Is ψ-Epistemic Theory at Stake?

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Brief Description of QM

$$\textit{State} \Rightarrow |\psi\rangle \; (\textit{Unit vector in } H)$$

$$\textit{Observable}: A \; (\textit{Hermitain operator on } H)$$

$$A = \sum a_i |\phi_i\rangle\langle\phi_i| = \sum a_i P_i, \quad \sum P_i = I$$

(1) Measurement result is one of the Eigen values.

$$
\begin{array}{cccccc}
|\psi\rangle & |\psi\rangle & |\psi\rangle & |\psi\rangle & |\psi\rangle & |\psi\rangle \\
|\psi\rangle & |\psi\rangle & |\psi\rangle & |\psi\rangle & |\psi\rangle & |\psi\rangle \\
|\psi\rangle & |\psi\rangle & |\psi\rangle & |\psi\rangle & |\psi\rangle & |\psi\rangle \\
\cdots & \cdots & \cdots & \cdots & \cdots &
\end{array}
$$

(2) $p(a_i|\psi) = \langle\psi|P_i|\psi\rangle = |\langle\psi|\phi_i\rangle|^2$ **(Born probability)**

(3) $|\psi\rangle \rightarrow |\phi_i\rangle$ **(Measurement collapse)**

Copenhagen Interpretation and Einstein

Bhor' statement:

It is wrong to think that the task of physics is to find out how nature is. Physics concerns what we say about nature.

Pauli's statement:

Observations not only disturb what has to be measured, they produce it …. We compel (the electron) to take a definite position. …We ourselves produce the result of measurement.

Question of reality independent of measurement is denied. ψ only represents pattern of the results for some future measurements.

Einstein:

The fact suggests that Einstein…….., but one wherein quantum states are solely representative of our knowledge. **(Harrigan & Spekkens, 2010)**

Two Pictures:

QM

$|0\rangle$ $|0\rangle$ $|0\rangle$ $|0\rangle$ $|0\rangle$ $|0\rangle$

$|0\rangle$ $|0\rangle$ $|0\rangle$ $|0\rangle$ $|0\rangle$ $|0\rangle$

... ...

... ...

Measure σ_x

50% up $(+1)$ 50% $down(-1)$

Measurement creates the result for σ_x.

Individual particle has definite value for both σ_x and σ_z.

$\sigma_z = 1$ $\sigma_x = 1$	$\sigma_z = 1$ $\sigma_x = -1$	$\sigma_z = 1$ $\sigma_x = -1$
$\sigma_z = 1$ $\sigma_x = 1$	$\sigma_z = 1$ $\sigma_x = -1$	$\sigma_z = 1$ $\sigma_x = 1$

... ...

Measure σ_x

50% up $(+1)$ 50% $down(-1)$

Measurement reveals the pre-existing value of σ_x.

Uncertainty and measurement incompatibility cannot discard the 2nd picture.

Formal HVT Model

(ψ-*supplemented*)

Quantum ensemble → **Scenario in HVT**

$\|\psi\rangle$	$\|\psi\rangle$	$\|\psi\rangle$	$\|\psi\rangle$	$\|\psi\rangle$
$\|\psi\rangle$	$\|\psi\rangle$	$\|\psi\rangle$	$\|\psi\rangle$	$\|\psi\rangle$
$\|\psi\rangle$	$\|\psi\rangle$	$\|\psi\rangle$	$\|\psi\rangle$	$\|\psi\rangle$
…	…	…	…	..
…	…	…	…	..

(ψ,λ_1)	(ψ,λ_2)	(ψ,λ_5)	(ψ,λ_3)	(ψ,λ_2)
(ψ,λ_7)	(ψ,λ_2)	(ψ,λ_8)	(ψ,λ_1)	(ψ,λ_{13})
(ψ,λ_1)	(ψ,λ_5)	(ψ,λ_9)	(ψ,λ_1)	(ψ,λ_2)
…	…	…	…	.. …
…	…	…	…	.. …

(ψ,λ) *decides the values of all observables.*

$$v_{\psi,\lambda}(A) = \text{ one of the Eigen value of } A$$

To be compatible with QM:

$$\int v_{\psi,\lambda}(A)\mu_\psi(\lambda)d\lambda = \langle A\rangle_\psi \iff \int v_{\psi,\lambda}(P_i)\mu_\psi(\lambda)d\lambda = \langle\psi|P_i|\psi\rangle$$

$$\textit{where } \mu_\psi(\lambda) \geq 0,\ \int \mu_\psi(\lambda)d\lambda = 1$$

ψ-Epistemic Model

(Quantum state is not part o reality)

Quantum ensemble				
$\|\psi\rangle$	$\|\psi\rangle$	$\|\psi\rangle$	$\|\psi\rangle$	$\|\psi\rangle$
$\|\psi\rangle$	$\|\psi\rangle$	$\|\psi\rangle$	$\|\psi\rangle$	$\|\psi\rangle$
$\|\psi\rangle$	$\|\psi\rangle$	$\|\psi\rangle$	$\|\psi\rangle$	$\|\psi\rangle$
…	…	…	…	..
…	…	…	…	..

$\longrightarrow$

Scenario in HVT				
λ_{13}	λ_{15}	λ_5	λ_1	λ_6
λ_1	λ_7	λ_5	λ_2	λ_8
λ_1	λ_{11}	λ_3	λ_1	λ_{10}
…	…	…	…	..
…	…	…	…	..

λ ***decides the values of all observables.***

$$v_\lambda(A) = \textit{ one of the Eigen value of } A$$

To be compatible with QM:

$$\int v_\lambda(A)\mu_\psi(\lambda)d\lambda = \langle A\rangle_\psi \;\Leftrightarrow \int v_\lambda(P_i)\mu_\psi(\lambda)d\lambda = \langle\psi|P_i|\psi\rangle$$

$$\textit{where } \mu_\psi(\lambda) \geq 0,\; \int \mu_\psi(\lambda)d\lambda = 1$$

It's not just flawed, it's silly!

Von Neumann claimed 'No' to the possibility of dispersion free description:

$$A + B = C$$

Condition imposed by Von Neumann:

$$\langle A \rangle_S + \langle B \rangle_S = \langle C \rangle_S$$

S being the state of the theory.

Eigen value of A + Eigen value of B = Eigen value of C

Verifiable relation:

$$\langle \psi | A | \psi \rangle + \langle \psi | B | \psi \rangle = \langle \psi | C | \psi \rangle$$

$$\int v_\lambda(A) \mu_\psi(\lambda) d\lambda + \int v_\lambda(B) \mu_\psi(\lambda) d\lambda - \int v_\lambda(C) \mu_\psi(\lambda) d\lambda = 0$$

$$\int [v_\lambda(A) + v_\lambda(B) - v_\lambda(C)] \mu_\psi(\lambda) d\lambda = 0$$

$v_\lambda(A) + v_\lambda(B) - v_\lambda(C) = 0$ *may be a sufficient condion for the above but hardly a necessary condition.*

Statistics for spin measurement

Every qubit state is eigen state of some spin observable $\boldsymbol{n}.\boldsymbol{\sigma}$.

Let $|\psi\rangle$ *is up eigen state of* $\boldsymbol{n}.\boldsymbol{\sigma}$.

$$|\psi\rangle\langle\psi| = \frac{1}{2}[I + \boldsymbol{n}.\boldsymbol{\sigma}], \ |\boldsymbol{n}| = 1$$

Then for spin measurement along vector $\boldsymbol{m}$:

$$p(+1|\psi) = \frac{1}{2}(1 + \boldsymbol{n}.\boldsymbol{m})$$

$$p(-1|\psi) = \frac{1}{2}(1 - \boldsymbol{n}.\boldsymbol{m})$$

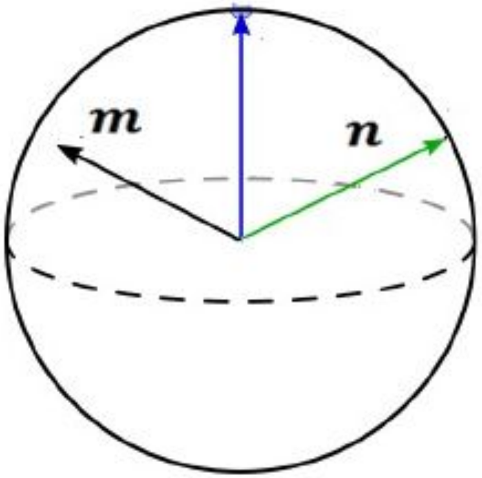
(Born rule)

$$\langle\psi|\boldsymbol{m}.\boldsymbol{\sigma}|\psi\rangle = \boldsymbol{n}.\boldsymbol{m}$$

Bell Model for qubit (A different version)

Completed state: $(\psi, \lambda) \equiv (n, \lambda)$

$\lambda \in [-\frac{1}{2}, \frac{1}{2}]$ *and distribution is uniform.*



Definite value for completed state:

$$v_{n,\lambda}[m.\sigma] = Sign(\lambda + \frac{1}{2}|n.m|\, Sign(n.m))$$

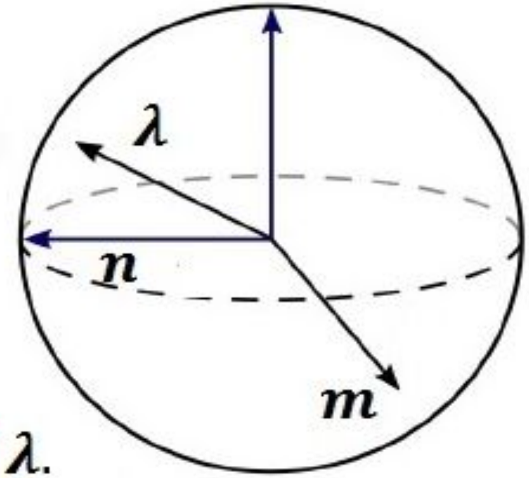
$Sign(x) = 1$ **when** $x \geq 0$ **and** $Sign(x) = -1$ **when** $x < 0$

Let $(n.m)$ *is negetive*

$$\langle m.\sigma \rangle = \int_{-\frac{1}{2}}^{\frac{1}{2}} \rho(\lambda) v_{n,\lambda}[m.\sigma]\, d\lambda = \int_{-\frac{1}{2}}^{\frac{1}{2}} Sign(\lambda + \frac{1}{2}|n.m|\, Sign(n.m))]d\lambda$$

$$= \int_{-\frac{1}{2}}^{\frac{1}{2}|n.m|} (-1) d\lambda + \int_{\frac{1}{2}|n.m|}^{\frac{1}{2}} (+1) d\lambda = -\frac{1}{2}[1 + |n.m|] + \frac{1}{2}[1 - |n.m|] = n.m$$

K-S Model $(\psi\text{−}Epistemic)$



$\boldsymbol{\lambda}$ *(Unit vector in Bloch sphere)* ***is state of individual particle.***

$$\psi \left(= \frac{1}{2}(1 + \sigma.n)\right)$$

$\boldsymbol{\psi}$ ***determines the distribution of*** $\boldsymbol{\lambda}$.

$$\rho_\psi(\lambda) = \frac{1}{\pi}\Theta(n.\lambda)(n.\lambda)$$

$$\begin{aligned} \Theta(x) &= 1 \; for \; x > 0 \\ &= 0 \; for \; x \leq 0 \end{aligned}$$

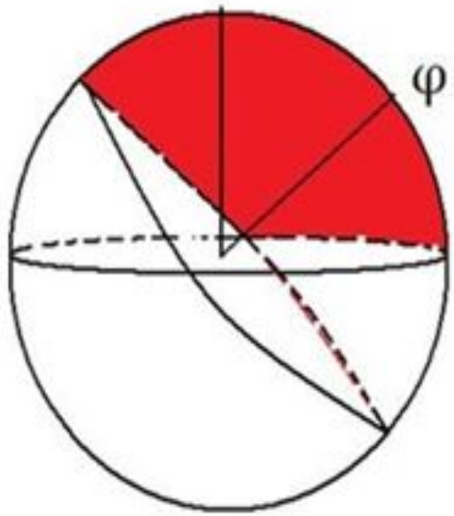
Probability for up result along direction m is given by

$$p_\lambda(\text{up}) = \Theta(m.\lambda)$$

$$p(\text{up}|\psi) = \frac{1}{\pi}\int d\lambda \; \Theta(m.\lambda)\,\Theta(n.\lambda)(n.\lambda) = \frac{1}{2}(1 + m.n)$$

Some Intresting Features of ψ- Epistemic Model

For two different quantum states ψ and φ there are common ontic states.



$$p(\lambda|\psi)p(\lambda|\varphi) = \frac{1}{\pi^2}\,\Theta(m.\lambda)\,\Theta(n.\lambda)(n.\lambda)(m.\lambda)$$

is non-zero for nonorthogonal ψ and φ .

* This can explain why non-orthogonal quantum states can not be reliably distinguished.

* This picure can also provide a solution to the notorious collapse postulate by giving the post-measurement state as a updating of probabilities.

Gleason's Theorem

The set of all projection operators $P(H)$.

μ is a probability measure on $P(H)$.

1) $0 \leq \mu(P) \leq 1$

2) $\mu(I) = 1$

3) $\mu(\sum P_i) = \sum \mu(P_i)$

where P_i are orthogonal projectors.

If $Dim(H) \geq 3$, then there exists a density operator ρ , such that

$$\mu(P) = Tr[\rho P]$$

So measurement contextuality was discovered in 1957 itself though it was not appreciated before Bell put the question with glaring clarity during 1964 to 1966.

HVT in higher dimensional Hilbert space

$$\sum_{i=1}^{3} P_i = I \qquad \text{where} \qquad P_i = |\phi_i\rangle\langle\phi_i|$$

$\{|\phi_1\rangle, |\phi_2\rangle, |\phi_3\rangle\}$ being an orthogonal basis B_1.

Another projective measurement: $P_1 + Q_2 + Q_3 = I$

Q_2 : Projector on $\frac{1}{\sqrt{2}}(|\phi_2\rangle + |\phi_3\rangle)$ $\quad Q_3$: Projector on $\frac{1}{\sqrt{2}}(|\phi_2\rangle - |\phi_3\rangle)$

$\{|\phi_1\rangle, \frac{1}{\sqrt{2}}(|\phi_2\rangle + |\phi_3\rangle), \frac{1}{\sqrt{2}}(|\phi_2\rangle - |\phi_3\rangle)\}$ being an orthogonal basis B_2

Measurements in B_1 and B_2 are different.

A HVT is called non-contextual if it assigns value to observables in a context independent way i.e. independent of other observables along with which it is measured.

In this case non-contextuality implies,

$$v_{B_1}(P_1) = v_{B_2}(P_1)$$

Now if there is non-contextual HVT, the probability measure on projectrs has to satisfy the following conditions;

1. $\mu(P) = Tr[\rho P]$ (Gleason's theorem)
2. $\mu(P) = 1 \; or \; 0$

for all projection operators P.

$Tr[\rho P]$ does not depend on the context in which P appears hence non-contextual.

But ρ being a density matrix, there is always a projector Q such that

$$Tr[\rho Q] \neq 1, 0$$

Various versions of Kochen-Specker theorem prove the same with finite no. of projectors.

Impossibility of Non-contextual HVT

18 Vectors in 4-dimension:

$\varphi_1 = (0,0,0,1)$	$\varphi_2 = (0,0,1,0)$	$\varphi_3 = (1,1,0,0)$
$\varphi_4 = (1,-1,0,0)$	$\varphi_5 = (0,1,0,0)$	$\varphi_6 = (1,01,0)$
$\varphi_7 = (1,0,-1,0)$	$\varphi_8 = (1,-1,1,-1)$	$\varphi_9 = (0,0,1,1)$
$\varphi_{10} = (1,1,1,1)$	$\varphi_{11} = (0,1,0,-1)$	$\varphi_{12} = (1,0,0,1)$
$\varphi_{13} = (1,0,0,-1)$	$\varphi_{14} = (0,1,-1,0)$	$\varphi_{15} = (1,1,-1,1)$
$\varphi_{16} = (1,1,1,-1)$	$\varphi_{17} = (-1,1,1,1)$	$\varphi_{18} = (1,-1,-1,1)$

Rules of Value Assignment:

1) $v(\phi_i) \equiv v(|\phi_i\rangle\langle\phi_i|) = 0, 1$

2) $\sum v(\phi_i) = 1, \{|\phi_i\rangle\}$ *form an orthogonal basis.*

$$v(\varphi_1) + v(\varphi_2) + v(\varphi_3) + v(\varphi_4) = 1$$

$$v(\varphi_1) + v(\varphi_5) + v(\varphi_6) + v(\varphi_7) = 1$$

$$v(\varphi_8) + v(\varphi_{18}) + v(\varphi_3) + v(\varphi_9) = 1$$

$$v(\varphi_8) + v(\varphi_{10}) + v(\varphi_7) + v(\varphi_{11}) = 1$$

$$v(\varphi_2) + v(\varphi_5) + v(\varphi_{12}) + v(\varphi_{13}) = 1$$

$$v(\varphi_{18}) + v(\varphi_{10}) + v(\varphi_{13}) + v(\varphi_{14}) = 1$$

$$v(\varphi_{15}) + v(\varphi_{16}) + v(\varphi_4) + v(\varphi_9) = 1$$

$$v(\varphi_{15}) + v(\varphi_{17}) + v(\varphi_6) + v(\varphi_{11}) = 1$$

$$v(\varphi_{16}) + v(\varphi_{17}) + v(\varphi_{12}) + v(\varphi_{14}) = 1$$

If added, the L.H.S. is even as every vector has appeared twice and the R.H.S. is odd.

It shows that non-contextual HVT, in general can not reproduce quantum mechanics.

ψ- Epistemic theory makes life simple

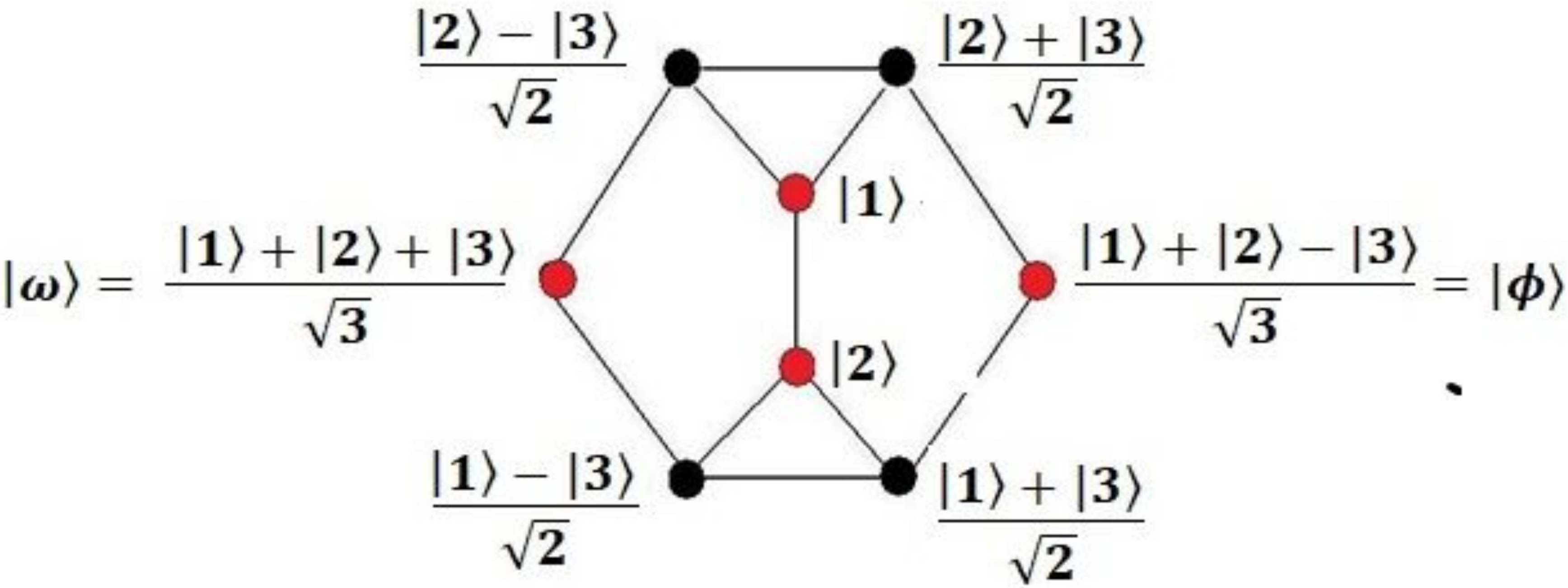
$|\phi\rangle$ *and* $|\omega\rangle$ *are non-orthogonal.*

There my exist λ *such that*

$\theta_\psi(\lambda) \neq 0$ **and** $\theta_\omega(\lambda) \neq 0$

We consider such HVT state λ and observe the following

For this HVT state , $v_\lambda(P_\psi) = 1 \; and \; v_\lambda(P_\varphi) = 1$



(Leifer & Spekkens, 2005, PRL)

Preparation Contextuality of qubit

(Spekkens, PRA, 2005)

Various decomposition of $\boldsymbol{\rho_n}$

$$\rho_n = \frac{1-q}{2}|\phi_n^{\perp}\rangle\langle\phi_n^{\perp}| + \frac{1+q}{2}|\phi_n\rangle\langle\phi_n|$$

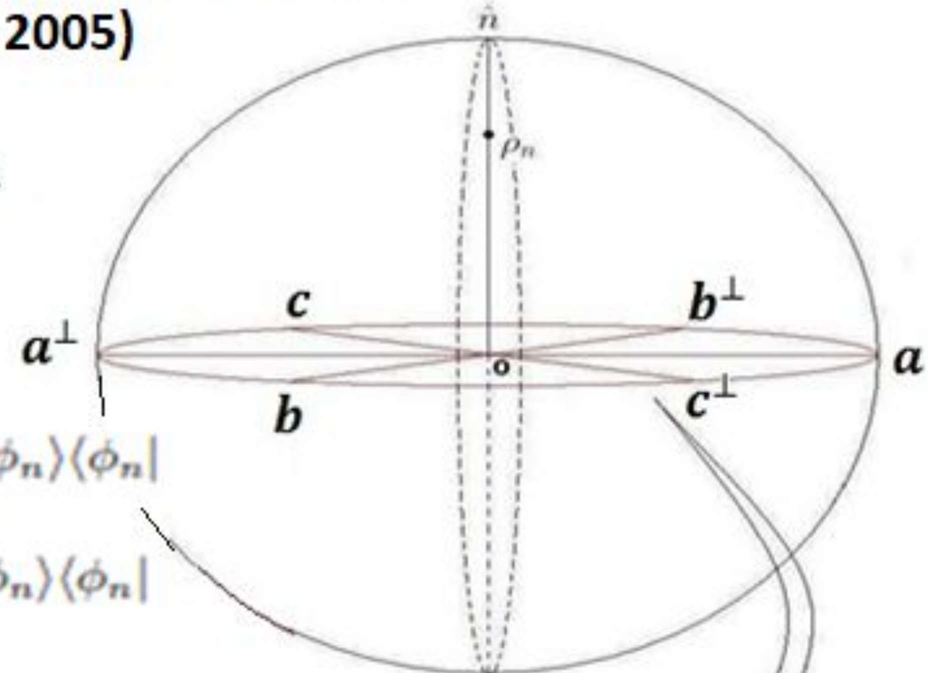
$$\rho_n = \frac{1-q}{2}(|\psi_a\rangle\langle\psi_a| + |\psi_a^{\perp}\rangle\langle\psi_a^{\perp}|) + q|\phi_n\rangle\langle\phi_n|$$

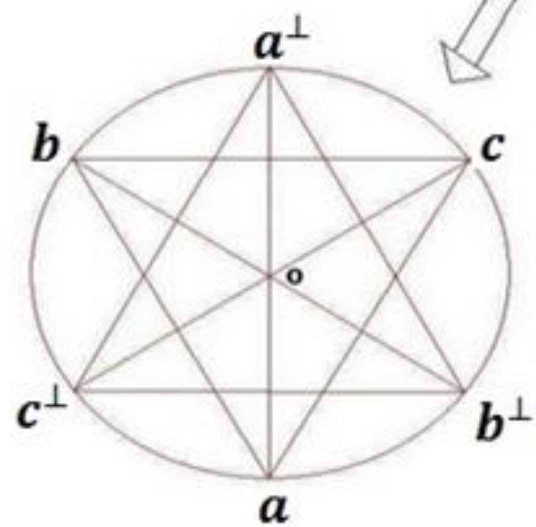
$$\rho_n = \frac{1-q}{2}(|\psi_b\rangle\langle\psi_b| + |\psi_b^{\perp}\rangle\langle\psi_b^{\perp}|) + q|\phi_n\rangle\langle\phi_n|$$

$$\rho_n = \frac{1-q}{2}(|\psi_c\rangle\langle\psi_c| + |\psi_c^{\perp}\rangle\langle\psi_c^{\perp}|) + q|\phi_n\rangle\langle\phi_n|$$

$$\rho_n = \frac{1-q}{3}(|\psi_a\rangle\langle\psi_a| + |\psi_b\rangle\langle\psi_b| + |\psi_c\rangle\langle\psi_c|) + q|\phi_n\rangle\langle\phi_n|$$

$$\rho_n = \frac{1-q}{3}(|\psi_a^{\perp}\rangle\langle\psi_a^{\perp}| + |\psi_b^{\perp}\rangle\langle\psi_b^{\perp}| + |\psi_c^{\perp}\rangle\langle\psi_c^{\perp}|) + q|\phi_n\rangle\langle\phi_n|.$$





Preparation non-contextuality implies:

$$\begin{aligned}
\mu(\lambda|\rho_n) &= \frac{1-q}{2}\mu(\lambda|\phi_n^\perp) + \frac{1+q}{2}\mu(\lambda|\phi_n) \\
&= \frac{1-q}{2}[\mu(\lambda|\psi_a) + \mu(\lambda|\psi_a^\perp)] + q\mu(\lambda|\phi_n) \\
&= \frac{1-q}{2}[\mu(\lambda|\psi_b) + \mu(\lambda|\psi_b^\perp)] + q\mu(\lambda|\phi_n) \\
&= \frac{1-q}{2}[\mu(\lambda|\psi_c) + \mu(\lambda|\psi_c^\perp)] + q\mu(\lambda|\phi_n) \\
&= \frac{1-q}{3}[\mu(\lambda|\psi_a) + \mu(\lambda|\psi_b) + \mu(\lambda|\psi_c)] + q\mu(\lambda|\phi_n) \\
&= \frac{1-q}{3}[\mu(\lambda|\psi_a^\perp) + \mu(\lambda|\psi_b^\perp) + \mu(\lambda|\psi_c^\perp)] + q\mu(\lambda|\phi_n)
\end{aligned}$$

Orthogonality implies:

$$\mu(\lambda|\phi_n)\mu(\lambda|\phi_n^\perp) = 0$$
$$\mu(\lambda|\psi_a)\mu(\lambda|\psi_a^\perp) = 0$$
$$\mu(\lambda|\psi_b)\mu(\lambda|\psi_b^\perp) = 0$$
$$\mu(\lambda|\psi_c)\mu(\lambda|\psi_c^\perp) = 0$$

All these equations imply:

$$\mu_{\phi_n}(\lambda) = \mu_{\phi_n^\perp}(\lambda) = \mu_{\phi_a}(\lambda) = \mu_{\phi_a^\perp}(\lambda) = \mu_{\phi_b}(\lambda) = \mu_{\phi_b^\perp}(\lambda)$$
$$= \mu_{\phi_c}(\lambda) = \mu_{\phi_c^\perp}(\lambda) = 0$$

(Spekkens, Phys.Rev.A, 2005 & M.Banik et al, Found. Phys., 2014)

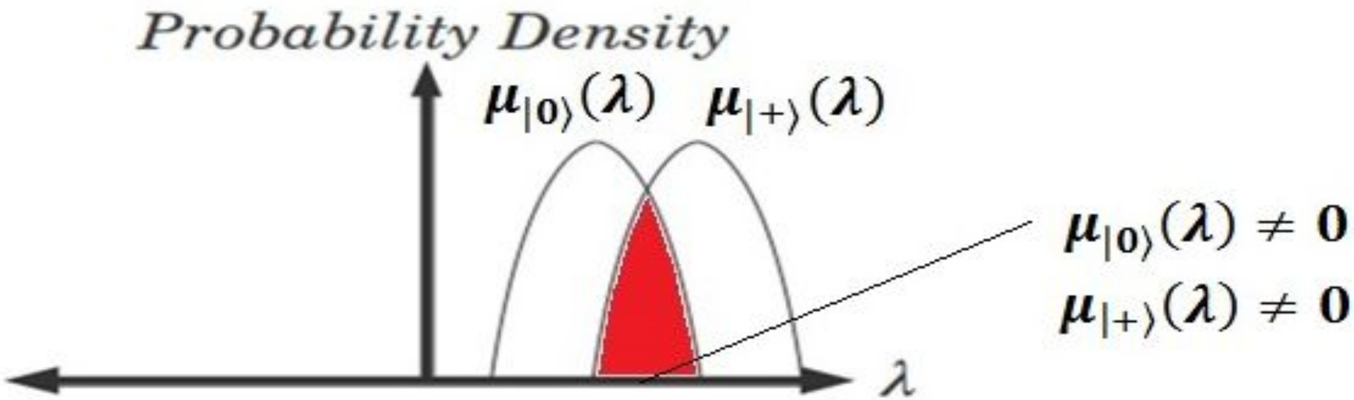
Bolt from the blue

(PBR-Result)

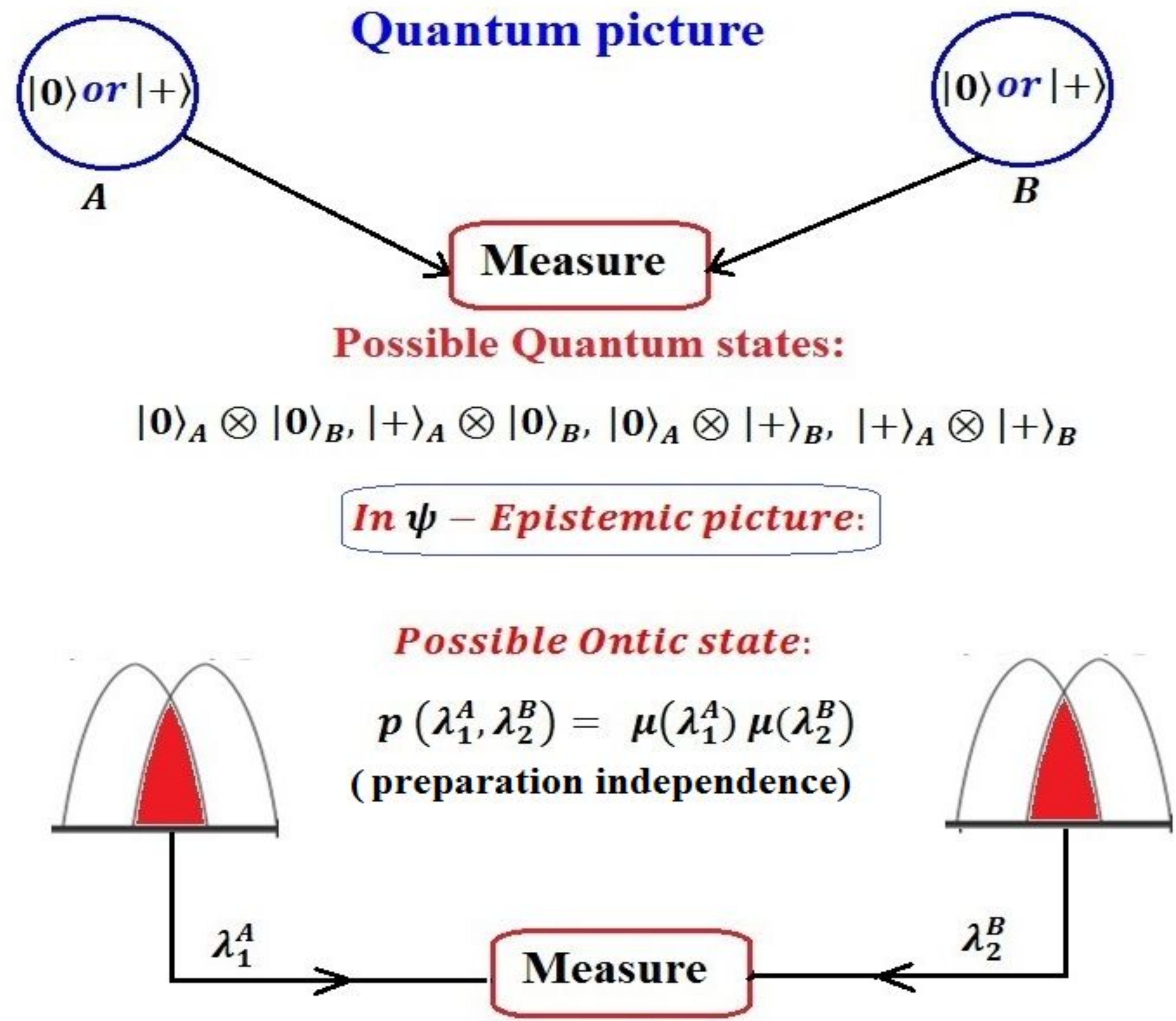
Consider the following states:

$$|0\rangle \; and \; |+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

***In ψ − Epistemic picture*:**



So it is expected that when the ontic state λ comes from the red portion, one will not be able to distinguish the quantum states.


Quantum picture
$|0\rangle$ or $|+\rangle$
A
$|0\rangle$ or $|+\rangle$
B
Measure
Possible Quantum states:
$|0\rangle_A \otimes |0\rangle_B, |+\rangle_A \otimes |0\rangle_B, |0\rangle_A \otimes |+\rangle_B, |+\rangle_A \otimes |+\rangle_B$
In ψ − Epistemic picture:
Possible Ontic state:
$p\left(\lambda_1^A, \lambda_2^B\right) = \mu(\lambda_1^A)\,\mu(\lambda_2^B)$
(preparation independence)
λ_1^A
λ_2^B
Measure

Argument based on $\psi - \textit{Epistemic}$ model would say that in such cases, measurement should not be able even to discard any one of the four states.

Measurement basis	State	Prob. of occurrence
$\frac{1}{\sqrt{2}}[\lvert 0\rangle \otimes \lvert 1\rangle + \lvert 1\rangle \otimes \lvert 0\rangle]$	$\lvert 0\rangle_A \otimes \lvert 0\rangle_B$	0
$\frac{1}{\sqrt{2}}[\lvert 0\rangle \otimes \lvert -\rangle + \lvert 1\rangle \otimes \lvert +\rangle]$	$\lvert 0\rangle_A \otimes \lvert +\rangle_B$	0
$\frac{1}{\sqrt{2}}[\lvert +\rangle \otimes \lvert 1\rangle + \lvert -\rangle \otimes \lvert 0\rangle]$	$\lvert +\rangle_A \otimes \lvert 0\rangle_B$	0
$\frac{1}{\sqrt{2}}[\lvert +\rangle \otimes \lvert -\rangle + \lvert -\rangle \otimes \lvert +\rangle]$	$\lvert +\rangle_A \otimes \lvert +\rangle_B$	0

So for every outcome one state can be excluded which goes against $\psi - \textit{Epistemic}$ theory of QM.

Towards ψ-Complete Theory?

$$p(x|a,b,\lambda) = p(x|a,\lambda)$$

$$p(y|a,b,\lambda) = p(y|b,\lambda)$$

λ *may include quantum state* ψ.

$$\int p(x,y|a,b,\lambda)\mu(\lambda)d\lambda = p(x,y|a,b,\psi_{AB})$$

$$\int p(x|a,\lambda)\mu(\lambda)d\lambda = p(x|a,\rho_A)$$

$$\int p(y|b,\lambda)\mu(\lambda)d\lambda = p(y|b,\rho_B)$$

We search for a theory for which

$$p(x|a,\lambda) \neq p(x|a,\rho_A)$$

Singlet statistics can not be reproduced by such theory.

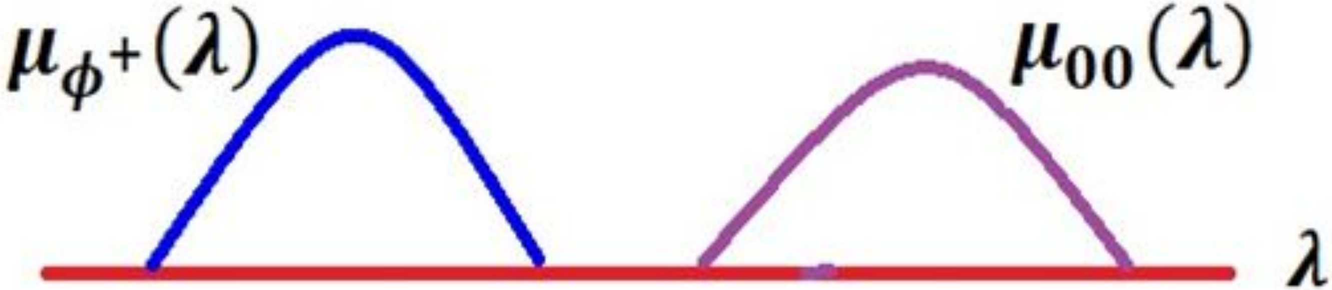
$$p(+1|\sigma_n,\lambda) = p\left(+1\middle|\sigma_n,\frac{I}{2}\right) = p(-1|\sigma_n,\lambda) = p\left(-1\middle|\sigma_n,\frac{I}{2}\right) = \frac{1}{2}$$

(Colbeck & Renner, PRL, 2008 and Liefer, Quanta, 2014.)

Great consequence of C-R result:

$|\phi^+\rangle$ $\quad Prob. of\ down\ result\ is \frac{1}{2}\ for\ all\ \lambda\ in\ its\ support$

$|00\rangle$ $\quad Prob. of\ down\ result\ is\ 0\ for\ all\ \lambda\ in\ its\ support$



To my knowledge, this is the first time, one could show without further assumption excepting no-signaling that two non-orthogonal states are ontologically distinct.

(Leifer, Quanta, 2014)

Some questions:

1) **How to see the preparation independence assumption of PBR result?**
2) **Does $\psi - epistemic$ theory really solve the measurement problem?**
3) **What is the final implication of Colbeck-Renner result?**

My Philosophical position:

I am materialist and hence believe in the existence of matter independent of our consciousness. I do not reduce matter to the sum of our sensations as preached by Positivists like Mach. The properties of matter belong to the category of Science and hence I am not afraid if rules of nature say that quantum particle may not possess definite position and momentum.

To me ψ is, of course, wave function of something that exists independently of my consciousness.