The (quantum) measurement problem

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Main learning:
No reason to give up realism –
but one should give up determinism

NG, Found. Ph. 42, 80 (2012)

The Q Measurement Problem

- How could an apparatus made out of ordinary matter not obey the superposition principle just because of a sticker saying "measurement apparatus"?

- If you believe in reductionism, then there should be some "magical" configurations of atoms and photons to which the superposition principle doesn't apply…
  … or the Schrödinger eq. is not the entire story.

arXiv:1602.01497 & 1701.08300

The deep measurement problem

- Physics is all about extracting information about

  How Nature Does it

- Extracting information = performing measurements.
- A physics theory must tell what is measurable and how – in principle – one should perform measurements.
- Hence the Quantum Measurement Problem is a serious physics problem:

  Without a resolution, Q theory is not physics.

- A resolution will lead to new physics.

arXiv:1602.01497 & 1701.08300

The deep measurement problem

1. Measurements have outcomes.

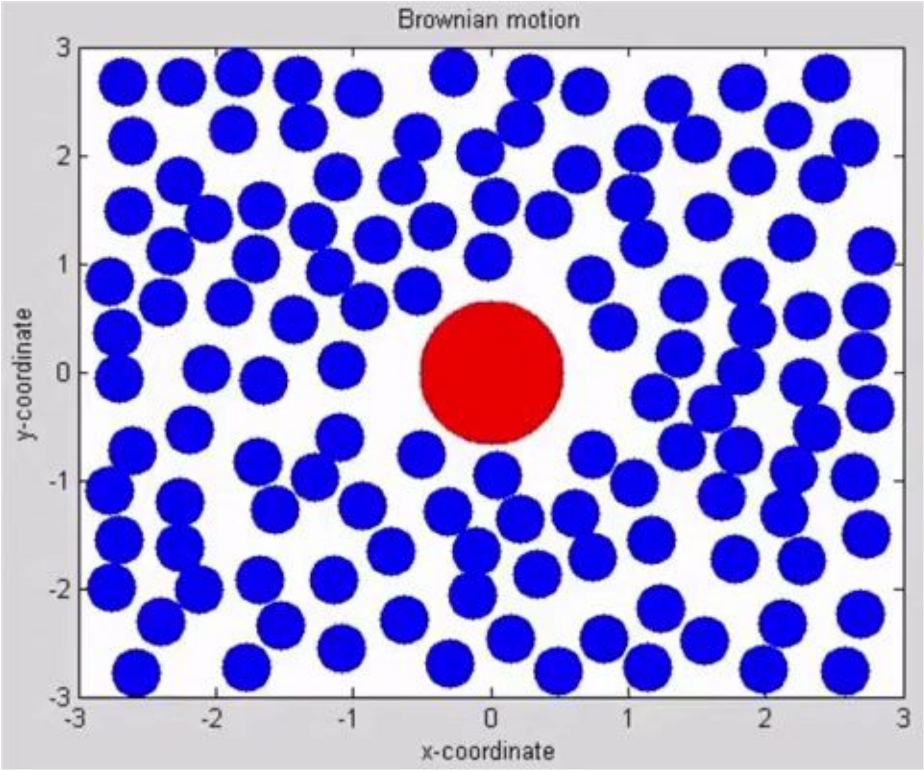
2. Realism: the world out there exists and functions independently of humans (humans can act on the world, but the world would also exist without us).

3. All physical theories require interpretations. Physics is not only about mathematical models and sophisticated technologies: understanding requires "telling stories how nature does it"

4. Don't attack the measurement problem head on, but combine it with good physics.

“Deterministic” chaos

- Chaotic classical dynamical systems are usually considered deterministic.
- But is this really so? Or is it the consequence of the standard interpretation of classical physics?



Example of a chaotic system

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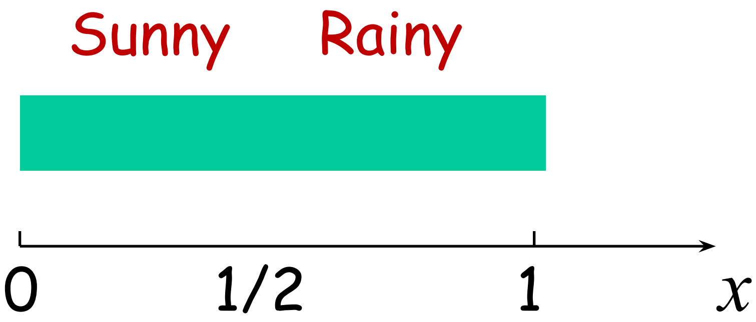

$$x \mapsto \begin{cases} 2x & \text{if } x < 1/2 \\ 2x-1 & \text{if } x \geq 1/2 \end{cases}$$

X binary = $0.b_1 b_2 b_3 b_4 \ldots . b_n \ldots .$

$\rightarrow$ X binary = $0.b_2 b_3 b_4 \ldots . b_n \ldots .$

Whether x lies in the left half or the right half after n steps depends on the n^{th} bit, b_n, of the initial condition x.

Is the billionth bits "physically real"? The question is not whether the billionth bit can be measured, but whether it corresponds to something physical?

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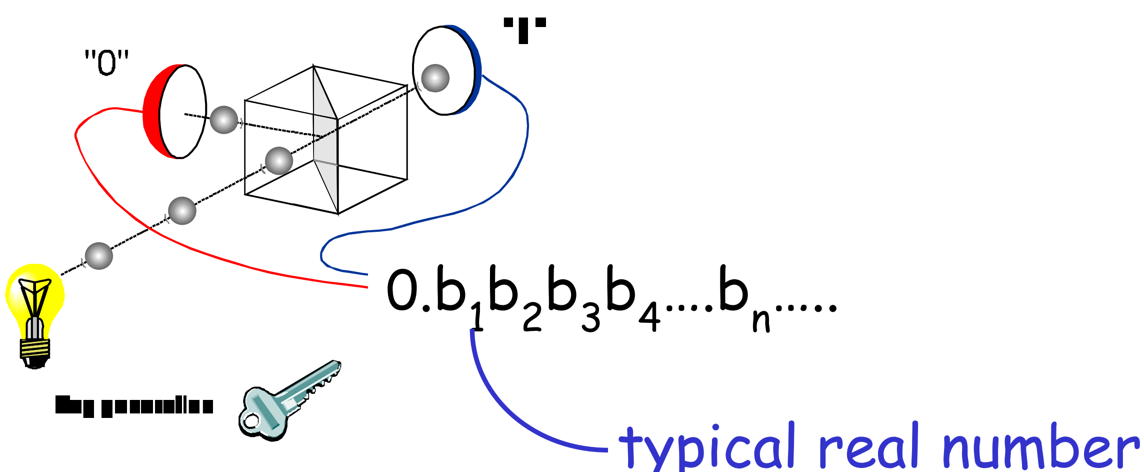
Typical real numbers

- All real numbers one encounters are exceptional:
  they have a name and are defined by a (finite) algorithm: $\sqrt{2}$, π, 77/125, etc.
- The bits of typical real numbers have no structure: the bits are random, as random as quantum measurement outcomes:

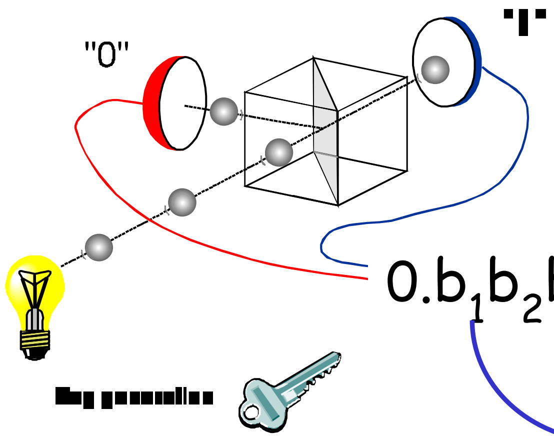


Typical real numbers

- Sinc… names and algo… ain an infi… tion.
- One… s to all que… uman lang…

When we say:
Let $x_0 \in \Re$
what we effectively say is:
let x_0 denote an infinite amount of information.

- The only good way of thinking of a typical real number is the unlimited string of outcomes of a true random number generator.



$0.b_1b_2b_3b_4....b_n.....$

typical real number

G. Chaitin, The Labyrinth of the Continuum, in Meta Math!, Vintage 2008

Finite volume ⇒ finite information

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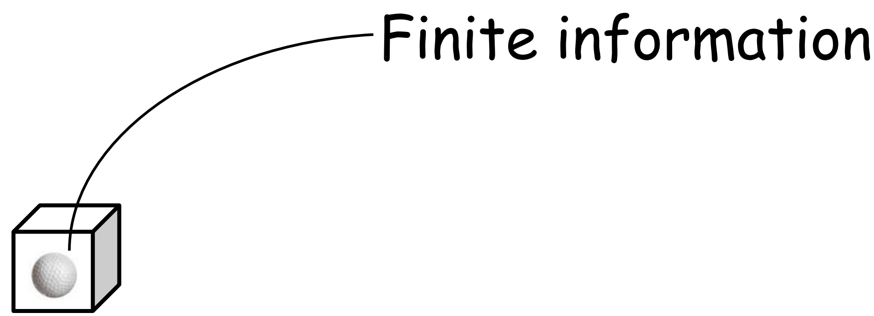
- A finite volume of space can not contain infinitely many bits of information.
- Hence, the position of a classical particle is not a real number.



Beckenstein bound? Yes, but one doesn't need quantum field theory and black holes. Finite information density also applies to Newtonian mechanics and to special relativity.

My intuition of Real Number

0.3906342870591187385492127064927337249582389510
846240185997352602438088375931280495553838500
253876418066556042229443891573679220895949251
6557407035711679933993248067873515575774013122
7431324693922270201711866887060460893183284727
6922985461713919713178282345782969482862331709
948195022204975468668227602507767970267765509
8004162611736118997682498364052959743959340385
72758526775122381194075126706025509511650595705
2699076723890903253959291426547215530138624443
1492940062575082558407172562542821798142848272
4352496992926362434998376619659525125108385861
6044677473288849351659508830828628585264092549
7236099171936642858390197572002307537290953804
45847567821641758542905395011953079755477916723
19340106851299062095688236614693906421190207033
712264516218230979428454131121504352853313616564
87306019819422629712856540790996892145044904136
74815159345054159983821378228976969913337362783

Classical mathematics Real Number

0.390634287059118738549212706492733724958238951 0
846240185997352602438088375931280495553838500
25387641806655604222944389157367922089594 9251

Instead of indeterministic events happening as time passes, all the indeterminism is coded in the initial condition.

692298546171391971317828234578296948286233170 9

Instead of God playing dice as time passes, God played all dies at the big-bang and coded all outcomes in the i.c.

Seems it's the mathematical language that forces us to speak of deterministic chaos !

72360991719366428583901975720023075372909538 04
45847567821641758542905395011953079755477916723
19340106851299062095688236614693906421190207033
71226451621823097942845413112150435285331361656 4
87306019819422629712856540790996892145044904136
74815159345054159983821378228976969913337362783

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Intuitionistic mathematics

Mathematical real numbers are Physical random numbers

real numbers are not really real

- Climate physics uses truncated numbers and stochastic remainders.
  Palmer, T. N. *Nat. Rev. Phys.* **1**, 463–471 (2019).

- And it should be these random numbers that lead us to believe in deterministic chaos ?!?

The mathematical language we speak has a huge influence on the world-view that physics presents to us.

C. Posy, Mathematical Intuitionism, Cambridge Univ. Press, 2020.
N. Gisin, Indeterminism in Physics and Intuitionistic Mathematics, Synthese 2021, doi.org/10.1007/s11229-021-03378-z

Whether Newtonian classical mechanics is deterministic or not, is not a scientific question; it depends on the physical significance one associates with mathematical real numbers,

i.e. it depends on interpretations

The measurement problem affects all indeterministic physics:

- when do potentialities become actual ?
- when does indeterminacy become determined ?
- when does indeterminacy lead to events ?

Supplementary variables

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- Instead of "God playing dice" when potentialities become actual, God played all dice at the initial time and coded all results in the initial condition.
- Postulating that the initial condition of all classical dynamical systems are faithfully described by mathematical real numbers is an elegant way of adding all future results, while making sure that they remain inaccessible for long enough a time.
- ⇒ The real numbers are the hidden (inaccessible) variables of classical mechanics !
  non-contextual
- *The fact is that almost all physicists accept real numbers – without noticing that they are hidden variables – while simultaneously reject Bohmian positions as unnecessary.*

N. Gisin, *Real Numbers are the Hidden Variables of Classical Mechanics*, Quantum Studies: Mathematics and Foundations 7, 197-201 (2020).

Impossible Measurements on Quantum Fields

RAFAEL D. SORKIN

Department of Physics, Syracuse University, Syracuse NY 13244-1130

arXiv:gr-qc/9302018

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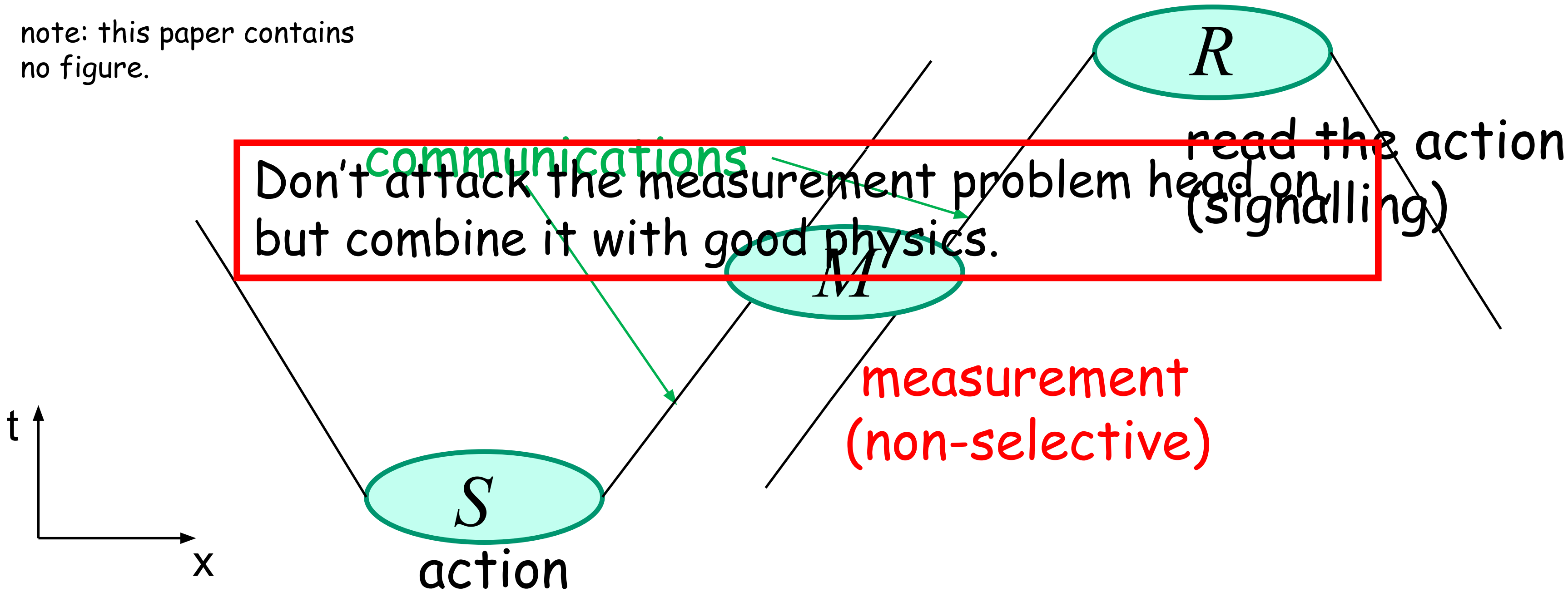
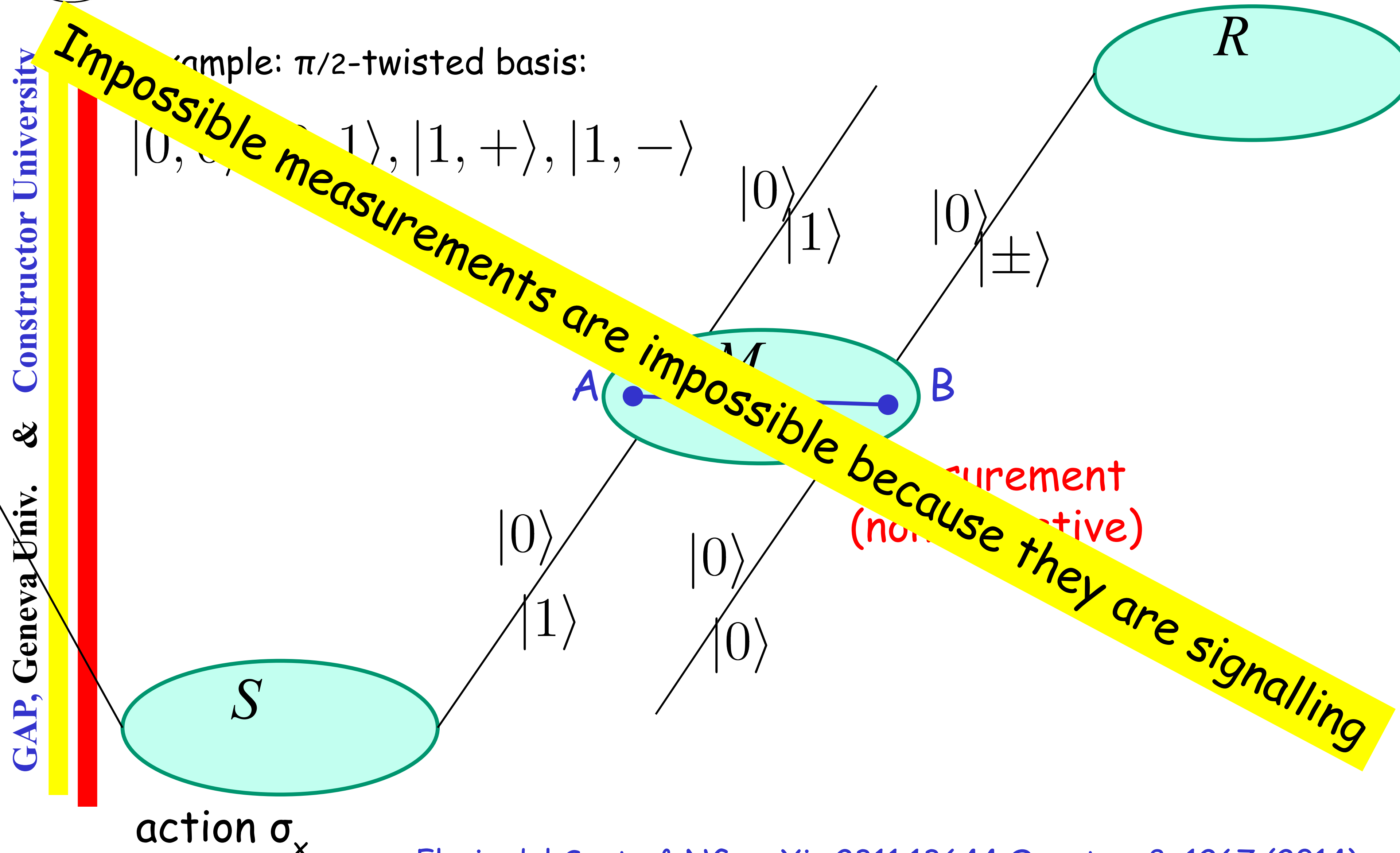
note: this paper contains no figure.



Don't attack the measurement problem head on but combine it with good physics.

- Clearly, it is not the communications that should be questioned, but the measurement.
- The measurement covers some space-like region, ifnot no impossible measurement. Hence, impossible measurements are measurements of NL (non-local) variables.

Impossible Measurements



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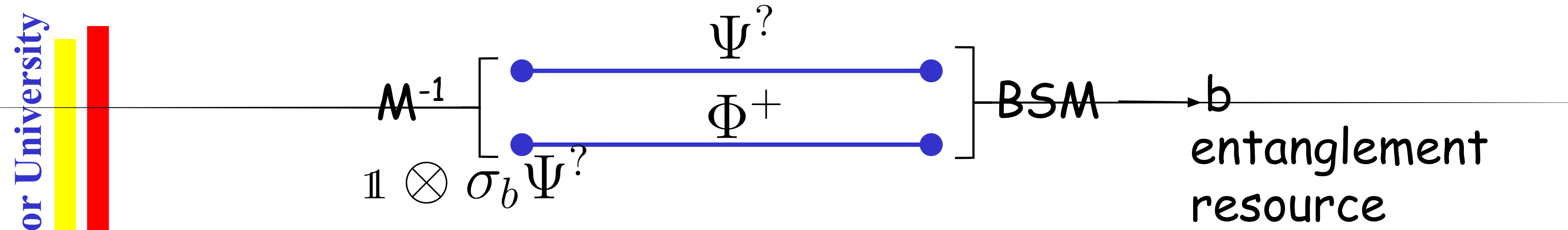
Flavio del Santo & NG, arXiv:2311.13644 Quantum 8, 1267 (2014)

Localizable NLM with 1 ebit



$$M^\dagger \cdot \mathbb{1} \otimes \sigma_b \cdot M = \tilde{P}_b \Phi_b \equiv P_b$$

= permutation + phases of 00,01,10,11

Such NLM are not ideal: they are not immediately reproducible, but they reproduce Born rule.

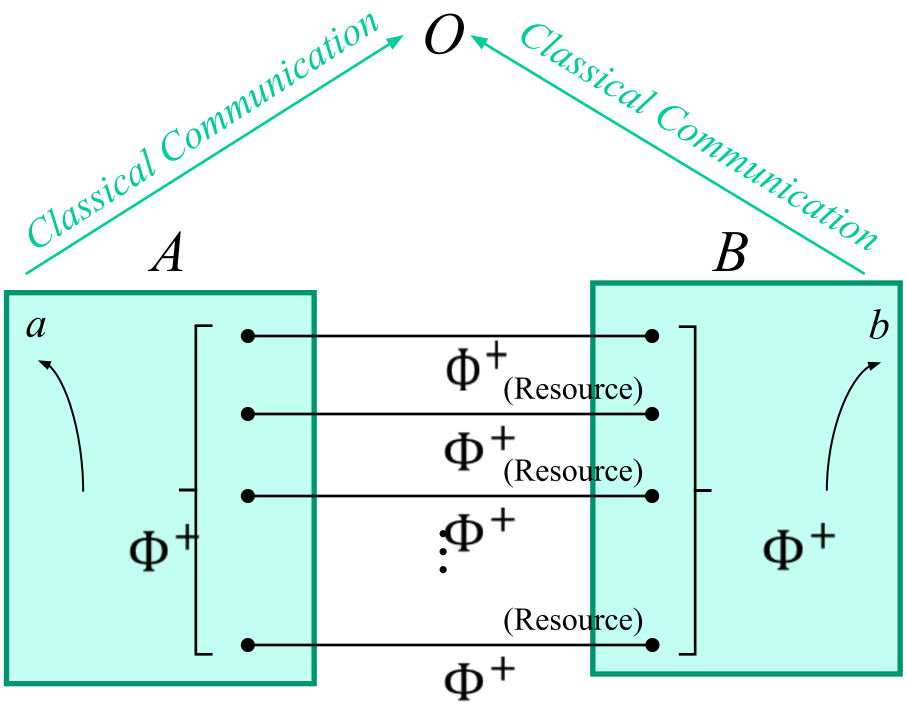
Theorem (1st level of the hierarchy)

The only measurements localizable with 1 ebit, but not with 0 ebits are the

1. π/2-twisted basis measurement, and the
2. Bell State Measurement.

Flavio del Santo & NG, Quantum 8, 1267 (2014)
Jef Pauwels, Alejandro Pozas Kerstjens, Flavio del Santo, NG, arXiv:2408.

Localizable Measurements



All the quantum to classical transitions are localized.

The classical data code the measurement result.

Main Theorem

Theorem: All measurements, any dimension, any nb of parties, are localizable. (Groisman-Reznik, Vaidman)

PHYSICAL REVIEW A **66**, 022110 (2002)

Measurements of semilocal and nonmaximally entangled states

Berry Groisman and Benni Reznik
School of Physics and Astronomy, Tel Aviv University, Tel Aviv 69978, Israel
(Received 15 November 2001; published 16 August 2002)

Consistency with relativistic causality narrows down dramatically the class of measurable observables. We argue that, by weakening the preparation role of ideal measurements, many of these observables become measurable. In particular, we show by applying entanglement assisted remote operations that all Hermitian observables of a (2×2)-dimensional bipartite system are measurable.

VOLUME 90, NUMBER 1 PHYSICAL REVIEW LETTERS week ending 10 JANUARY 2003

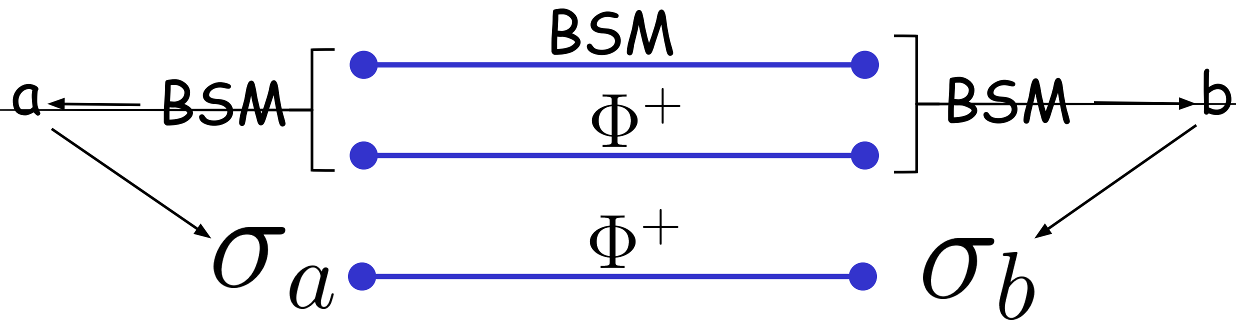
Instantaneous Measurement of Nonlocal Variables

Lev Vaidman
[1]*School of Physics and Astronomy, Raymond and Beverly Sackler Faculty of Exact Sciences, Tel-Aviv University, Tel-Aviv 69978, Israel*
(Received 14 December 2001; revised manuscript received 15 April 2002; published 2 January 2003)

It is shown, under the assumption of the possibility to perform an arbitrary local operation, that all nonlocal variables related to two or more separate sites can be measured instantaneously, except for a finite time required for bringing to one location the classical records from these sites which yield the result of the measurement. It is a verification measurement: it yields reliably the eigenvalues of the nonlocal variables, but it does not prepare the eigenstates of the system.

In a nutshell: "impossible" measurements are localizable, hence possible, but can't be ideal.

Ideal BSM



Now, the BSM is localized and ideal (up to some swaps).

Theorem: (Popescu-Vaidman)
The BSM is the only joint NL measurement of 2 qubits that can be measured ideally without signalling.

Note: the BSM is not a typical measurement, but is exceptional !

⇒ one should not base our intuition of joint measurements on the BSM !

Flavio del Santo & NG, Quantum 8, 1267 (2014)
Jef Pauwels, Alejandro Pozas Kerstjens, Flavio del Santo, NG, arXiv:2408

Localizable NLM with 3 ebit (2nd level)

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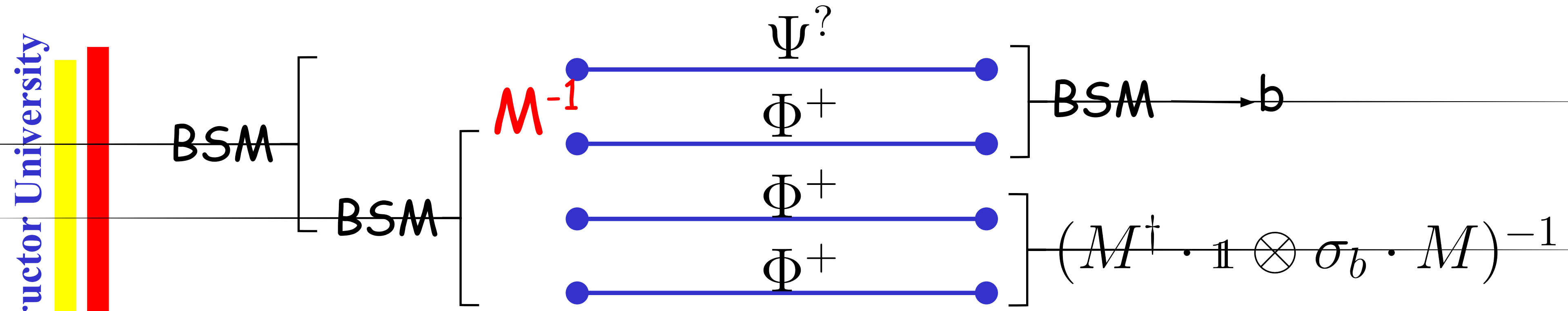

$$(M^\dagger \cdot 1\!\!1 \otimes \sigma_b \cdot M) \cdot \sigma_{a_1} \otimes \sigma_{a_2} \cdot (M^\dagger \cdot 1\!\!1 \otimes \sigma_b \cdot M)$$

= permutation+phases of 00,01,10,11

Theorem (2nd level of the hierarchy)

The only measurements localizable with 3 e-bit, but not with 1 e-bits are the:

1. partial BSM $\{\psi^\pm, |00\rangle, |11\rangle\}$
2. elegant joint measurement (EJM), see next slide,
3. another member of the EJM family
4. two members of the BS family

arXiv:2307:06998

PR Research 6,023085(2024)

Jef Pauwels, Alejandro Pozas Kerstjens, Flavio del Santo, NG, arXiv:2408.

The Elegant Joint Measurement (EJM)

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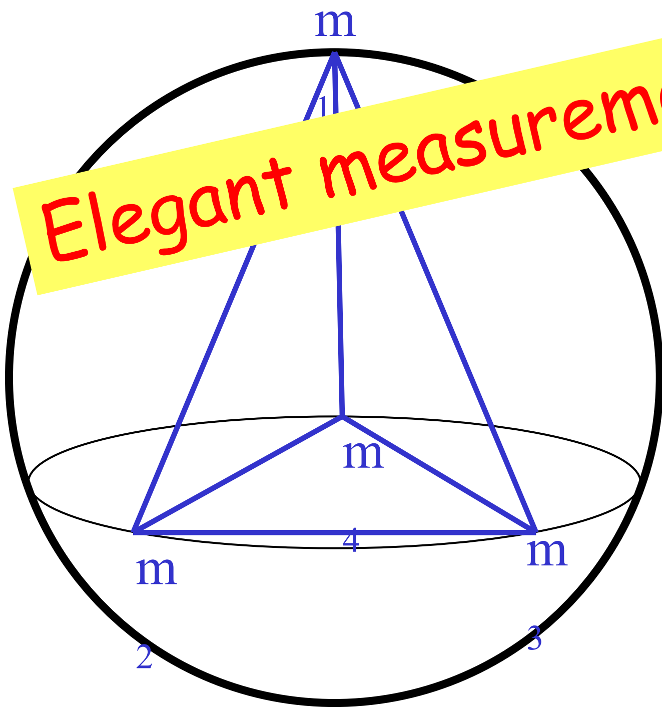
Elegant measurements for n qubits ?

Look fo[r 4 …]gled and
[…]gonal states
[w]ith same degrees of entanglement
and with partial states along the
vertices of the tetrahedron.



$$\left|\Phi_j\right\rangle = c_0\left|\vec{m}_j, -\vec{m}_j\right\rangle + q_0\left|-\vec{m}_j, \vec{m}_j\right\rangle$$

$$= \frac{\sqrt{3}+1}{2\sqrt{2}}\left|\vec{m}_j, -\vec{m}_j\right\rangle + \frac{\sqrt{3}-1}{2\sqrt{2}}\left|-\vec{m}_j, \vec{m}_j\right\rangle$$

$\left\langle \Phi_j \middle| \Phi_i \right\rangle = \delta_{ji}$ & c_0, q_0 are real

$$Tr_B\left(\left|\Phi_j\right\rangle\left\langle\Phi_j\right|\right) = \frac{1}{2}\left(1 + \frac{\sqrt{3}}{2}\vec{m}_j\right) \qquad Tr_A\left(\left|\Phi_j\right\rangle\left\langle\Phi_j\right|\right) = \frac{1}{2}\left(1 - \frac{\sqrt{3}}{2}\vec{m}_j\right)$$

NG, Entropy, «25 years of Q teleportation», 21,325 (2019)
Tavakoli, Pozas-Kerstjens, NG, PRL 126, 220401 (2021)

The Schrödinger eq is not the full story. Not more than Newton's eqs.

- When do potentialities become actual?

1. Spontaneously: It is merely Nature that spontaneously and continually produces new information, eg GRW, QSD, intuitionistic mathematics.
2. When some special conditions are met, eg when the gravitational field gets into "large" superpositions, as in the Diosi-Penrose model.
3. When a higher order level requires it, eg Copenhagen interpretation.

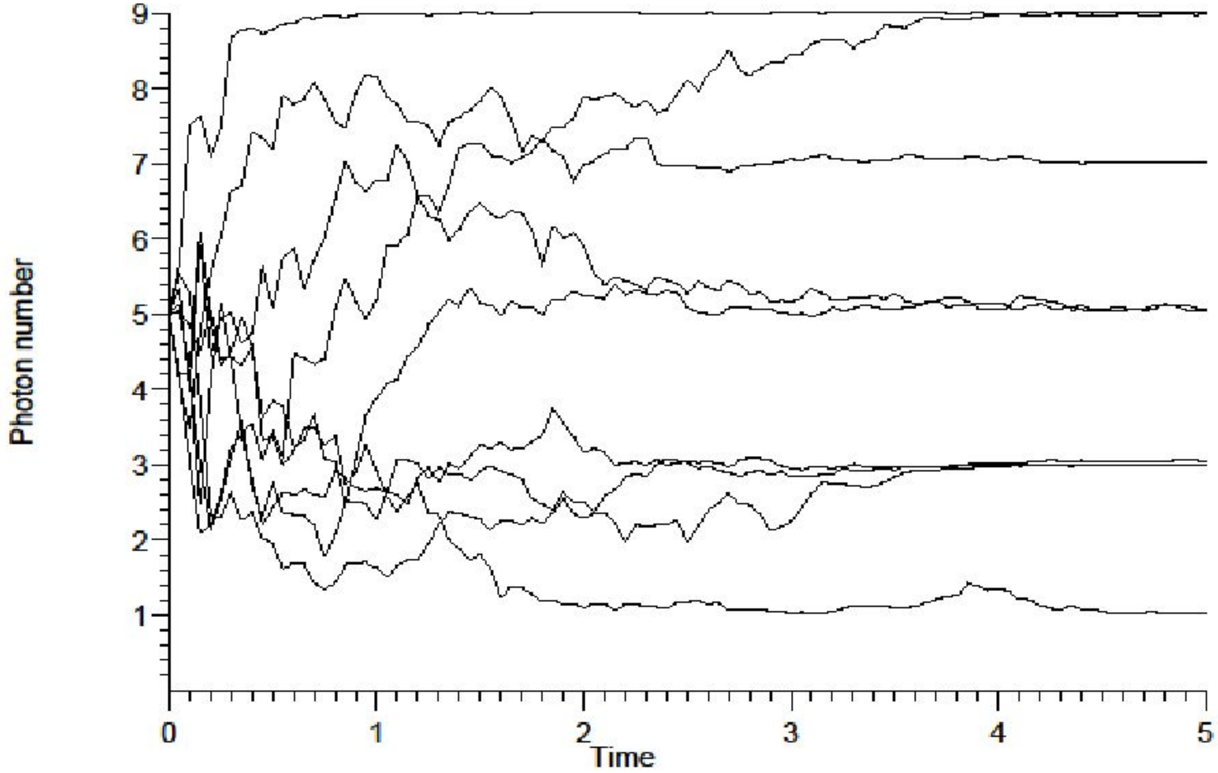
Only options 1 & 2 are compatible with reductionism.

Don't attack the measurement problem head on, but combine it with good physics.

1. Modified Schrödinger Equation

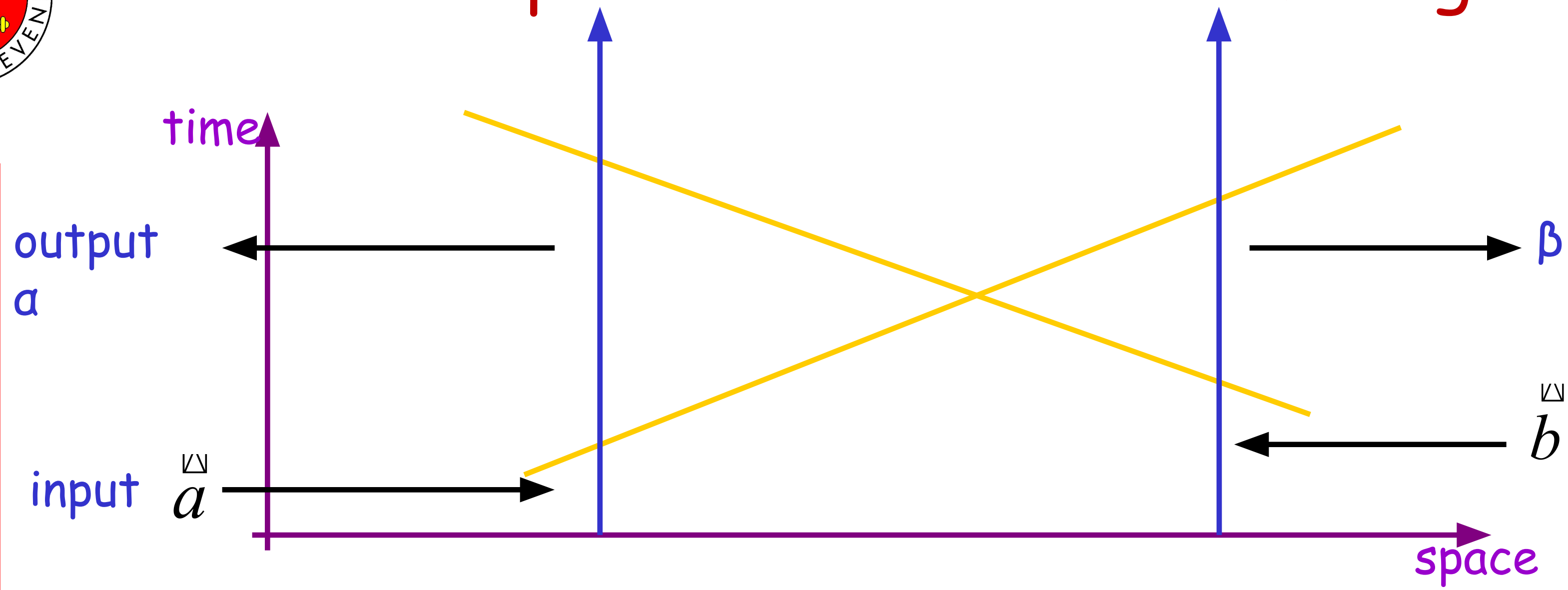
1. The measurement problem arises from the linearity of the Schrödinger equation.
2. The Schrödinger eq. is the only linear eq. that preserves the Hilbert space structure (Wigner's th).
3. Hence, we need a non-linear modification.
4. All deterministic non-linear modifications activate quantum non-locality (activate signalling) (NG, Helv.Phys.Acta 1989)
5. We need a stochastic non-linear Schrödinger eq. that does not activate signalling,(NG, PRL 52, 1657, 1984)
When averaged over the stochastic process, the resulting density matrix follows a closed form master eq.

Stochastic Schrödinger equations GRW - QSD



- ✔ Akin to Brownian motion in Hilbert space, each solution converges to an eigenstate, with the Q probabilities.
- ✔ Predicts new Physics (hence, at least it could be wrong !)
- ✔ Useful for numerical simulations of open Q systems.

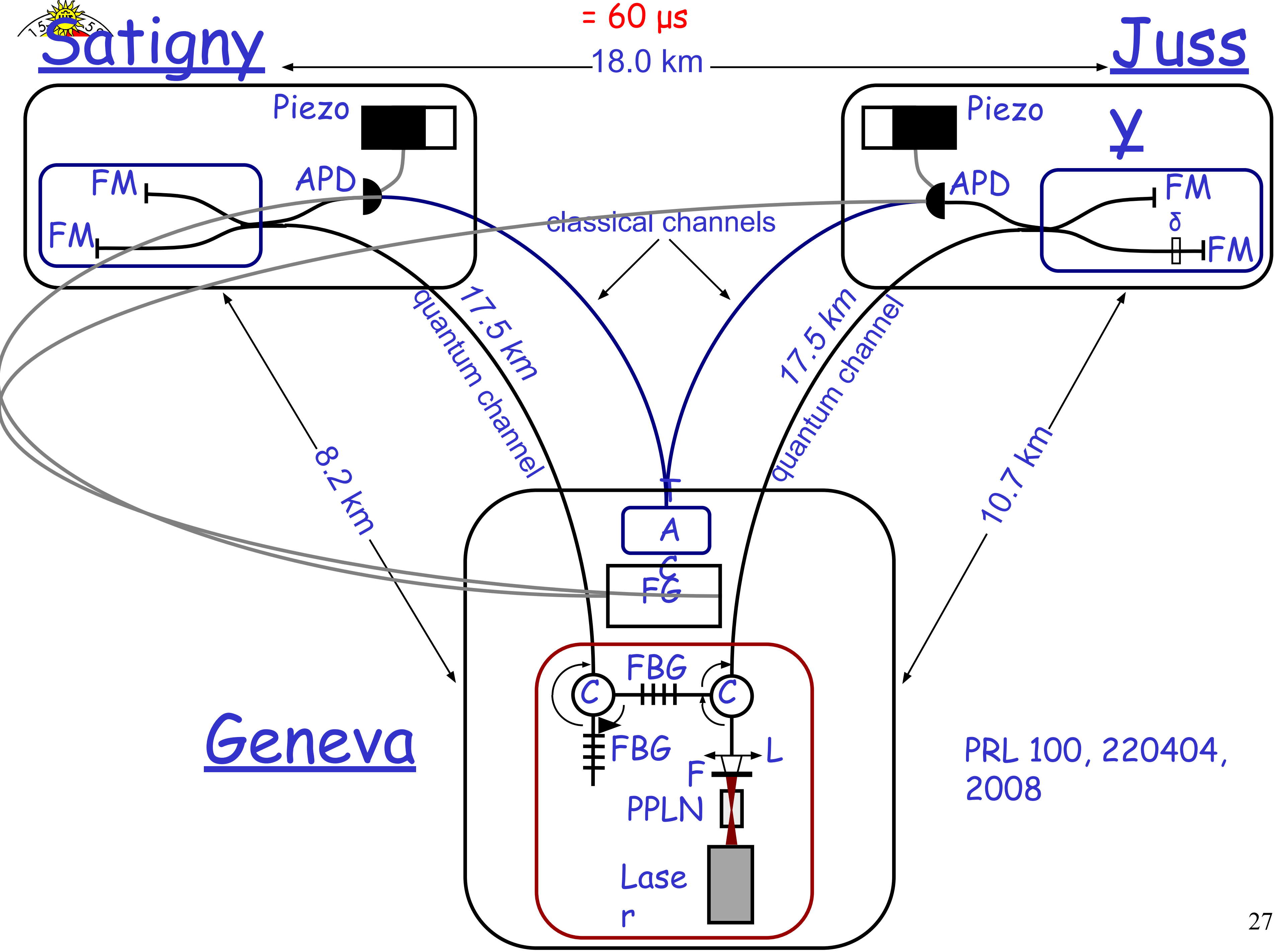
2. When special conditions are met: gravity



- **When is a quantum measurement finished ?**
- Possibly only once a macroscopic mass has significantly moved, as advocated e.g. by Diosi and Penrose.
- In usual Bell tests, detection events only trigger the motion of electrons of insufficient mass to finish the measurement process.

Adrian Kent noticed that according to this plausible assumption, no Bell test so far ensured space-like separation !

A. Kent, arXiv:gr-qc/0507045



PRL 100, 220404, 2008

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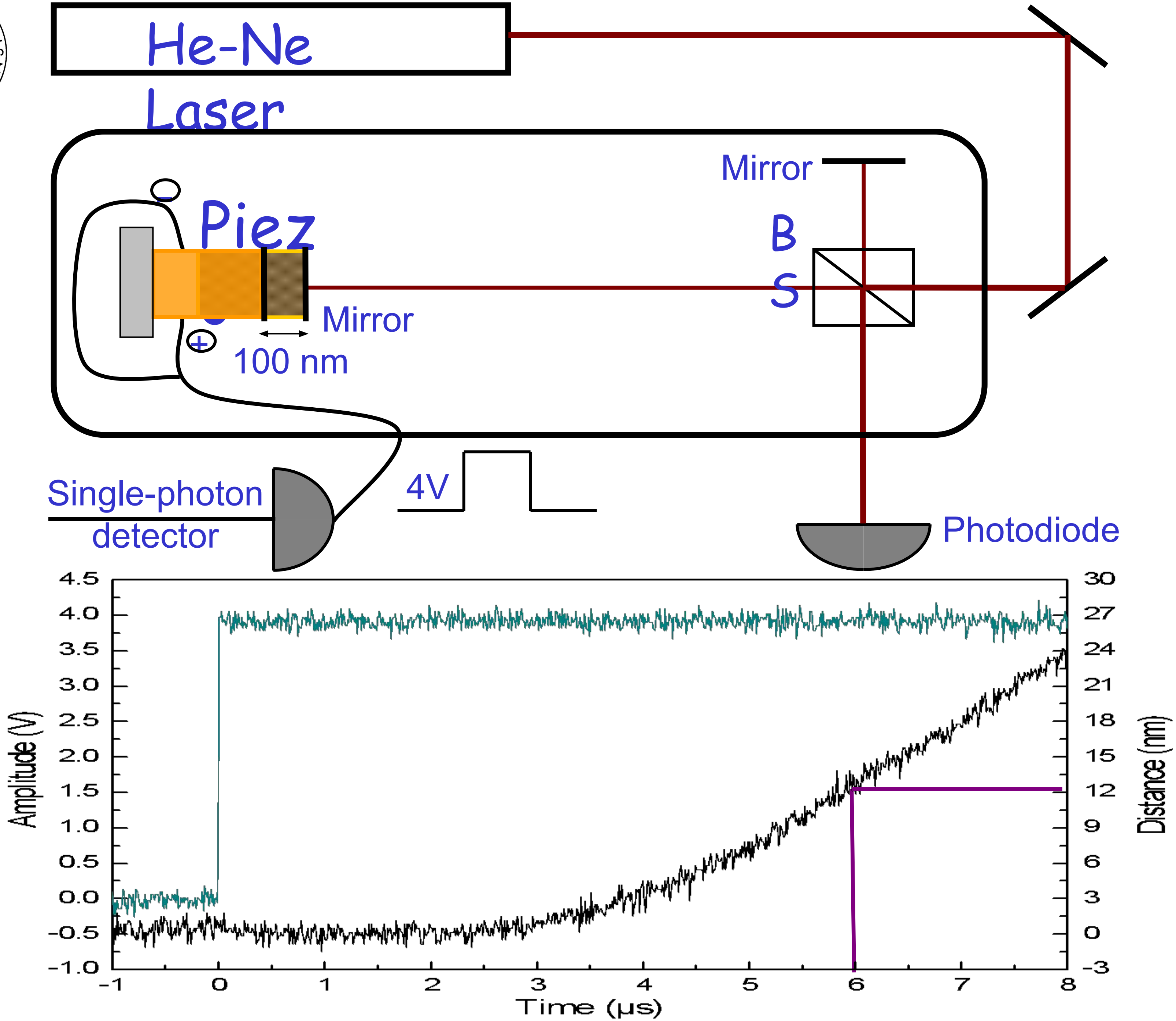

Diosi-Penrose formula for collapse time of the superposition: $\psi_1+\psi_2$

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- Diósi's equation

$$\tau_d^{-1} = \frac{Gm^2}{2\hbar} \iint d^3r\, d^3r' \frac{\left(|\psi_1(r)|^2 - |\psi_2(r)|^2\right)\left(|\psi_1(r')|^2 - |\psi_2(r')|^2\right)}{|r-r'|}$$

- For a parallelepiped mirror

$$\tau_d = \frac{3\hbar V}{2\pi G m^2 d^2}$$

- Numerical application in our case

$$\tau_d = 1\mu s$$

$m = 2\cdot 10^{-6}\,kg$

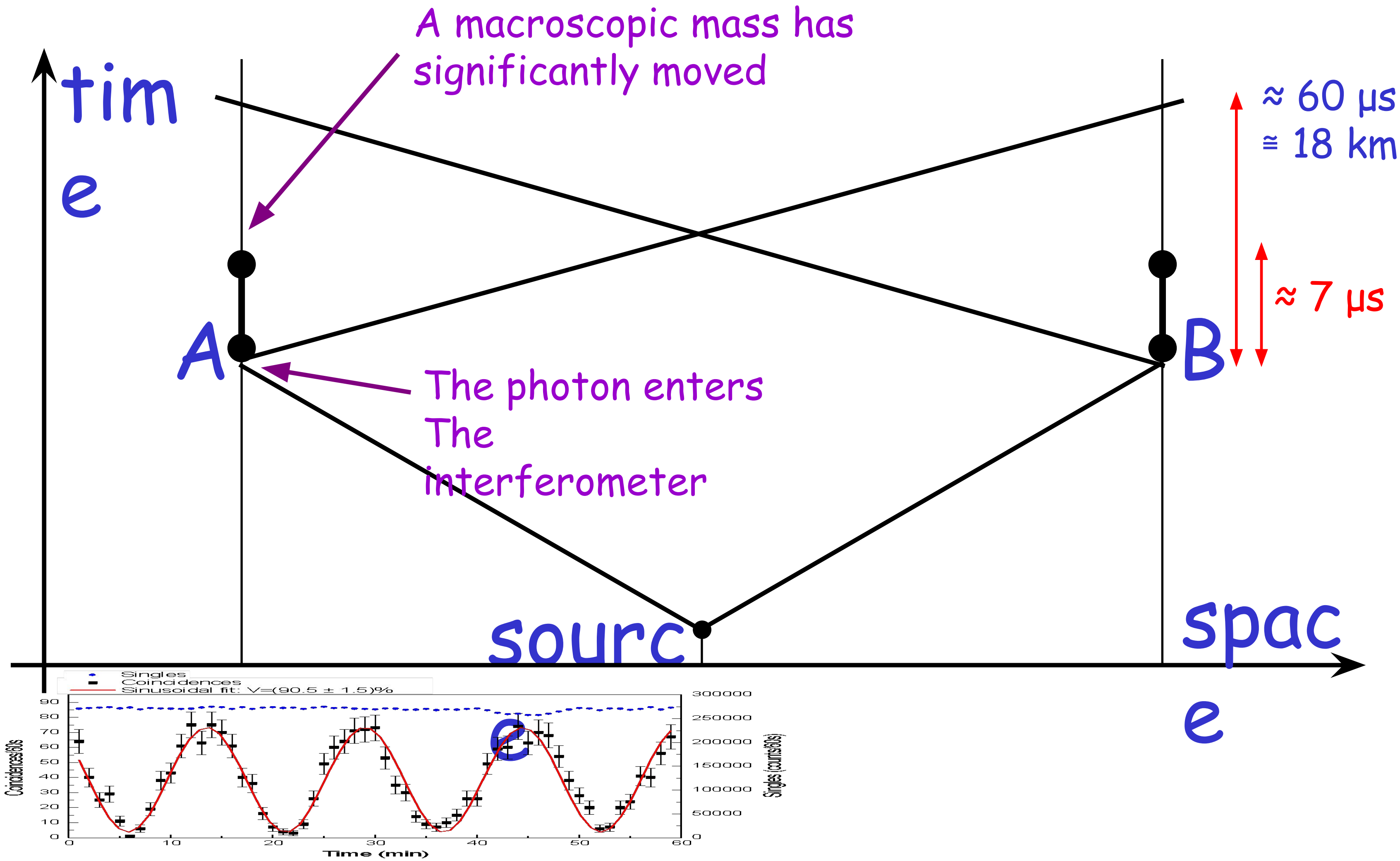
$d = 12.6\,nm$

$V = 3\,mm \times 2\,mm \times 0.15\,mm = 9\cdot 10^{-10}\,m^3$

S. Adler, J.Phys. A40, 755 (2007)

Bell test with true space-like separation

PRL 100, 220404, 2008



Visibility > 90% ⇒ nonlocal correlations between truly space-like separated events.

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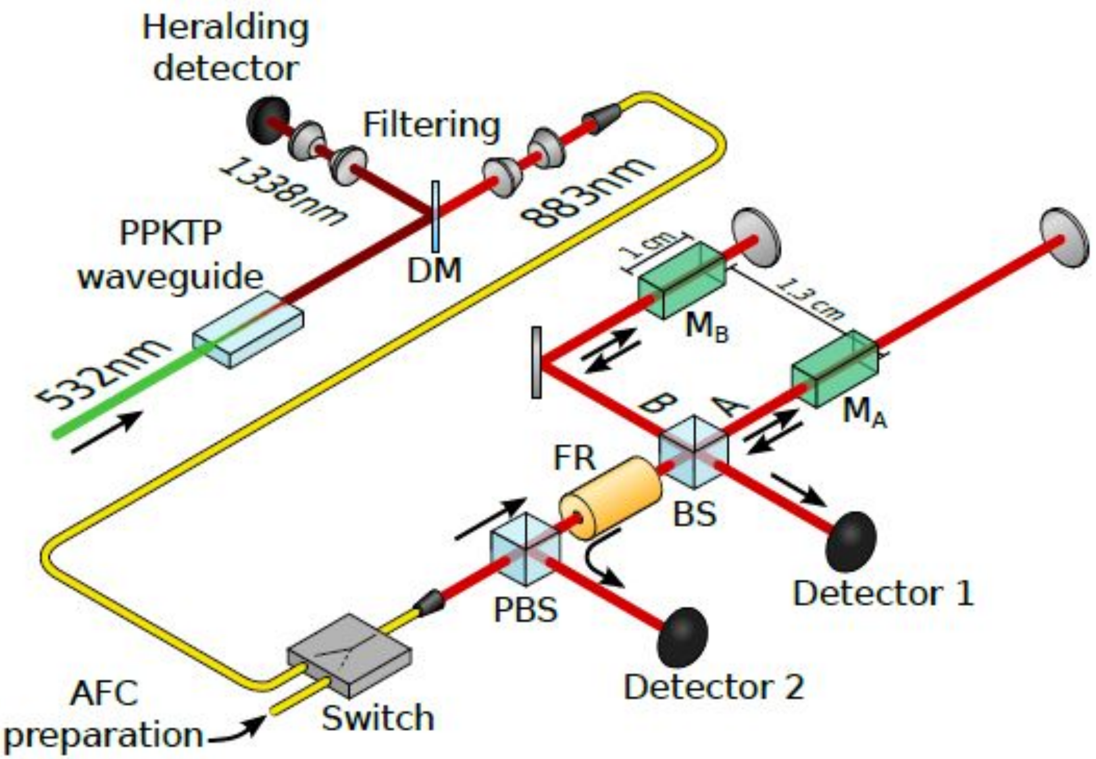
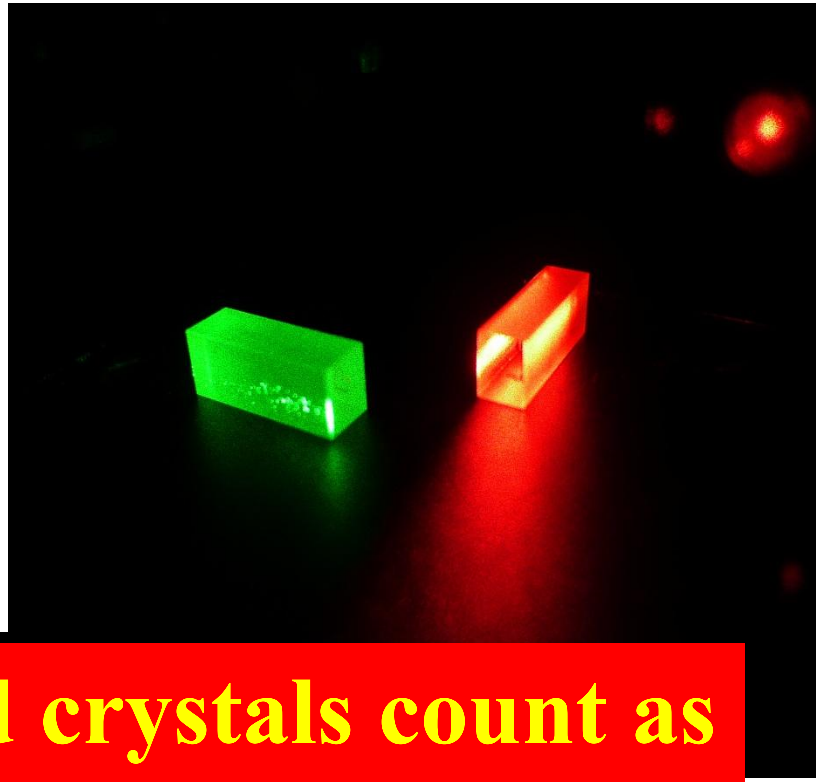
3. When a higher order requires it: macroscopicity





Do these two entangled crystals count as Macroscopic Quantumness ?

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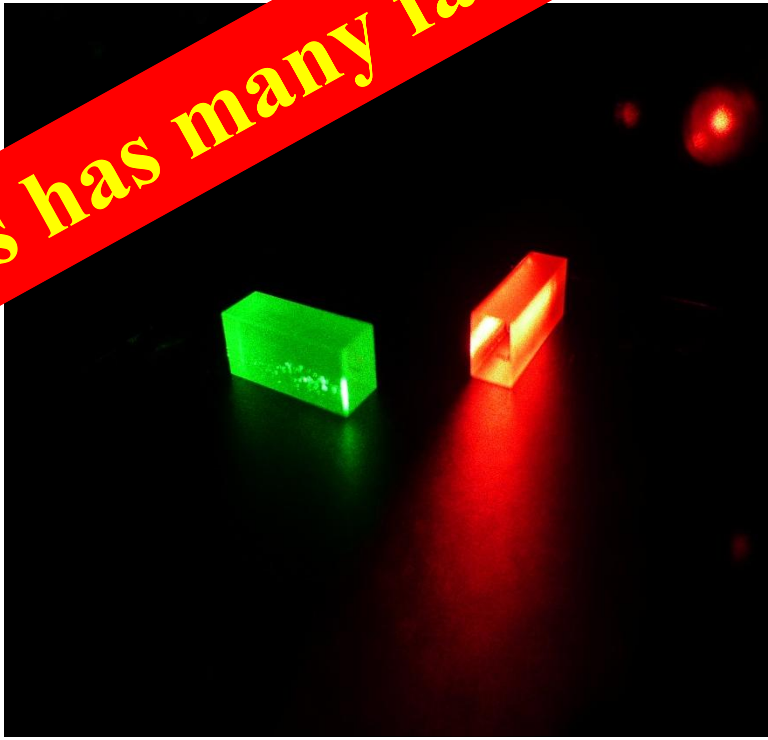
What is macroscopic ?
What is quantum ?

Macroscopic Quantumness has many facets

- Quantum = entanglement.
- Do these 2 crystals coun
  as large entangleme
  No !
- Billion s in a
  m pic object,
  only” one
  - delocalized - excitation



Nature Photonics 6, 234-7, 2012

- How difficult is it to prove quantumness of macroscopic states, P. Sekatski, N. Gisin and N. Sangouard, PRL 113, 090403 (2014).
- The size of quantum superposition as measured with “classical” detectors, P. Sekatski, N. Sangouard and N. Gisin, PRA 89, 012116 (2014).

«Macroscopic Quantumness»

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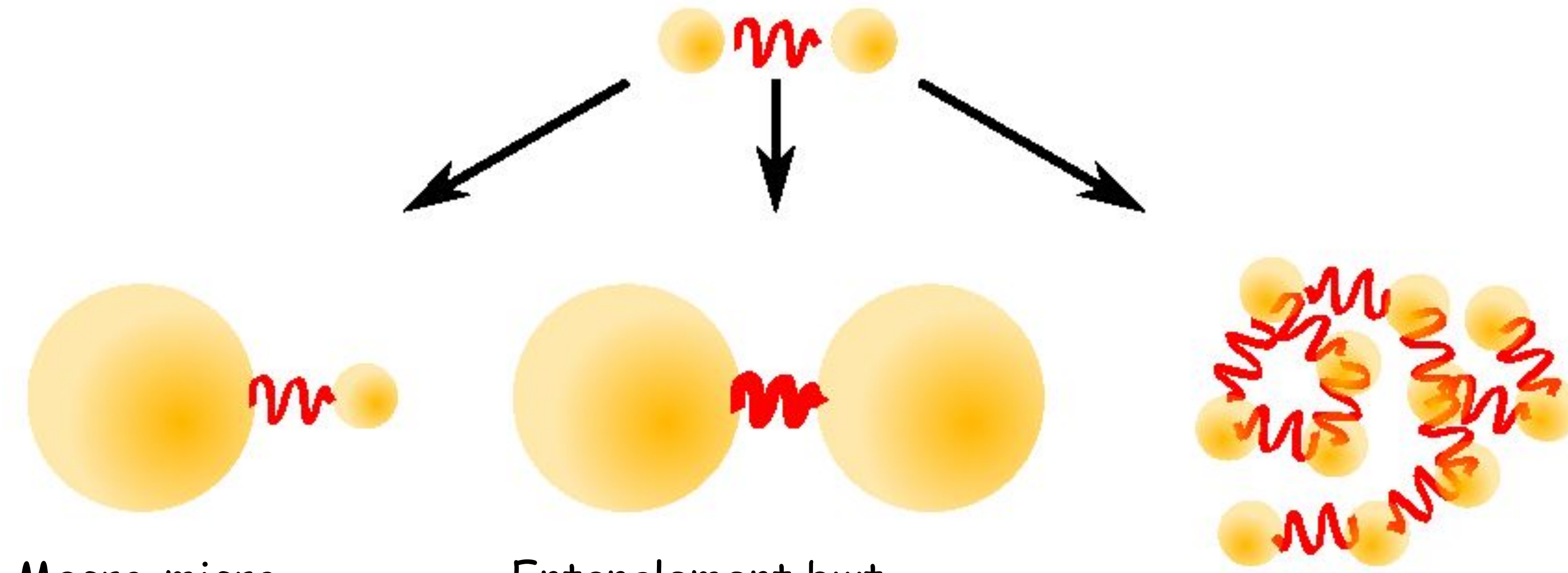


Macro-micro entanglement

Nature Photonics [**6**](#), **234-7 (2012)**
Up to 500 photons

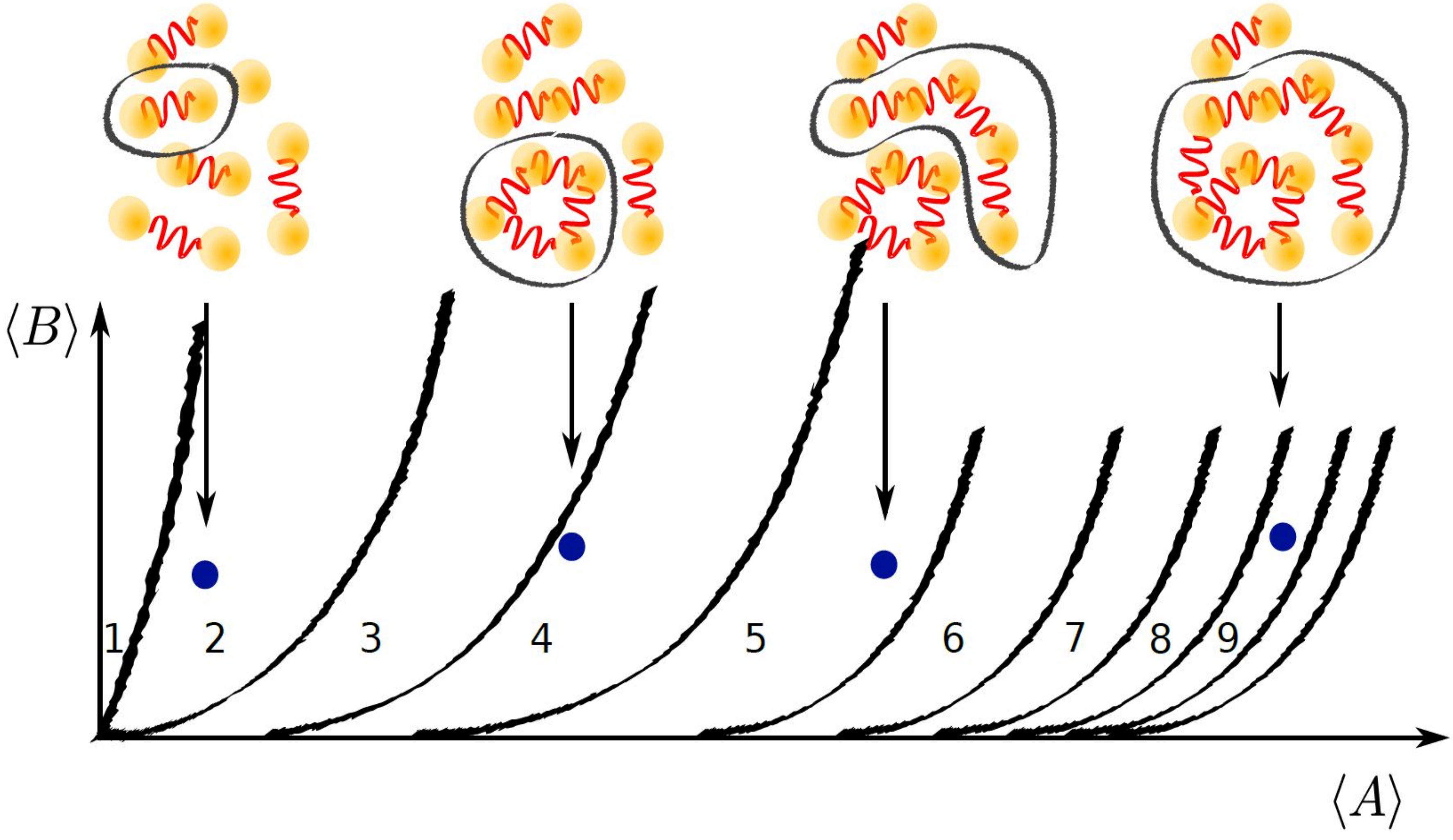
Entanglement bwt large objects, i.e. massive objects, high-dim. Hilbert spaces, etc

PRL 118, 110501 (2017)
Up to 4 e-bits

Entanglement bwt large collections of systems

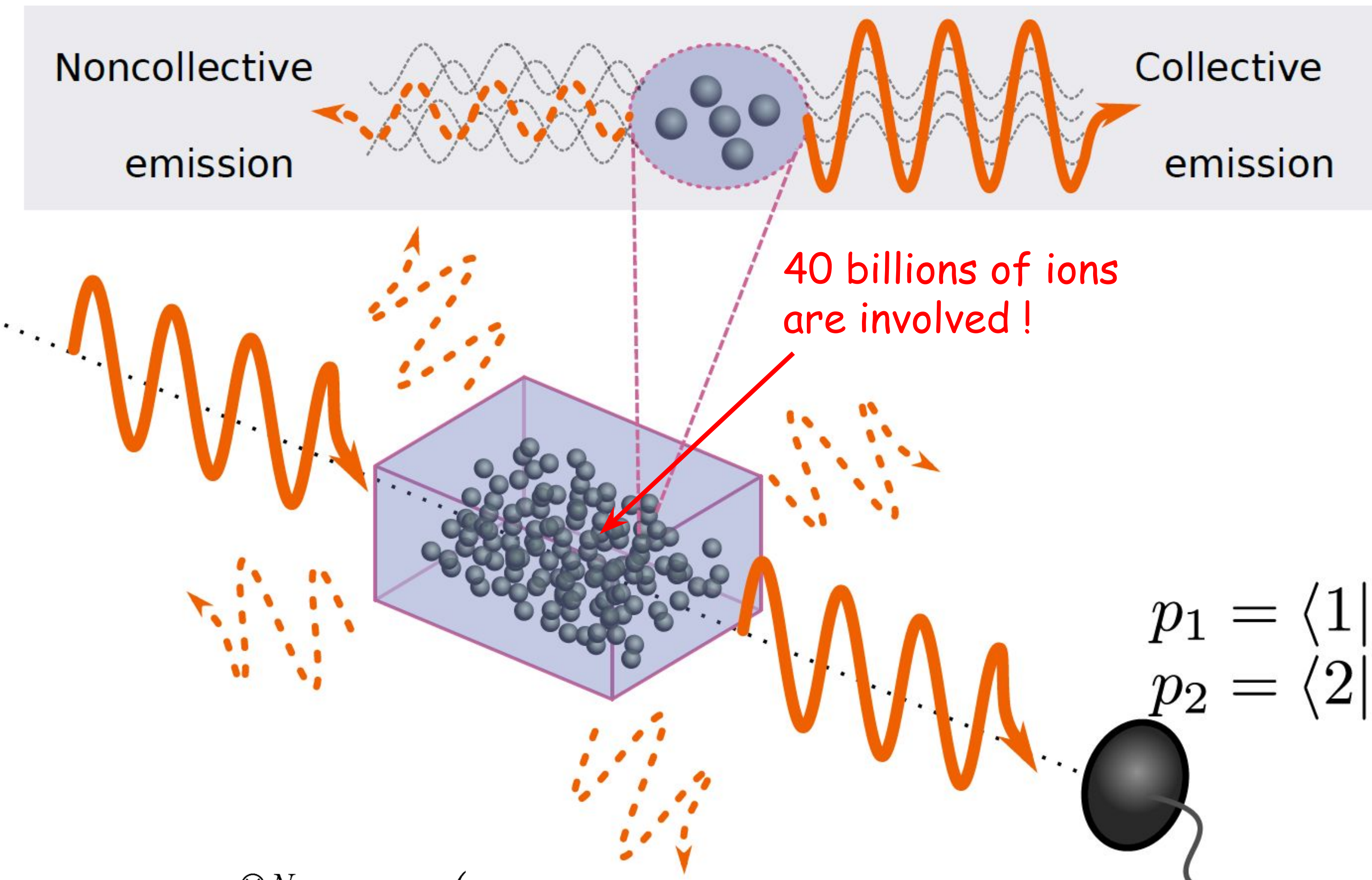
Nature Commun.
8, 907 (2017)

Certify from experimental data the minimal entanglement depth



Nature Commun. *8, 907 (2017)*

Entanglement bwt large collections of systems



$$p_1 = \langle 1|\rho|1\rangle$$
$$p_2 = \langle 2|\rho|2\rangle$$

$$|1\rangle \otimes |g\rangle^{\otimes N} \rightarrow |0\rangle\big(|eg...g\rangle + |geg...g\rangle + ... + |g...geg\rangle + |g...ge\rangle\big)$$

Nature Commun. *8, 907 (2017)*

Certification of entanglement bwt at least 16 millions of ions !

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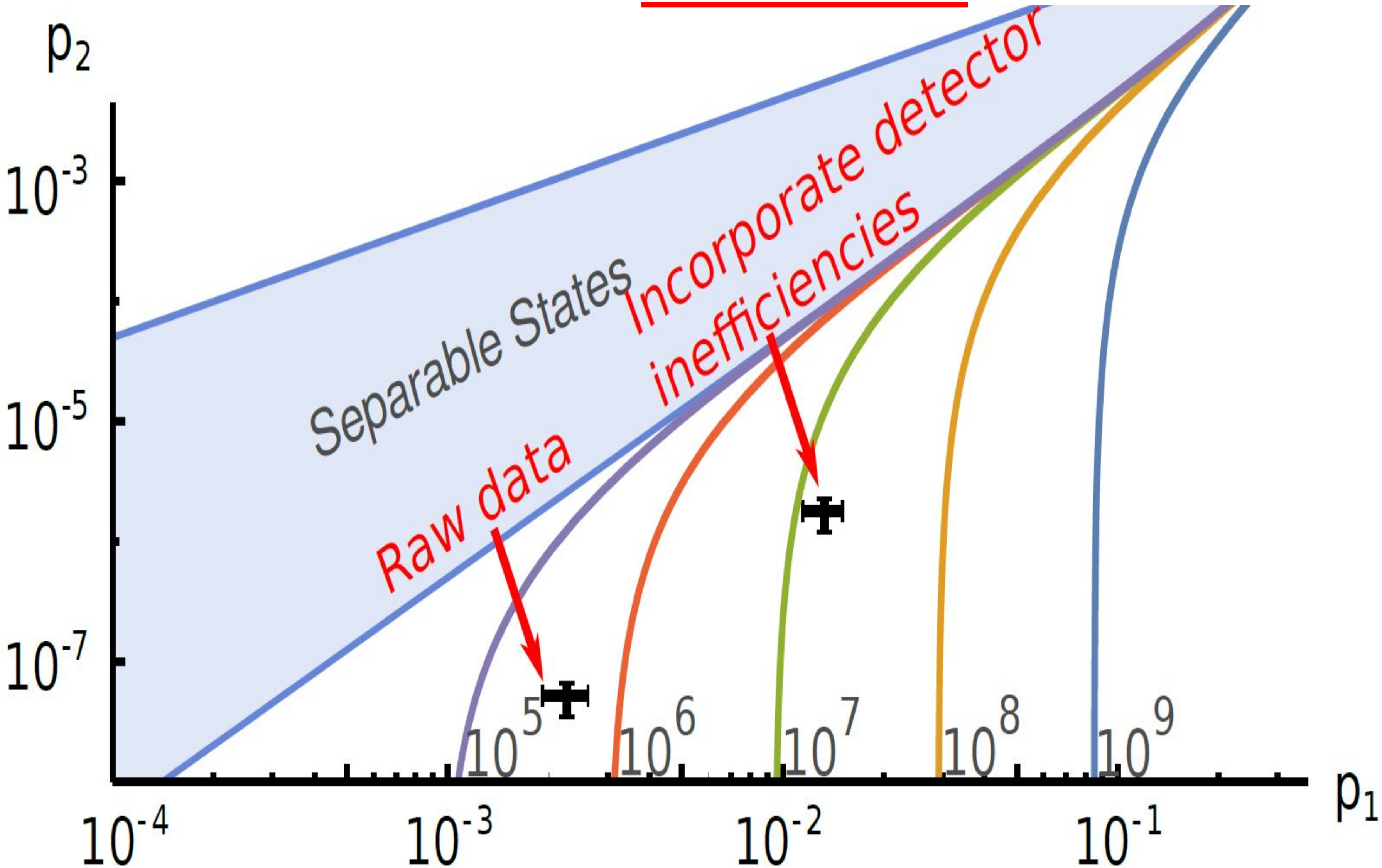

Nature Commun. *8, 907 (2017)*

Conclusion

1. A theory without a measurement model is not physics.
2. Nature is able to produce non-necessary events, hence to produce new information.
3. Ideal measurements are exceptional: the projection postulate applies rarely (though Born rule follows from the linearity of Hilbert spaces).
4. Don’t attack the measurement problem head on, but combine it with good physics: The solutions to the measurement problem will lead to new physics.

Collapse.

else?

… it is likely that it’s all more complicated than that.
… there is lot of good physics along the way.