lower rate when d/s goes to infinity than for square cross section when $a/s \rightarrow \infty$ (section F.2), e.g. $G \cong 2\pi s \cdot 0{,}918$ at $d/s = 10$ for an infinite bar of circular cross section.

H.2) Probe Array in Center of Cross Section.

$$\rho = G \cdot \frac{V}{I}$$



No calculation of the geometric factor has been published.

<u>For large diameters</u>, G can be approximated with that for an infinite plane sample (section D.1). The influence of the <u>finite diameter</u> is the greater, the smaller t. Therefore we have the upper limit for the influence of the periferi in the correction factor C_o of section I.1 for a thin, circular slice, and we can write

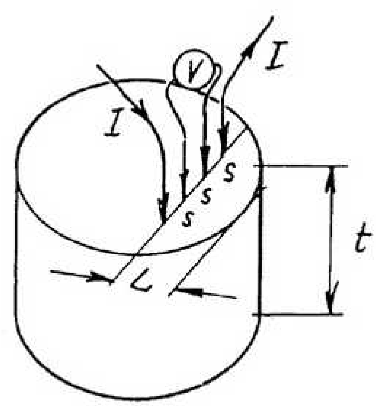
$$\rho = G\,\frac{V}{I}, \qquad 2\pi s\cdot T_1(\tfrac{t}{s})\cdot C_o(\tfrac{d}{s}) \leqq G \leqq 2\pi s\cdot T_1(\tfrac{t}{s})$$

where:

$2\pi s\cdot T_1(\frac{t}{s})$ is the geometric factor for an infinite plane sample of thickness t, and $C_o(\frac{d}{s})$ is the diameter correction for a thin circular slice of diameter d, when measuring in the center.

$T_1(\frac{t}{s})$ is found in section D.1, and $C_o(\frac{d}{s})$ in section I.1.

H.3) Probe Array Perpendicular to a Diameter at a fixed Distance from the Periferi.



This configuration has not been treated in the literature.

By a reasoning analogous to that in the previous section H.2, we conclude that

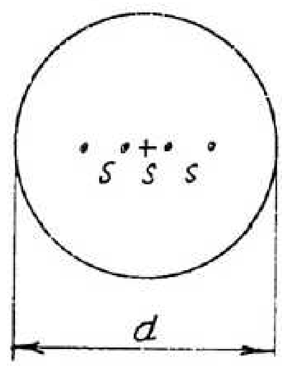
$$\rho = G\,\frac{V}{I}, \text{ where } 2\pi s\cdot T_1(\tfrac{t}{s})K_3(\tfrac{L}{s},\tfrac{d}{s}) \leqq G \leqq 2\pi s\cdot D_T(\tfrac{L}{s},\tfrac{t}{s})$$

where:

$2\pi s\cdot D_T(\frac{L}{s},\frac{t}{s})$ is the geometric factor for a semi-infinite plane sample of thickness t, when the probe array is parallel to the edge at a distance L, (see section E.4), and $2\pi s\cdot T_1(\frac{t}{s})$ is the geometric factor for an infinite plane sample of thickness t (see section D.), and $K_3(\frac{L}{s},\frac{t}{s})$ is the contour correction for the shown configuration, when $t \ll s$ (see section I.4).

I. THIN, CIRCULAR SLICE.

I.1) Measurement in the Center.



thickness $t < \frac{s}{2}$

This case has been treated by Smits (e). The result is:

$$\rho = G\,\frac{V}{I}, \qquad G = \frac{\pi}{\ln 2}\cdot t\cdot C_o\left(\frac{d}{s}\right) = 4{,}5324\cdot t\cdot C_o\left(\frac{d}{s}\right) \tag{18}$$

where:

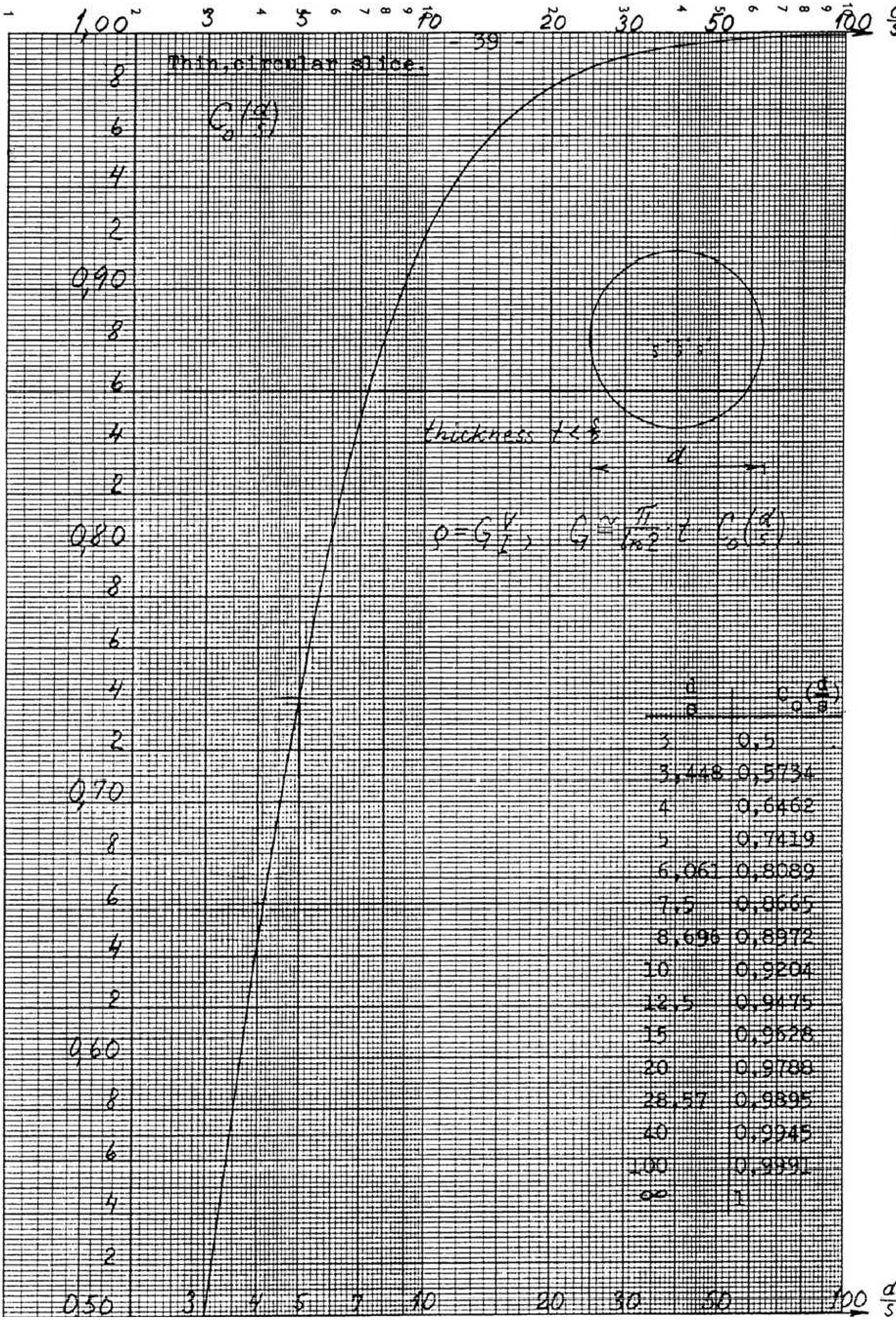
$\frac{\pi}{\ln 2}\cdot t = 4{,}5324\cdot t$ is the geometric factor for an infinitely large, thin slice (section D.2)

$$C_o\left(\frac{d}{s}\right) = \frac{1}{1 + \frac{1}{\ln 2}\ln\left[\frac{1 + 3\left(\frac{s}{d}\right)^2}{1 - 3\left(\frac{s}{d}\right)^2}\right]} \tag{19}$$

is the additional correction for a finite diameter $C_o \to 1$ as $d/s \to \infty$.

The magnitude $\frac{\pi}{\ln 2}\cdot C_o = 4{,}5324\cdot C_o$ has been tabulated by Smits (e) and later more extensively by Swartzendruber (h). $C_o\left(\frac{d}{s}\right)$ is shown at page 39 and $\frac{\pi}{\ln 2}\cdot C_o\left(\frac{d}{s}\right)$ at page 40.

Thin, circular slice.



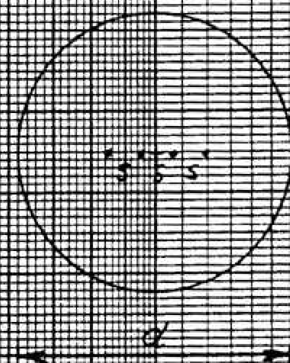
$\rho = G\frac{V}{I}$, $G \cong \frac{\pi}{\ln 2} \cdot t \cdot C_0(\frac{d}{s})$.

$\frac{d}{s}$	$C_0(\frac{d}{s})$
3	0,5
3,448	0,5734
4	0,6462
5	0,7419
6,061	0,8089
7,5	0,8665
8,696	0,8972
10	0,9204
12,5	0,9475
15	0,9628
20	0,9788
28,57	0,9895
40	0,9945
100	0,9991
∞	1

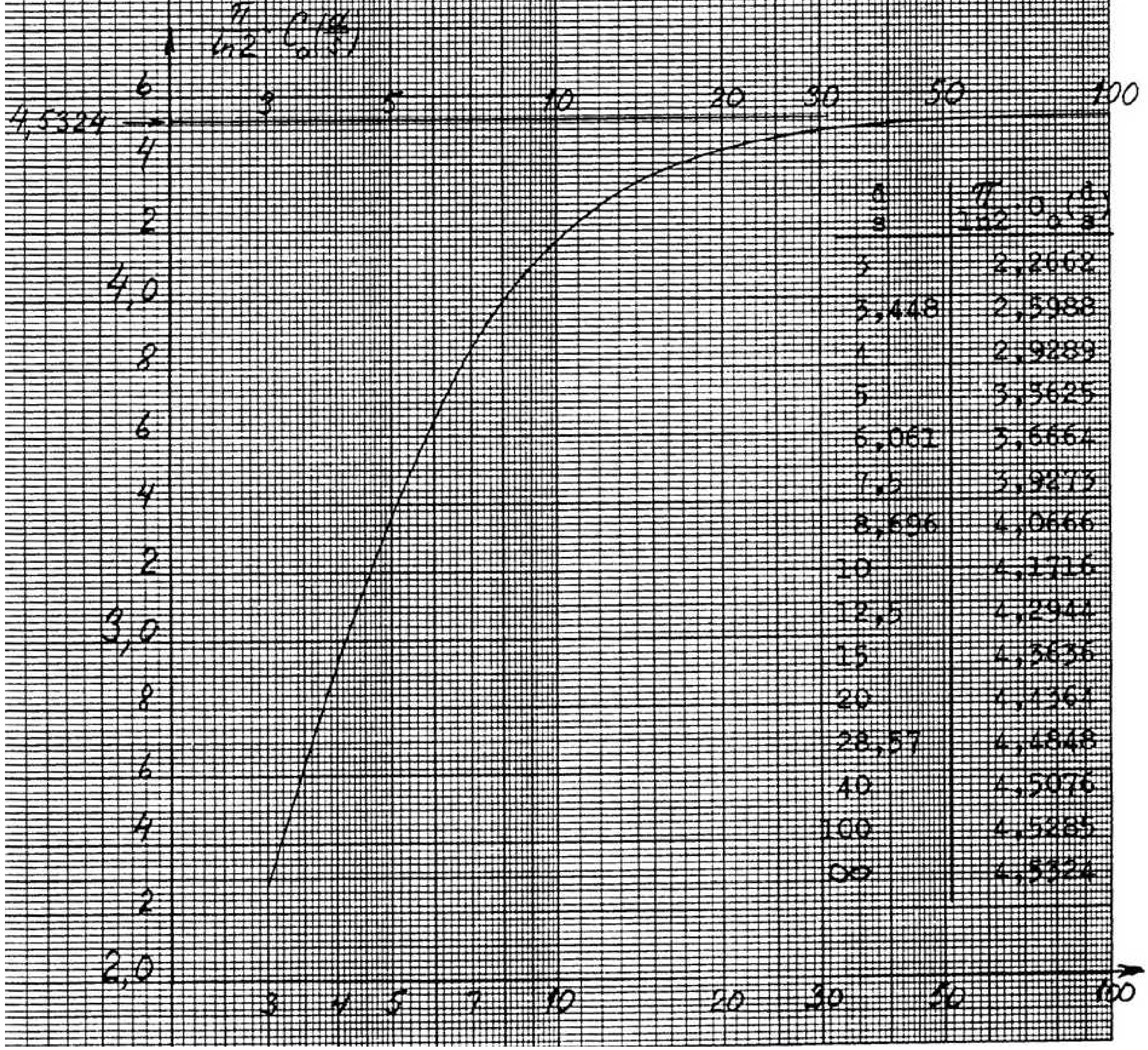
Thin, circular slice.

$\rho = G \frac{V}{I}$,

$G \simeq \frac{\pi}{\ln 2} \, t \, C_0\left(\frac{d}{s}\right)$

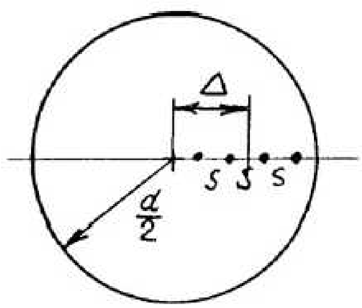


thickness $t < \frac{s}{2}$



$\frac{d}{s}$	$\frac{\pi}{\ln 2} \cdot C_0\left(\frac{d}{s}\right)$
3	2,2662
3,448	2,5988
4	2,9289
5	3,3625
6,061	3,6664
7,5	3,9273
8,696	4,0666
10	4,1716
12,5	4,2944
15	4,3636
20	4,4364
28,57	4,4848
40	4,5076
100	4,5285
∞	4,5324

I.2) Probe Array on a Diameter.



thickness $t < \frac{s}{2}$

The geometric factor for the case, when the probes are lying on a diameter, but displaced from the center of the slice, has been calculated by Logan (i), and tabulated in detail by Swartzendruber (h).

The resistivity is given by:

$$\rho = G \cdot \frac{V}{I}, \qquad G = \frac{\pi}{\ln 2} \cdot t \cdot C_1\left(\frac{\Delta}{d}, \frac{d}{s}\right) \tag{20}$$

where

$\frac{\pi}{\ln 2} \cdot t = 4{,}5324 \cdot t$ is the geometric factor for an infinitely large, thin slice (section D.2),

and:

$$C_1\left(\frac{\Delta}{d}, \frac{d}{s}\right) = \frac{1}{1 + \frac{1}{2\ln 2} \ln \frac{\left[1 - \left(\frac{2\Delta}{d} + \frac{s}{d}\right)\left(\frac{2\Delta}{d} - 3\frac{s}{d}\right)\right]\left[1 - \left(\frac{2\Delta}{d} - \frac{s}{d}\right)\left(\frac{2\Delta}{d} + 3\frac{s}{d}\right)\right]}{\left[1 - \left(\frac{2\Delta}{d} - \frac{s}{d}\right)\left(\frac{2\Delta}{d} - 3\frac{s}{d}\right)\right]\left[1 - \left(\frac{2\Delta}{d} + \frac{s}{d}\right)\left(\frac{2\Delta}{d} + 3\frac{s}{d}\right)\right]}} \tag{21}$$

$C_1(\frac{s}{d}, \frac{\Delta}{d})$ may be written:

$$C_1(\frac{s}{d}, \frac{\Delta}{d}) = C_1(\frac{s}{d}, 0) \cdot K_1(\frac{\Delta}{d}, \frac{d}{s}) = C_o(\frac{s}{d}) \cdot K_1(\frac{s}{d}, \frac{d}{s})$$ where:

$C_o(\frac{s}{d})$ is the geometric factor for measurement in the center of a slice with diameter d (section I.1), and:

$K_1(\frac{\Delta}{d}, \frac{d}{s})$ is the additional correction for a displacement Δ of the probes from the center.

$K_1(\frac{\Delta}{d}, \frac{d}{s}) \longrightarrow 1$ as $\Delta \longrightarrow 0$.

K_1 was computed as a function of $\frac{\Delta}{d}$ for various $\frac{d}{s}$, on the basis of reference (h).

The result is tabulated below and plotted at page 43.

$K_1(\frac{\Delta}{d}, \frac{d}{s})$

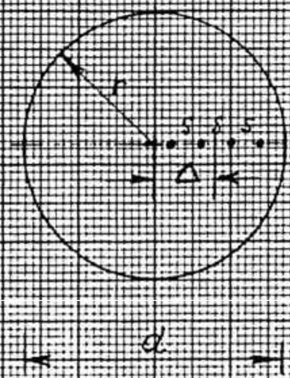
$\frac{\Delta}{d}$	$\frac{d}{s} = 5$	$\frac{d}{s} = 10$	$\frac{d}{s} = 12{,}5$	15,38	20	40
0	1	1	1	1	1	1
0,05	0,9936	0,9983	0,9989	0,9993	0,9996	0,9999
0,1	0,9730	0,9929	0,9954	0,9969	0,9982	0,9995
0,15	0,9343	0,9827	0,9888	0,9925	0,9956	0,9989
0,2	0,8677	0,9653	0,9775	0,9850	0,9911	0,9977
0,25		0,9355	0,9582	0,9721	0,9834	0,9958
0,3		0,8811	0,9228	0,9485	0,9693	0,9922
0,35		0,7653	0,8483	0,8988	0,9395	0,9847
0,4				0,7602	0,8574	0,9639
0,45						0,8556

Thin, circular slice.

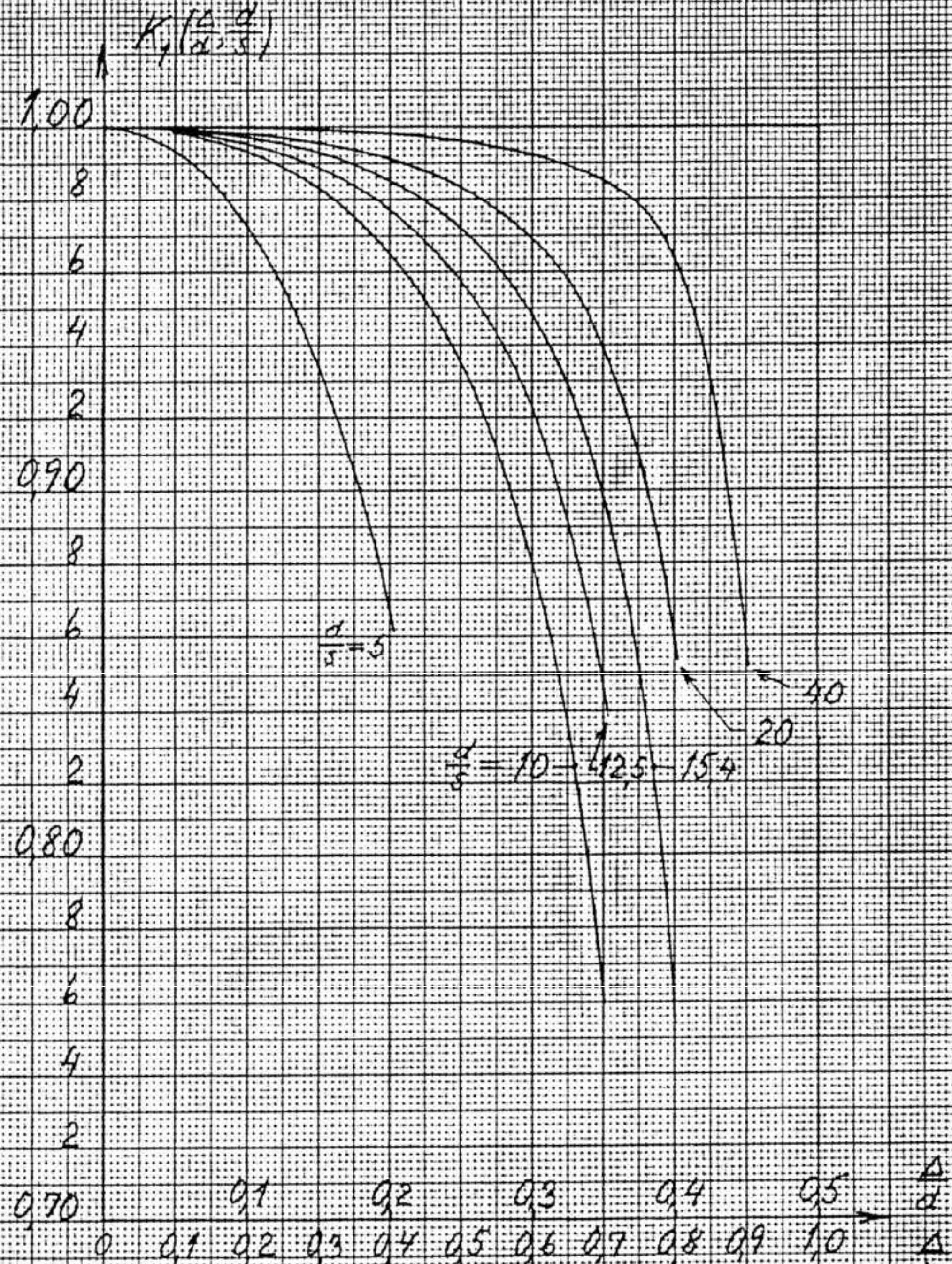
$$\rho = G\,\frac{V}{I}$$

$$G \approx \frac{\pi}{\ln 2}\, t \cdot C_1\left(\frac{\Delta}{d}, \frac{d}{s}\right)$$

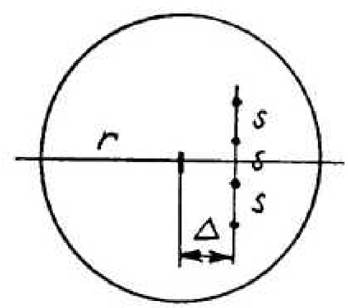
$$= \frac{\pi}{\ln 2}\, t\, C_0\left(\frac{d}{s}\right) K_1\left(\frac{\Delta}{d}, \frac{d}{s}\right).$$



thickness $t < \frac{s}{2}$



I.3) Probe Array Perpendicular to a Diameter.



thickness $t < \frac{s}{2}$

Swartzendruber (j) published the geometric factor for this geometri:

$$\rho = G\,\frac{V}{I}, \qquad G = \frac{\pi}{\ln 2}\cdot t \cdot C_2\left(\frac{s}{d},\frac{\Delta}{d}\right) \tag{22}$$

where:

$$C_2\left(\frac{s}{d},\frac{\Delta}{d}\right) = \frac{1}{1 + \frac{1}{2\ln 2}\ln\frac{\alpha_1\alpha_2}{4\alpha_3\alpha_4}} \tag{23}$$

$$\alpha_1 = (V_2 - V_1)^2 + (U_2 + U_1)^2$$

$$\alpha_2 = (V_2 + V_1)^2 + (U_2 + U_1)^2$$

$$\alpha_3 = (V_2 - V_1)^2 + (U_2 - U_1)^2$$

$$\alpha_4 = (V_2 + V_1)^2 + (U_2 - U_1)^2$$

$$U_1 = \frac{3\frac{s}{r}}{(1+\frac{\Delta}{r})^2 + \frac{9}{4}(\frac{s}{r})^2}$$

$$U_2 = \frac{\frac{s}{r}}{(1+\frac{\Delta}{r})^2 + \frac{1}{4}(\frac{s}{r})^2}$$

$$V_1 = \frac{1 - (\frac{\Delta}{r})^2 - \frac{9}{4}(\frac{s}{r})^2}{(1+\frac{\Delta}{r})^2 + \frac{9}{4}(\frac{s}{r})^2}$$

$$V_2 = \frac{1 - (\frac{\Delta}{r})^2 - \frac{1}{4}(\frac{s}{r})^2}{(1+\frac{\Delta}{r})^2 + \frac{1}{4}(\frac{s}{r})^2}$$

This complicated expression has been tabulated extensively in ref. (h). We choose to present the results in the following form:

$$C_2(\frac{s}{d},\frac{\Delta}{d}) = C_2(\frac{s}{d}, 0) \cdot K_2(\frac{\Delta}{d},\frac{d}{s}) = C_o(\frac{s}{d}) \cdot K_2(\frac{\Delta}{d},\frac{d}{s}) \quad \text{where:}$$

$C_o(\frac{s}{d})$ is the geometric factor for measurement in the center of a slice with diameter d (section I.1) and $K_2(\frac{\Delta}{d},\frac{d}{s})$ is the additional correction for a displacement Δ of the probes from the center.

$K_2(\frac{\Delta}{d},\frac{d}{s}) \rightarrow 1$ as $\Delta \rightarrow 0$.

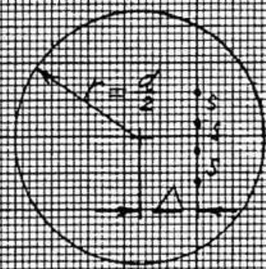
$K_2(\frac{\Delta}{d},\frac{d}{s})$ is tabulated below and plotted at page 46.

$\frac{\Delta}{r}$	$\frac{d}{s} = 5$	$\frac{d}{s} = 10$	$\frac{d}{s} = 12,5$	$\frac{d}{s} = 15,38$	$\frac{d}{s} = 20$	$\frac{d}{s} = 40$
0,1	0,9957	0,9985	0,9990	0,9993	0,9996	0,9999
0,2	0,9827	0,9936	0,9957	0,9971	0,9982	0,9995
0,3	0,9598	0,9847	0,9896	0,9929	0,9957	0,9989
0,4	0,9256	0,9701	0,9796	0,9859	0,9914	0,9978
0,5	0,8783	0,9468	0,9631	0,9744	0,9842	0,9959
0,6	0,8186	0,9092	0,9354	0,9543	0,9714	0,9924
0,7	0,7439	0,8470	0,8862	0,8169	0,9464	0,9852
0,8		0,7442	0,7937	0,8391	0,8891	0,9663
0,9		0,6607	0,6312	0,6717	0,7334	0,8883

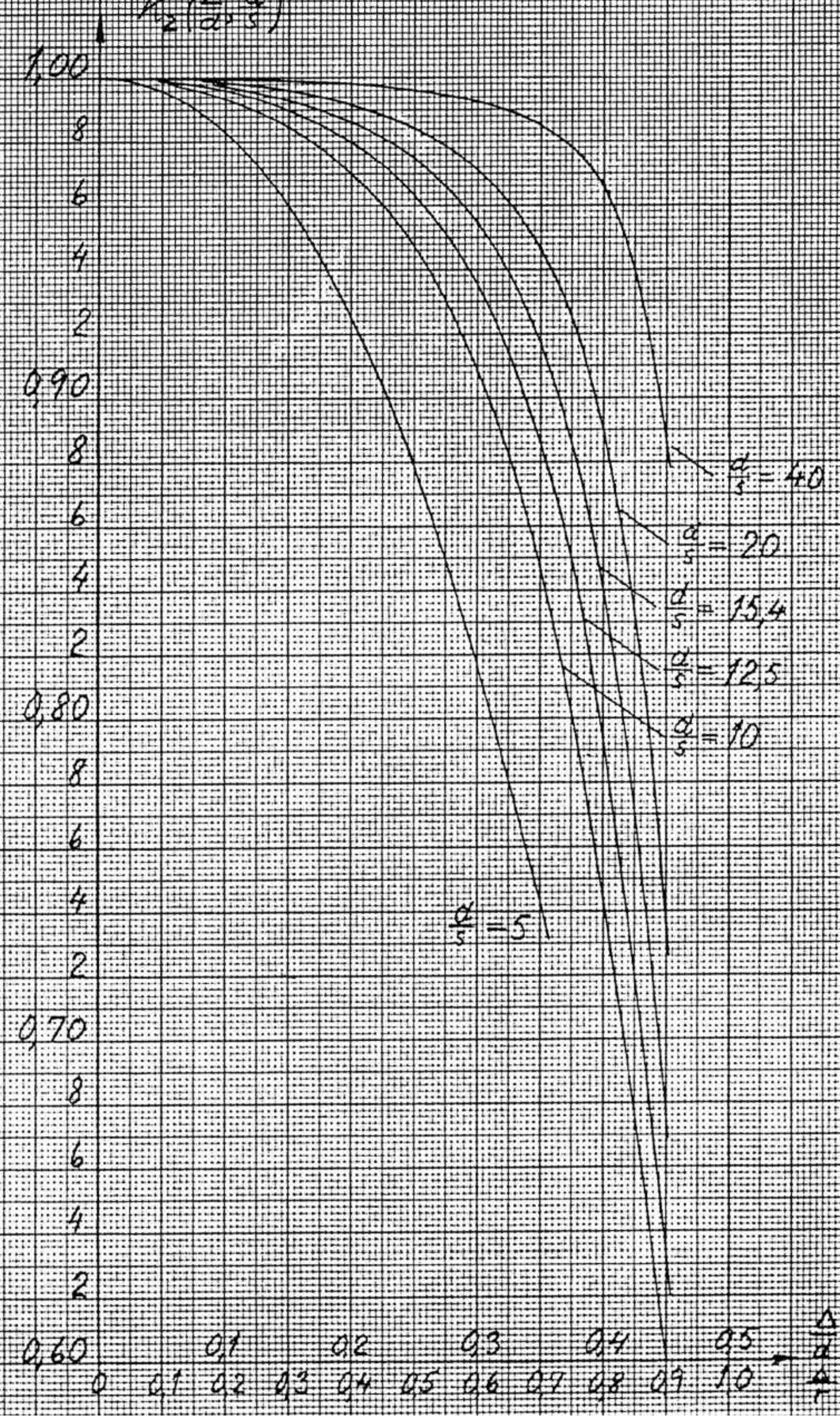
Thin, circular slice.

$$\rho = G\,\frac{V}{I}$$

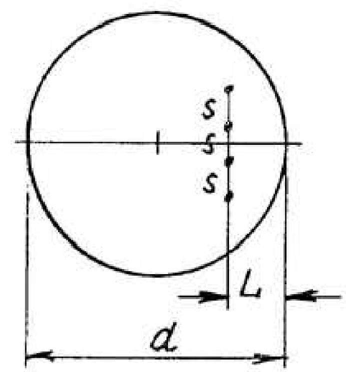
$$G \approx \frac{\pi}{\ln 2}\cdot t\cdot C_2\left(\frac{\Delta}{d},\frac{d}{s}\right) = \frac{\pi}{\ln 2}\cdot t\cdot C_0\left(\frac{d}{s}\right)\cdot k_2\left(\frac{\Delta}{d},\frac{d}{s}\right)$$



thickness $t < \frac{s}{2}$



I.4) <u>Probe Array Perpendicular to a Diameter, at a Fixed Distance from the Periferi.</u>



thickness $t < \frac{s}{2}$

This special case of section I.3. is separately treated because of the common practice of specifying the resistivity of a slice by the values found in the center and at a specified distance from the edge.

The geometric factor is given by equations (22) and (23) of section I.3., but a different presentation is convenient here, in which the distance L is fixed and the diameter d continuously variable. We write:

$$\rho = G \cdot \frac{V}{I}, \qquad G = \frac{\pi}{\ln 2} \cdot t \cdot K_3\left(\frac{L}{s}, \frac{d}{s}\right) \qquad (24)$$

where:

$\frac{\pi}{\ln 2} \cdot t = 4{,}5324 \cdot t$ is the geometric factor for a infinitely large, thin slice (section D.2.)

$K_3\left(\frac{L}{3}, \frac{d}{s}\right)$ is the additional correction to apply for the shown arrangement of the probes on a circular slice of diameter d. The following table was computed on the basis of reference (h) putting $\Delta = r - L$. $K_3\left(\frac{L}{s}, \frac{d}{s}\right)$ is shown at page 48 and 49.

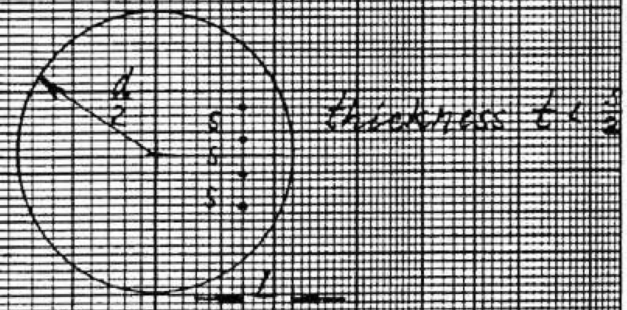
$K_3(\frac{L}{s}, \frac{d}{s})$

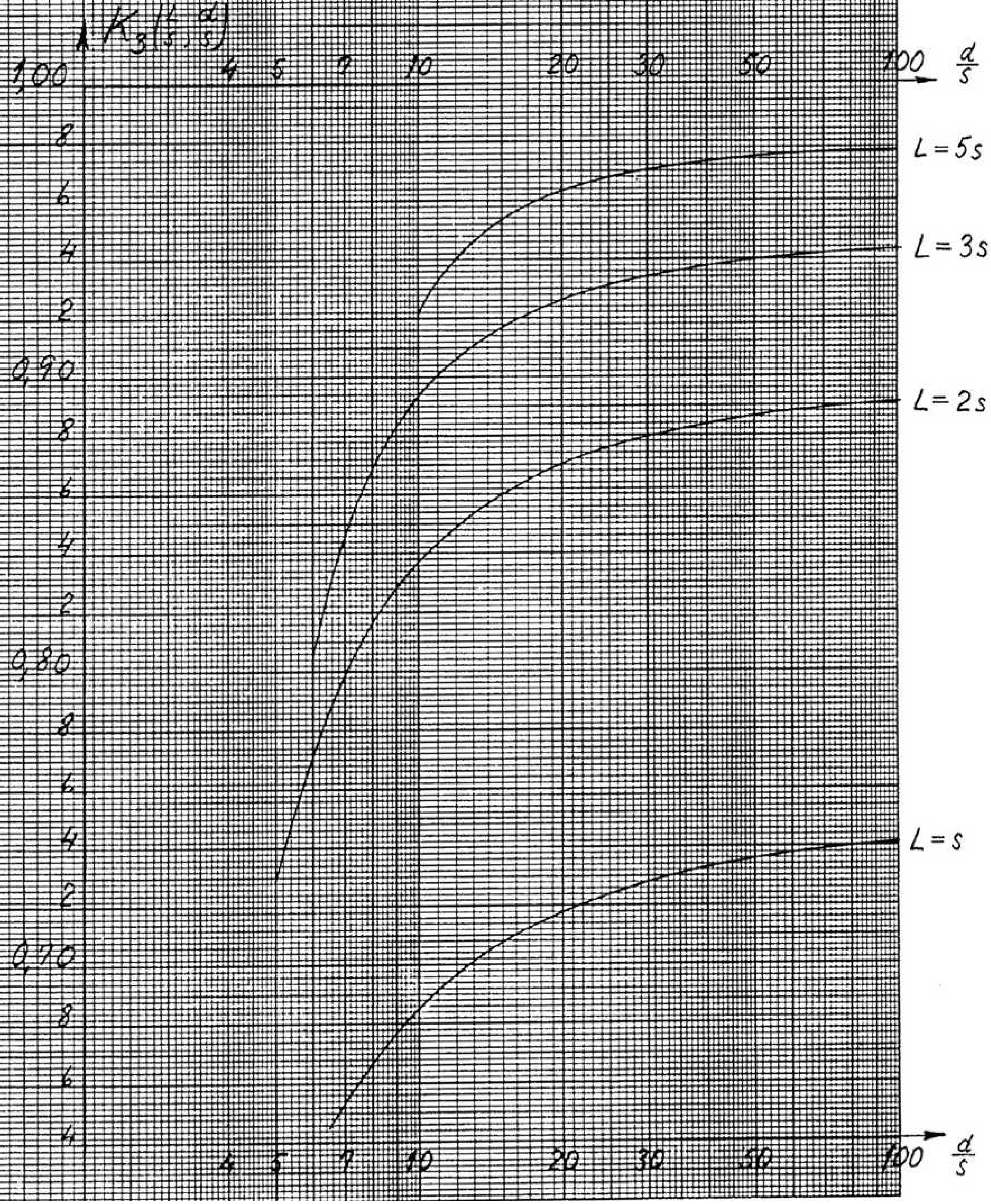
$\frac{d}{s}$	L = s	L = 2s	L = 3s	L = 5s
4		0,6462		
5	0,6059	0,7290		
6			0,805	
6,897	0,6516	0,7964	0,8418	
10	0,6849	0,8368	0,8928	0,9204
12,5	0,6986	0,8514	0,9083	0,9434
15,38	0,7084	0,8612	0,9180	0,9549
20	0,7178	0,8702	0,9263	0,9633
28,57	0,7269	0,8785	0,9335	0,9696
40	0,7328	0,8836	0,9377	0,9730
100	0,7413	0,8907	0,9434	0,9771
200	0,7441	0,8929	0,9451	0,9782
∞	0,7468	0,8951	0,9468	0,9793

Thin, circular slice.

$$\rho = G \frac{V}{I}$$

$$G \approx \frac{\pi}{\ln 2} \cdot t \cdot K_3\left(\frac{L}{s}, \frac{d}{s}\right)$$





When measuring alternately in the center and at a distance from the periferi of the slices, a different presentation of the geometric factor is convenient: We write:

$$\rho = G\,\frac{V}{I},$$

$$G = \frac{\pi}{\ln 2} \cdot t \cdot K_3\left(\frac{L}{s},\frac{d}{s}\right) = \frac{\pi}{\ln 2} \cdot C_o\left(\frac{d}{s}\right) \cdot K_4\left(\frac{L}{s},\frac{d}{s}\right)$$

where

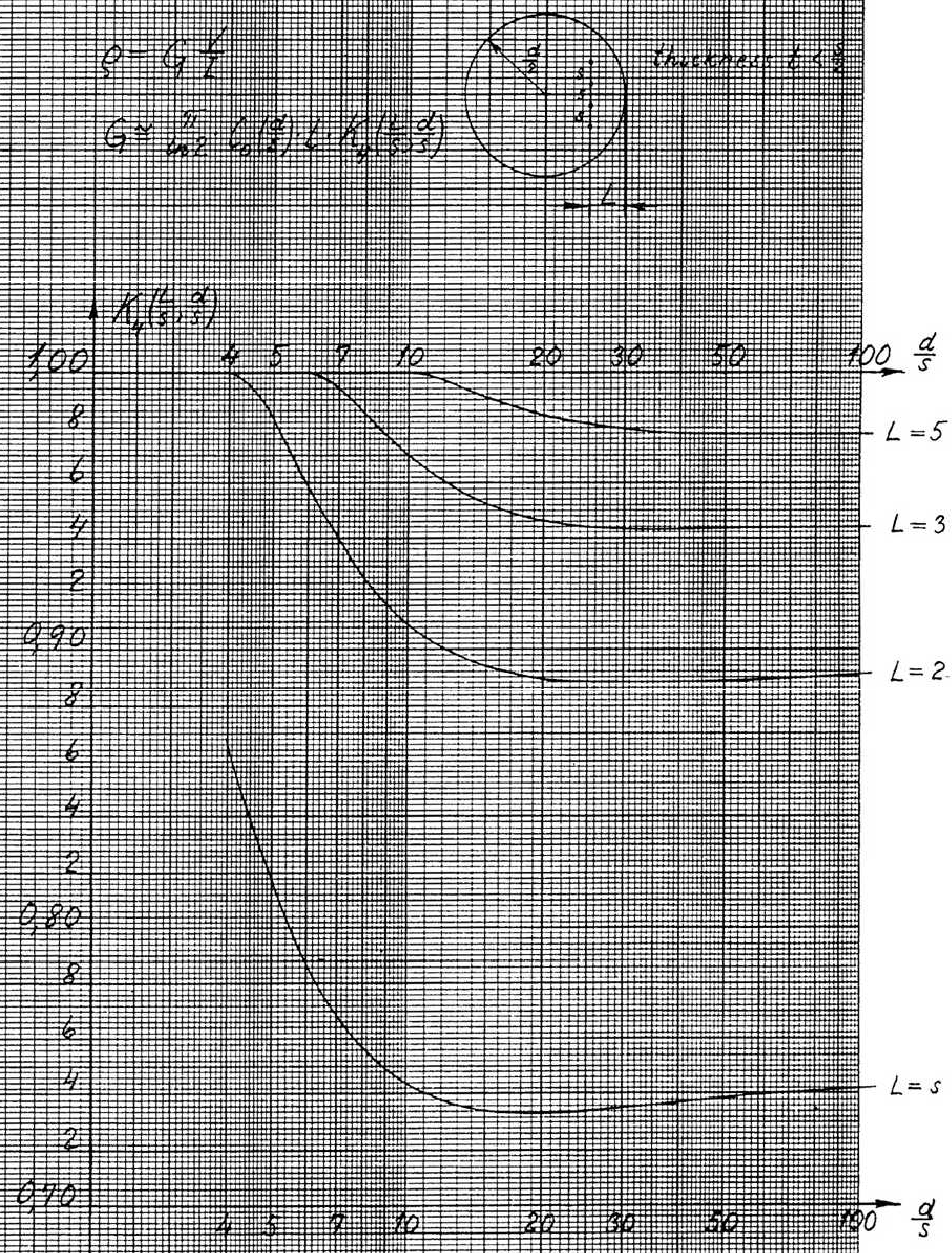
$\frac{\pi}{\ln 2} \cdot C_o\left(\frac{d}{s}\right) \cdot t$ is the geometric factor when measuring in the center of a circular slice of diameter d and thickness $t < \frac{s}{2}$ (see section I.1), and $K_4\left(\frac{L}{s},\frac{d}{s}\right)$ is the additional correction to apply when measuring at a distance L from the periferi of the slice.

$K_4\left(\frac{L}{s},\frac{d}{s}\right)$ is tabulated below and plotted at page 52.

$K_4(\frac{L}{s}, \frac{d}{s})$

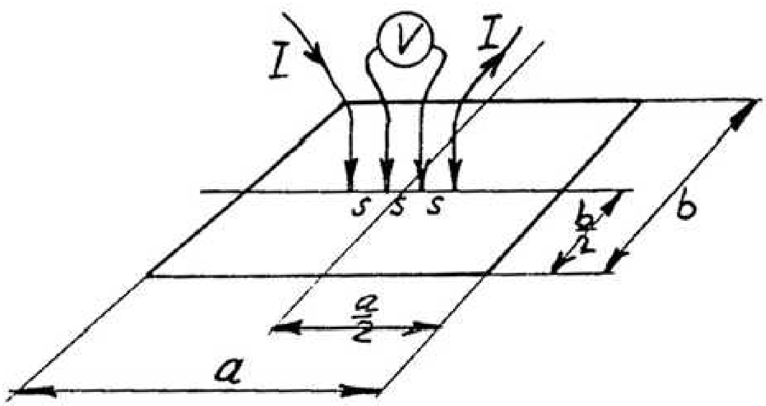
$\frac{s}{r}$	$\frac{d}{s}$	L = s	L = 2s	L = 3s	L = 5s
0,50	4	0,8657	1		
0,45	4,444	0,8409	0,9952		
0,40	5	0,8168	0,9827		
0,35	5,714	0,7942	0,9652		
	6			1	
0,29	6,897	0,7704	0,9416	0,9952	
0,25	8	0,7571	0,9262	0,9852	
0,20	10	0,7442	0,9092	0,9701	1
0,16	12,5	0,7373	0,8986	0,9587	0,9957
0,13	15,38	0,7343	0,8928	0,9516	0,9899
0,10	20	0,7334	0,8891	0,9464	0,9842
0,07	28,57	0,7346	0,8878	0,9434	0,9799
0,05	40	0,7367	0,8883	0,9428	0,9783
0,02	100	0,7419	0,8914	0,9442	0,9779
0,01	200	0,7442	0,8931	0,9453	0,9785
0	∞	0,7468	0,8951	0,9468	0,9793

Thin, circular slice.



K) THIN, RECTANGULAR SLICE.

K.1) Rectangular Slice.



thickness $t < \frac{s}{2}$

This configuration has been treated by Smits (e). The resistivity is given by;

$$\rho = G \frac{V}{I}, \qquad G = \frac{\pi}{\ln 2} \cdot t \cdot R_1\left(\frac{b}{s}, \frac{a}{b}\right) \qquad (25)$$

where

$\frac{\pi}{\ln 2} \cdot t = 4{,}5324 \cdot t$ is the geometric factor for an infinitely large slice of thickness $t \ll s$, and

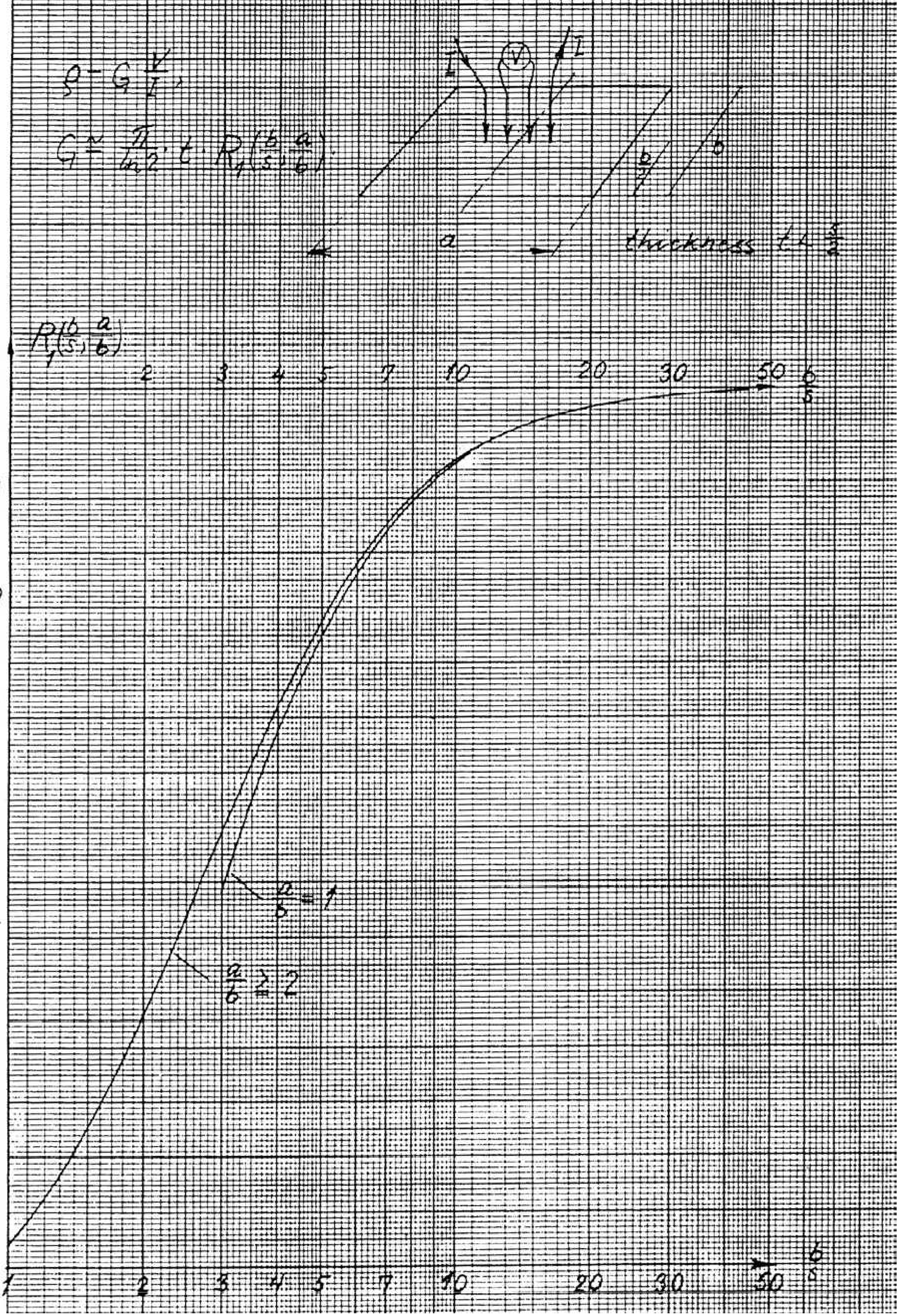
$R_1\left(\frac{b}{s}, \frac{a}{b}\right)$ is the additional correction to apply because of the finite, rectangular shape.

R_1 is tabulated at page 54 and shown at page 55. $\frac{\pi}{\ln 2} \cdot R_1$ is tabulated at page 56 and shown at page 57.

$R_1(\frac{b}{s}, \frac{a}{b})$

$\frac{b}{s}$	$\frac{a}{b} = 1$	$\frac{a}{b} = 2$	$\frac{a}{b} = 3$	$\frac{a}{b} \geqq 4$
1			0,2204	0,2205
1,25			0,2751	0,2751
1,5		0,3263	0,3286	0,3286
1,75		0,3794	0,3803	0,3803
2		0,4292	0,4297	0,4297
2,5		0,5192	0,5194	0,5194
3	0,5422	0,5957	0,5958	0,5958
4	0,6870	0,7115	0,7115	0,7115
5	0,7744	0,7887	0,7888	0,7888
7,5	0,8846	0,8905	0,8905	0,8905
10	0,9313	0,9345	0,9345	0,9345
15	0,9682	0,9696	0,9696	0,9696
20	0,9822	0,9830	0,9830	0,9830
40	0,9955	0,9957	0,9957	0,9957
∞	1	1	1	1

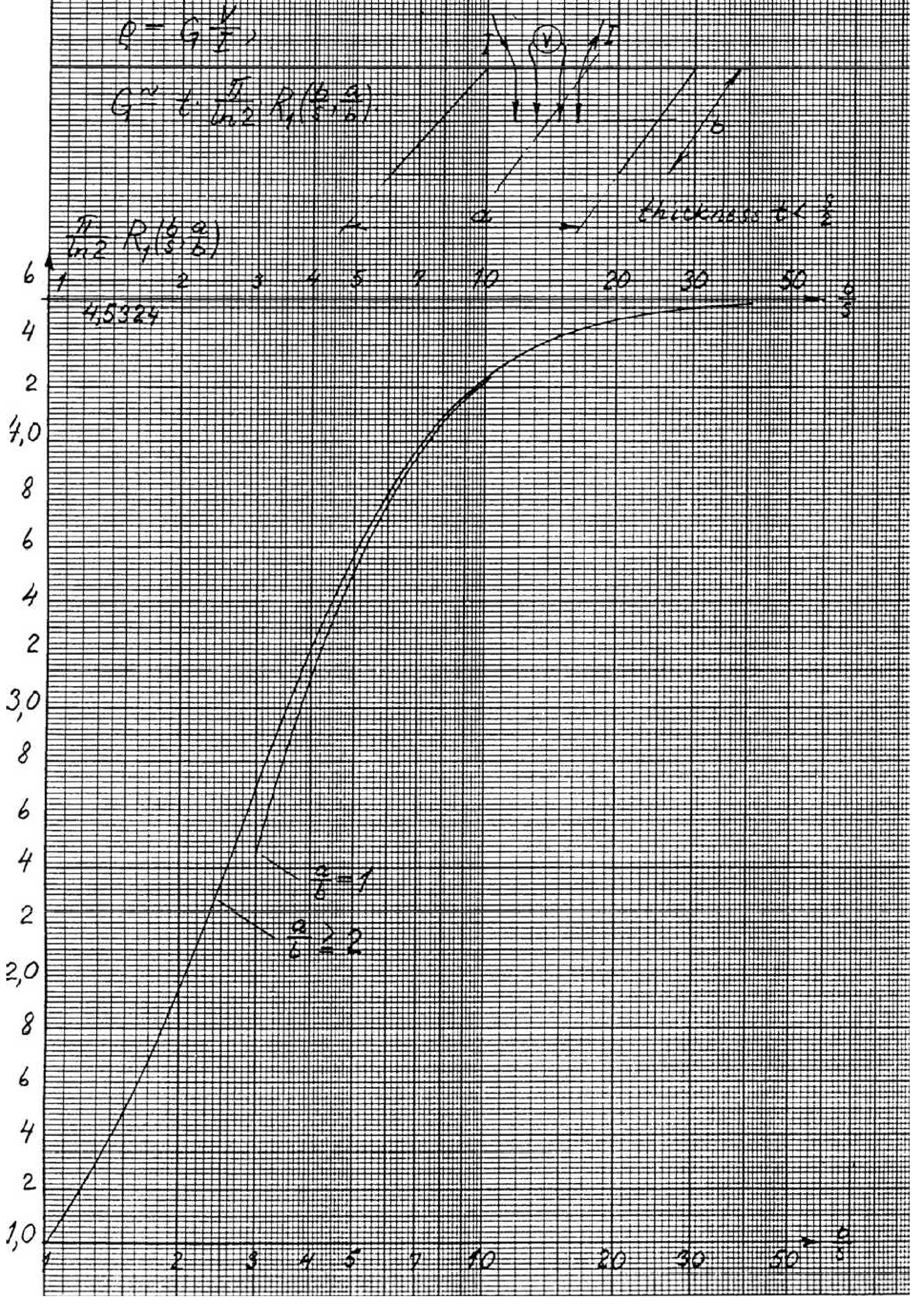
Thin, rectangular slice.



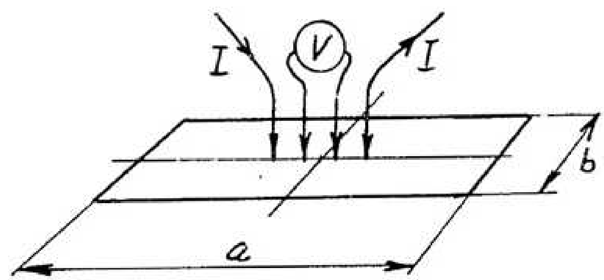
$$\frac{\pi}{\ln 2}\cdot R_1\left(\frac{b}{s},\frac{a}{b}\right)$$

$\frac{b}{s}$	$\frac{a}{b} = 1$	$\frac{a}{b} = 2$	$\frac{a}{b} = 3$	$\frac{a}{b} \geq 4$
1			0,9988	0,9994
1,25			1,2467	1,2468
1,5		1,4788	1,4893	1,4893
1,75		1,7196	1,7238	1,7238
2		1,9454	1,9475	1,9475
2,5		2,3532	2,3541	2,3541
3	2,4575	2,7000	2,7005	2,7005
4	3,1137	3,2246	3,2248	3,2248
5	3,5098	3,5749	3,5750	3,5750
7,5	4.0095	4,0361	4,0362	4,0362
10	4,2209	4,2357	4,2357	4,2357
15	4,3882	4,3947	4,3947	4,3947
20	4,4516	4,4553	4,4553	4,4553
40	4,5120	4,5129	4,5129	4,5129
∞	4,5324	4,5324	4,5324	4,5324

Thin, rectangular slice.



K.2) Narrow, Rectangular Slice.



thickness $t < \frac{s}{2}$

When b is smaller than 3 to 4 times s, it is convenient to express the resistivity is this way:

$$\rho = G \cdot \frac{V}{I}, \qquad G = t \cdot \frac{b}{s} \cdot \quad R_2(\frac{b}{s},\frac{a}{b}) \tag{26}$$

If $R_2 = 1$, we can write the resistance $\frac{V}{I}$ as

$\frac{V}{I} = \rho \cdot \frac{s}{b \cdot t}$, which is the resistance of a conductor of resistivity ρ ,length s and area $b \cdot t$.

So, $R_2 = 1$ corresponds to constant current density in the sample between the voltage probes. As $\frac{b}{s}$ increases, the current density becomes lower far from the probes, and R_2 decreases. R_2 was computed and tabulated by Smits (e). The results are tabulated below and plotted at page 59.

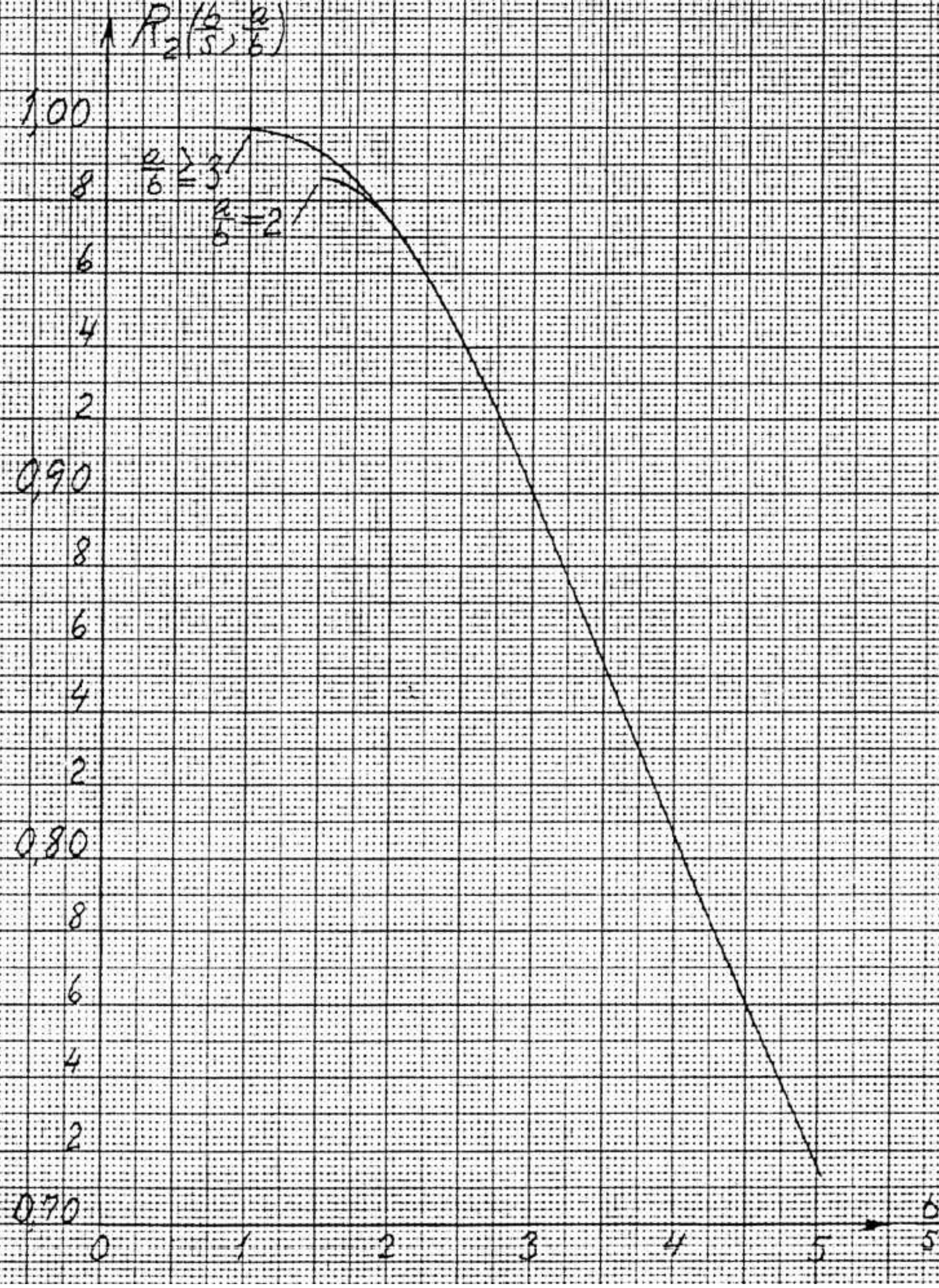
$R_2(\frac{b}{s},\frac{a}{b})$

$\frac{b}{s}$	$\frac{a}{b} = 1$	$\frac{a}{b} = 2$	$\frac{a}{b} = 3$	$\frac{a}{b} \geq 4$
1			0,9988	0,9994
1,25			0,9973	0,9974
1,5		0,9859	0,9929	0,9929
1,75		0,9826	0,9850	0,9850
2		0,9727	0,9737	0,9737
2,5		0,9413	0,9416	0,9416
3	0,8192	0,9000	0,9002	0,9002
4	0,7784	0,8061	0,8062	0,8062
5	0,7020	0,7150	0,7150	0,7150

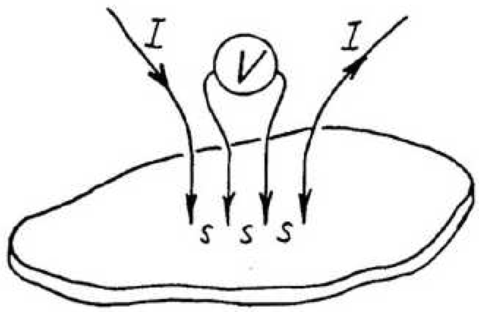
Thin and narrow rectangular slice.

$$\rho = G\,\frac{V}{I},$$

$$G \cong t\,\frac{b}{s}\cdot R_2\!\left(\frac{b}{s},\frac{a}{b}\right)$$



L) GENERAL CONSIDERATIONS ON FINITE SLICES.



1) In the sections I. 1-4 and K. 1-2 we have given geometric factors for thin slices ($t \ll s$) of finite extension. With the exception of section K.2, the resistivity was always written in the form:

$$\rho = G \frac{V}{I}, \qquad G = \frac{\pi}{\ln 2} \cdot t \cdot C, \tag{27}$$

where:

$\frac{\pi}{\ln 2} \cdot t = 4,5324 \cdot t$ is the geometric factor for an infinitely large, thin sample ($t \ll s$) and C is the additional correction for the finite size, dependent on the limiting contour. The given formulae and curves for the factor C are exaxt only in the limit $\frac{t}{s} \rightarrow 0$.

2) In section D_2, the geometric factor is given for an infinitely large slice of thickness $t \leq 2s$. The resistivity is written as:

$$\rho = G \frac{V}{I}, \qquad G = \frac{\pi}{\ln 2} \cdot t \cdot T_2(\frac{t}{s}) \tag{28}$$

where

$\frac{\pi}{\ln 2} \cdot t = 4,5324 \cdot t$ is the geometric factor for an infinitely large slice of thickness $t \ll s$, and $T_2(\frac{t}{s})$ is the additional correction for greater thickness. $T_2 \rightarrow 1$ as $\frac{t}{s} \rightarrow 0$.

When $0 \leq t \leq \frac{s}{2}$, then $0,9974 \leq T_2 \leq 1$.

That means, that the current distribution in the slice is changed only very slightly by the increase in thickness from zero to $\frac{1}{2} \cdot s$. We can put $T_2 = 1$ for all practical purposes in this interval.

3) For a slice of finite extension and thickness the resistivity can be written:

$$\rho = G\,\frac{V}{I}, \qquad G = \frac{\pi}{\ln 2} \cdot t \cdot T_2 \cdot C \cdot F(t,c), \qquad (29)$$

where

$\frac{\pi}{\ln 2} \cdot t \cdot T_2$ is the afore-mentioned geometric factor for an infinite slice of thickness t, C is the correction factor for the limiting contour, for a slice of thickness $t \ll s$ and

$F(t,c)$ is an additional correction factor depending on both thickness and contour. From section E.3 we realize that $F(t,c)$ may be both smaller and greater than unity. $F(t,c)$ approaches unity in the two limits:

1) $F \to 1$ as $\frac{t}{s} \to 0$, in which case $T_2 \to 1$
2) $F \to 1$ as the slice becomes large, in which case $C \to 1$.

In case 1 the slice is so thin that the current distribution is essentially that found in a sheet, which is the assumption under which C is calculated.
In case 2 the slice is so large that the contour does not effect the current distribution, which is the condition for the validity of T_2.
As a change in current distribution is reflected in a deviation from unity in the corresponding geometry factor, we can conclude:

1) Thin slice.

The sample is so thin, that the thickness correction $T_2(\frac{t}{s})$ is close to unity.

The resistivity may then be written:

$$\rho = G\,\frac{V}{I}, \qquad G = \frac{\pi}{\ln 2} \cdot t \cdot T_2(\tfrac{t}{s}) \cdot C \qquad (30)$$

where:
The thickness factor T_2 is given in section D.2.
The contour factor C is given in sections I.1-4 and K.1 for various configurations.
(30) is correct within a few procent, when $t \leqq s$, in

which case $0,92 \leqq T_2 \leqq 1$.
When $t \leqq \frac{s}{2}$, then $0,9974 \leqq T_2 \leqq 1$, and (30) can for all practical purposes be reduced to

$$\rho = G \frac{V}{I}. \qquad G = \frac{\pi}{\ln 2} \cdot t \cdot C$$

2) Large slice.

The sample is so large that the contour factor C is close to unity. The resistivity may be written:

$$\rho = G \frac{V}{I} \qquad G = \frac{\pi}{\ln 2} \cdot t \cdot T_2\left(\frac{t}{s}\right) \cdot C \qquad (31)$$

where:

The thickness factor T_2 is given in section D.2 and the contour factor C is given in sections I.1-4 and K.1 for various configurations.

The condition for C to be close to unity must be decided for each special configuration.

As the factor F(t,c) in formula (29) despicts a second order effect, it will deviate less from unity than C, when C is close to unity.

3) General case.

We must apply the complete expression (29).

$$\rho = G \frac{V}{I}, \qquad G = \frac{\pi}{\ln 2} \cdot t \cdot T_2\left(\frac{t}{s}\right) \cdot C \cdot F(t,c)$$

where

the thickness factor T_2 is found in section D.2 and the contour factor C is given in sections I.1-4 and K.1 for various configurations.

F(t,c) is the additional correction factor depending on both thickness and contour. F(t,c) may be smaller and greater than unity, and is generally not known.

When the slice is rectangular, the geometric factor for a bar of rectangular cross section as given in section F.1 may suffice.

When the slice is circular and of thickness $t > s$, the geometric factor can only be estimated by sensible approximations (see sections H.2-4).

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j) L.J. Swartzendruber. Solid State Electronics. 7 (1964) p. 413-422.

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