# 60 years of Bell inequalities:
## towards understanding Quantum Entanglement


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**Group of Applied Physics, University of Geneva**

**Constructor University, Geneva, Switzerland**


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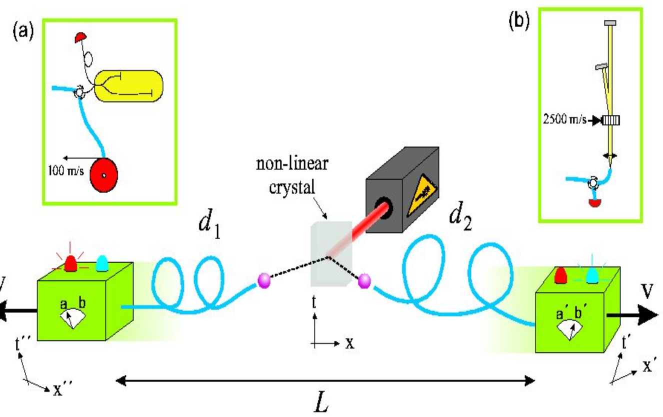


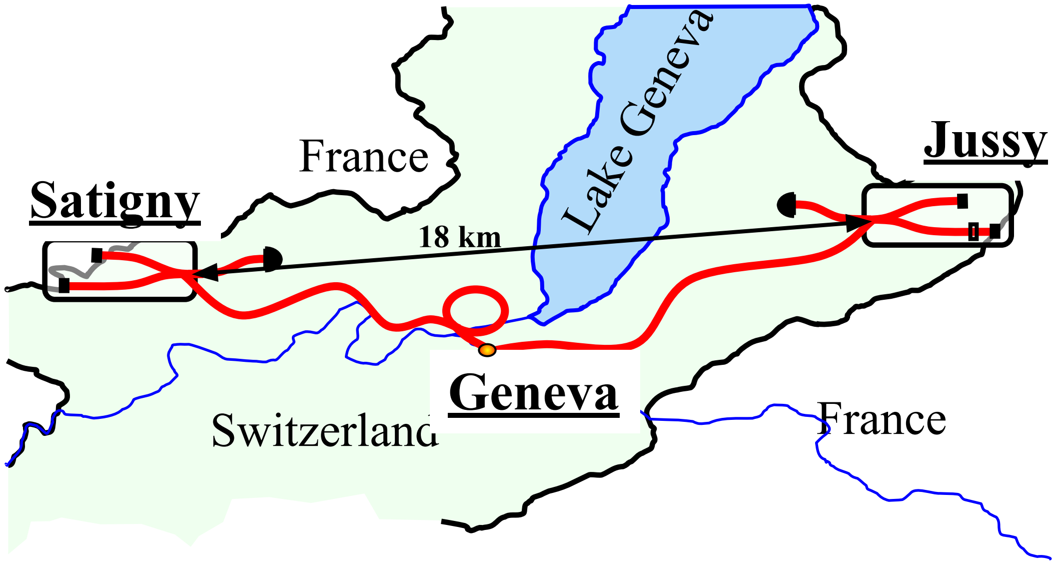


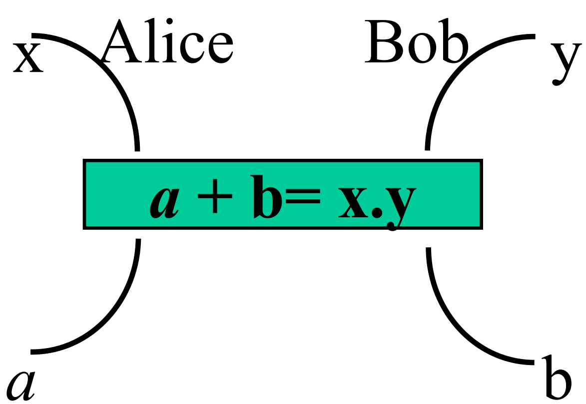


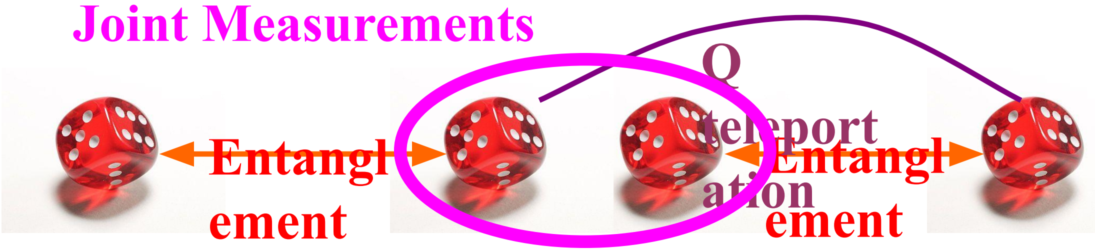

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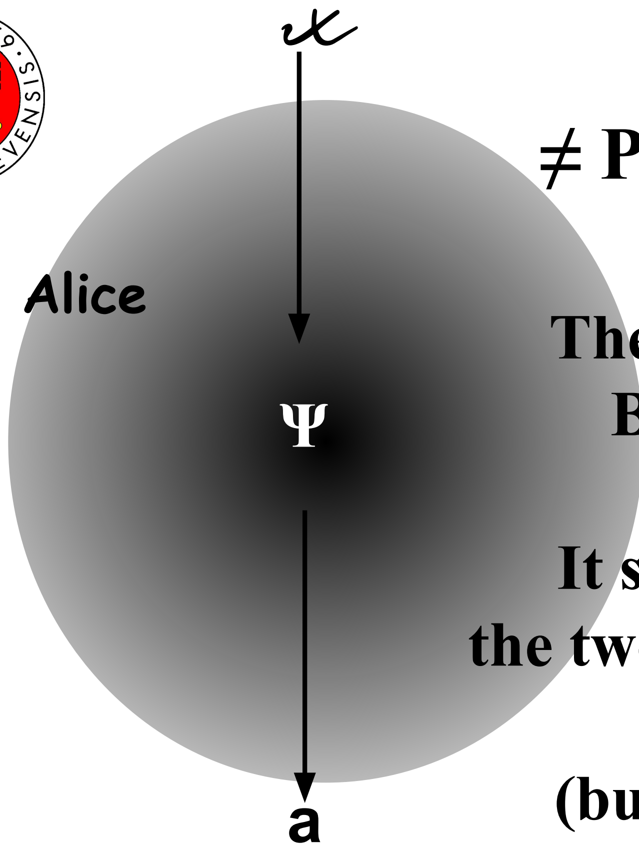
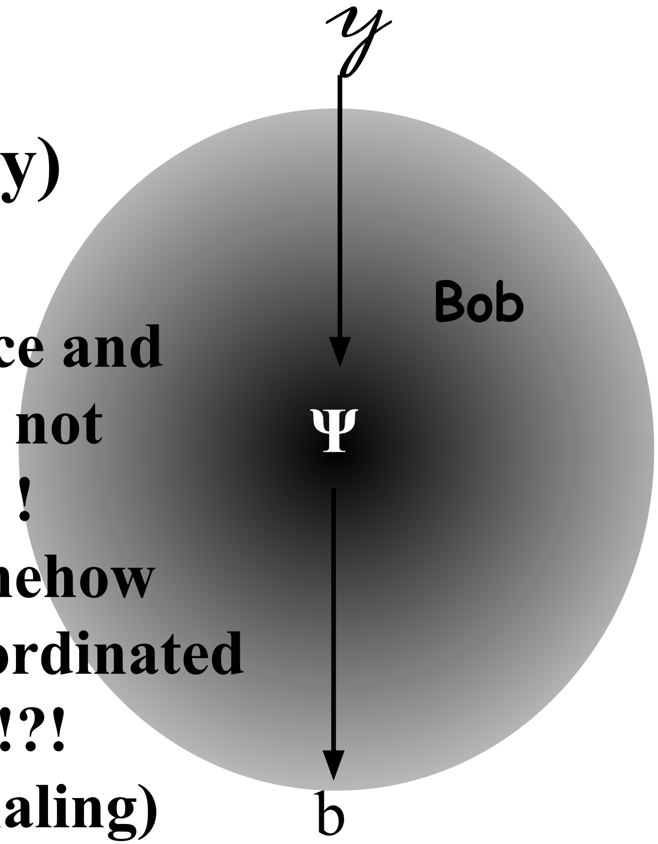



$$P_\Psi(a,b|x,y)$$
$$\neq P_\Psi(a|x)\cdot P_\Psi(b|y)$$




**The events at Alice and Bob's sides are not independent !**
**It seems that somehow the two sides are coordinated or "interact" !?!**
**(but without signaling)**

## Spatially separated systems are not logically separated.

## ⇒ Quantum Physics is nonlocal

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# During my early carrier as a physicist... when I was about 6 months old… I learned the hard way that in order to interact with an object I had either to crawl to it or to throw something at it.


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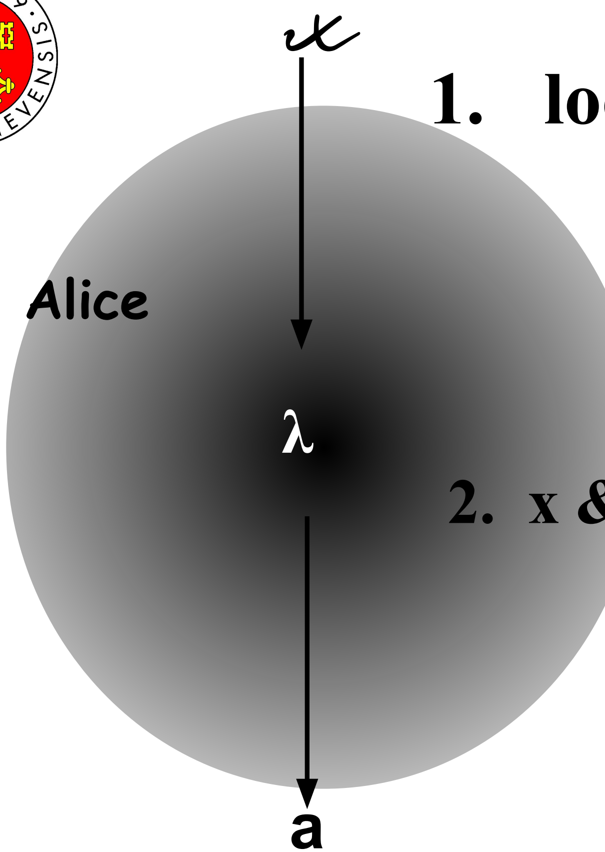
# Assumptions:



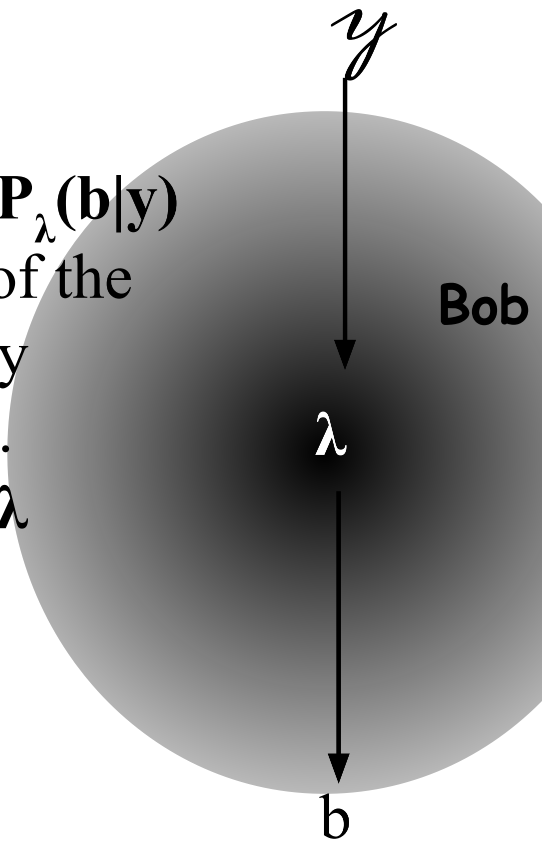

1. **locality:**

   $$\mathbf{P_\lambda(a,b|x,y) = P_\lambda(a|x)\cdot P_\lambda(b|y)}$$

   where λ=physical state of the systems according to any possible **future** theories.

2. **x & y are independent of λ**

   $I(x:\lambda)=I(y:\lambda)=0$




## Conclusion: Bell inequalities

**NG, Non-realism : deep thought or a soft option ? quant-ph/0702021**

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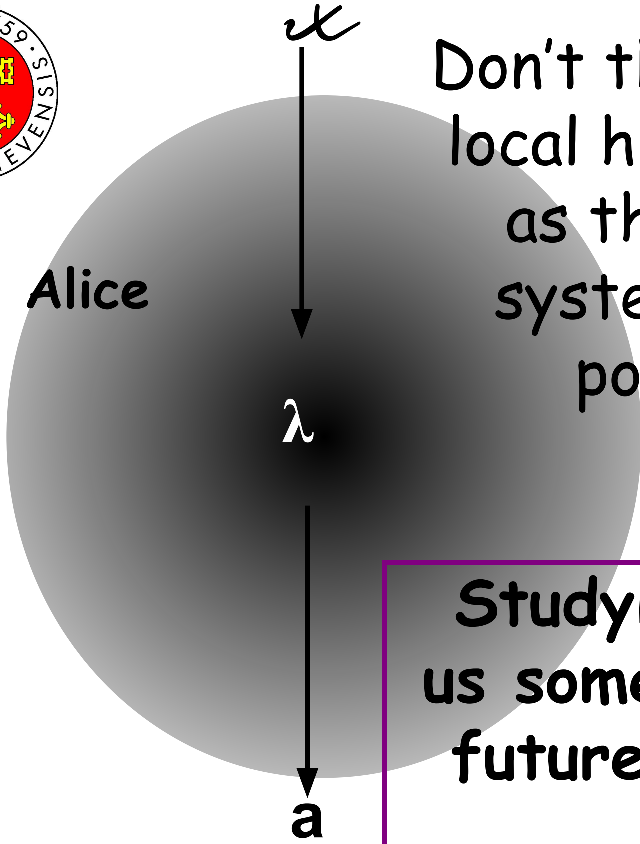
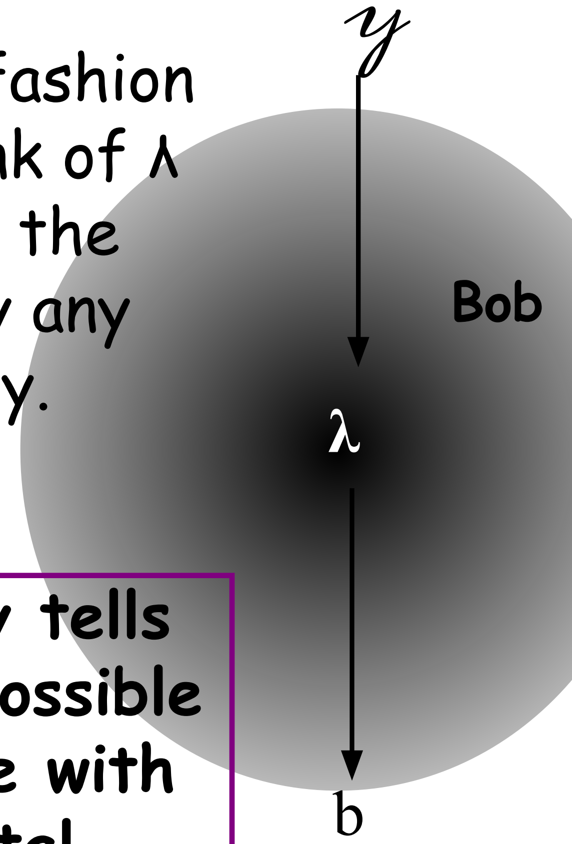



Don't think of λ as an old fashion local hidden variable. Think of λ as the physical state of the systems as described by any possible future theory.

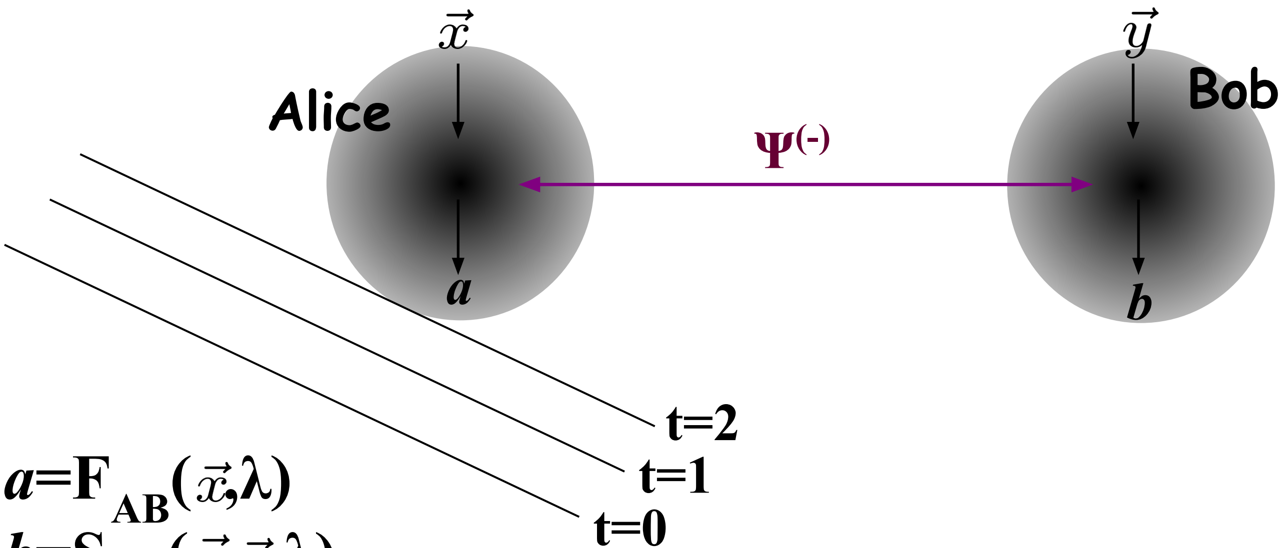



**Studying Bell's inequality tells us something about any possible future theory compatible with today's experimental observations.**

NG, Non-realism : deep thought or a soft option ? quant-ph/0702021

# Deterministic nonlocal hidden variables

**Let's try to add randomness, given from the beginning, to turn stochastic events into deterministic ones: $\lambda=\{\Psi, r_a, r_b\}$.**

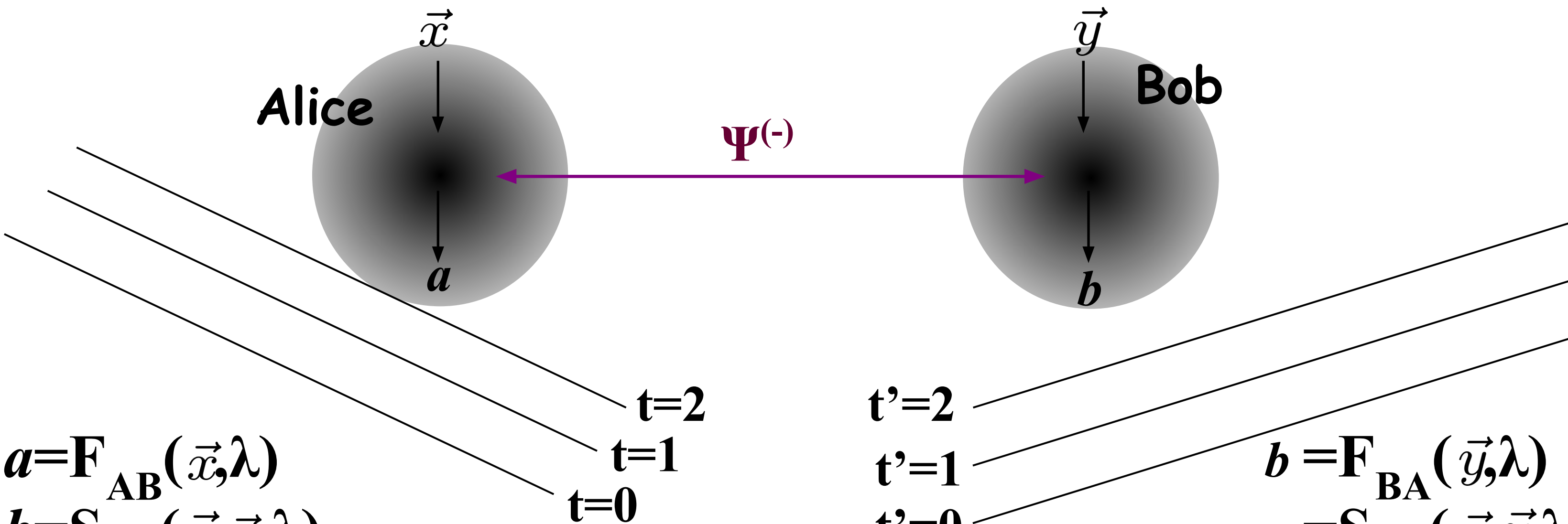



$a = F_{AB}(\vec{x}, \lambda)$

$b = S_{AB}(\vec{y}, \vec{x}, \lambda)$

# Deterministic nonlocal hidden variables

**Let's try to add randomness, given from the beginning, to turn stochastic events into deterministic ones: $\lambda=\{\Psi, r_a, r_b\}$.**




$a=F_{AB}(\vec{x},\lambda)$

$b=S_{AB}(\vec{y},\vec{x},\lambda)$

$b=F_{BA}(\vec{y},\lambda)$

$a=S_{BA}(\vec{x},\vec{y},\lambda)$

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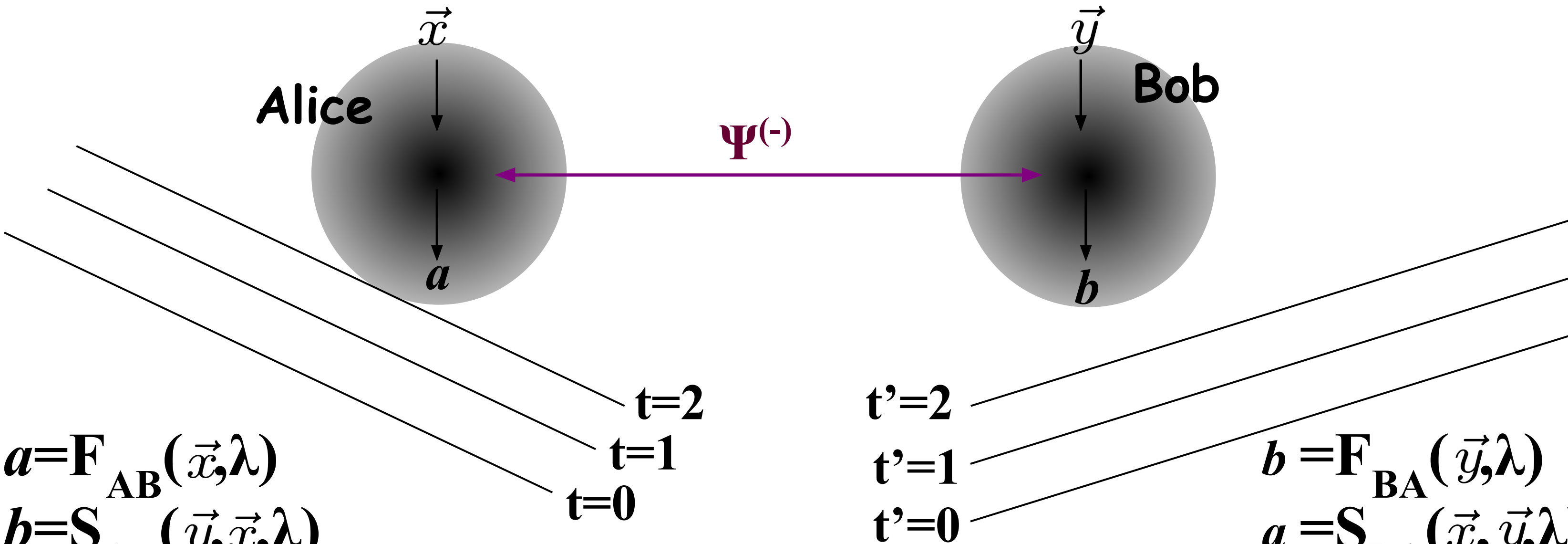



$a = F_{AB}(\vec{x},\lambda)$
$b = S_{AB}(\vec{y},\vec{x},\lambda)$

$b = F_{BA}(\vec{y},\lambda)$
$a = S_{BA}(\vec{x},\vec{y},\lambda)$

**Could there be λ, $F_{AB}$, $S_{AB}$, $F_{BA}$ and $S_{BA}$ s.t.**

$$F_{AB}(\vec{x},\lambda) = S_{BA}(\vec{x},\vec{y},\lambda) \quad ?$$

**<u>Theorem</u>: NO !**

**Proof: $S_{BA}$ would be independent of $\vec{y}$**
**⇒ locality ⇒ Bell inequality.**

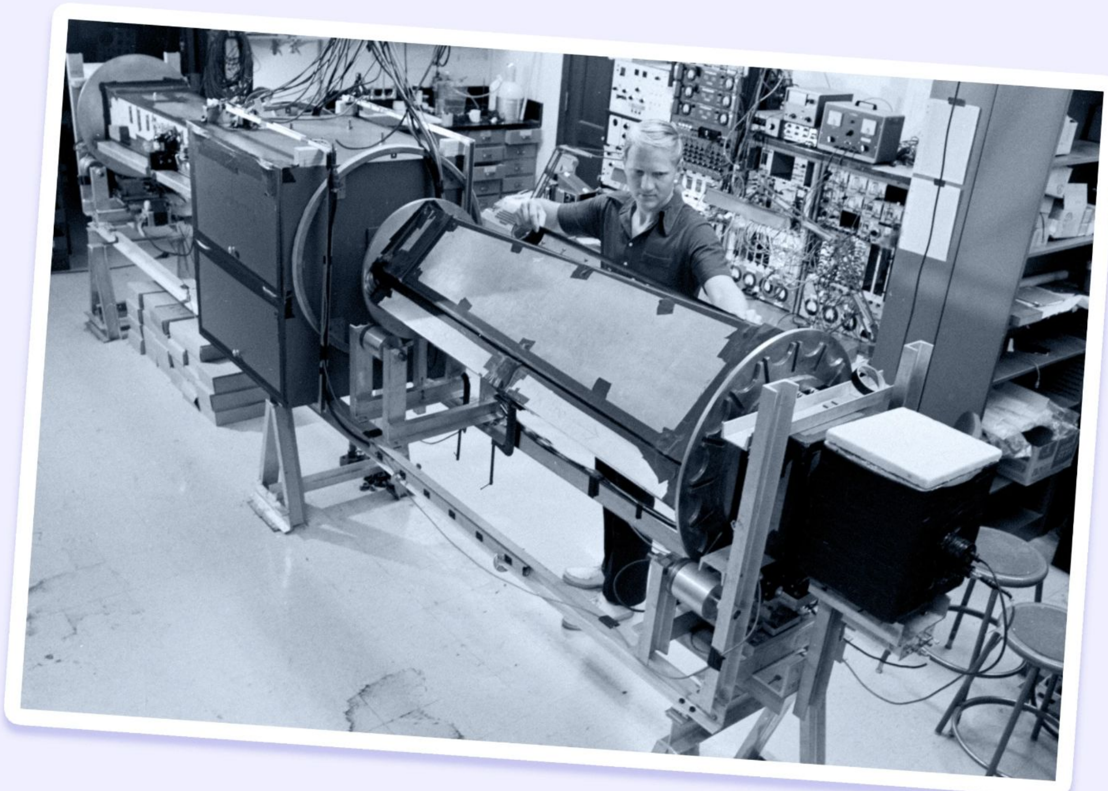
**Impossibility of covariant deterministic nonlocal hidden-variable extensions of quantum theory**
**NG, PRA 83, 020102, 2011**

**Quantum correlations can't be described with local variables, nor can they be described with deterministic nonlocal variables:**

**⇒ <u>need non-local randomness</u>**

# The Clauser – Freedman experiment

https://www.quantamagazine.org/metaphysical-experiments-test-hidden-assumptions-about-reality-20240730/...



The physicist John Clauser attends to the experiment he and Stuart Freedman built to test Bell's theorem in the 1970s.

Courtesy of Lawrence Berkeley National Laboratory

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# Tests of Bell's inequality: Aspect's 1982 experiment

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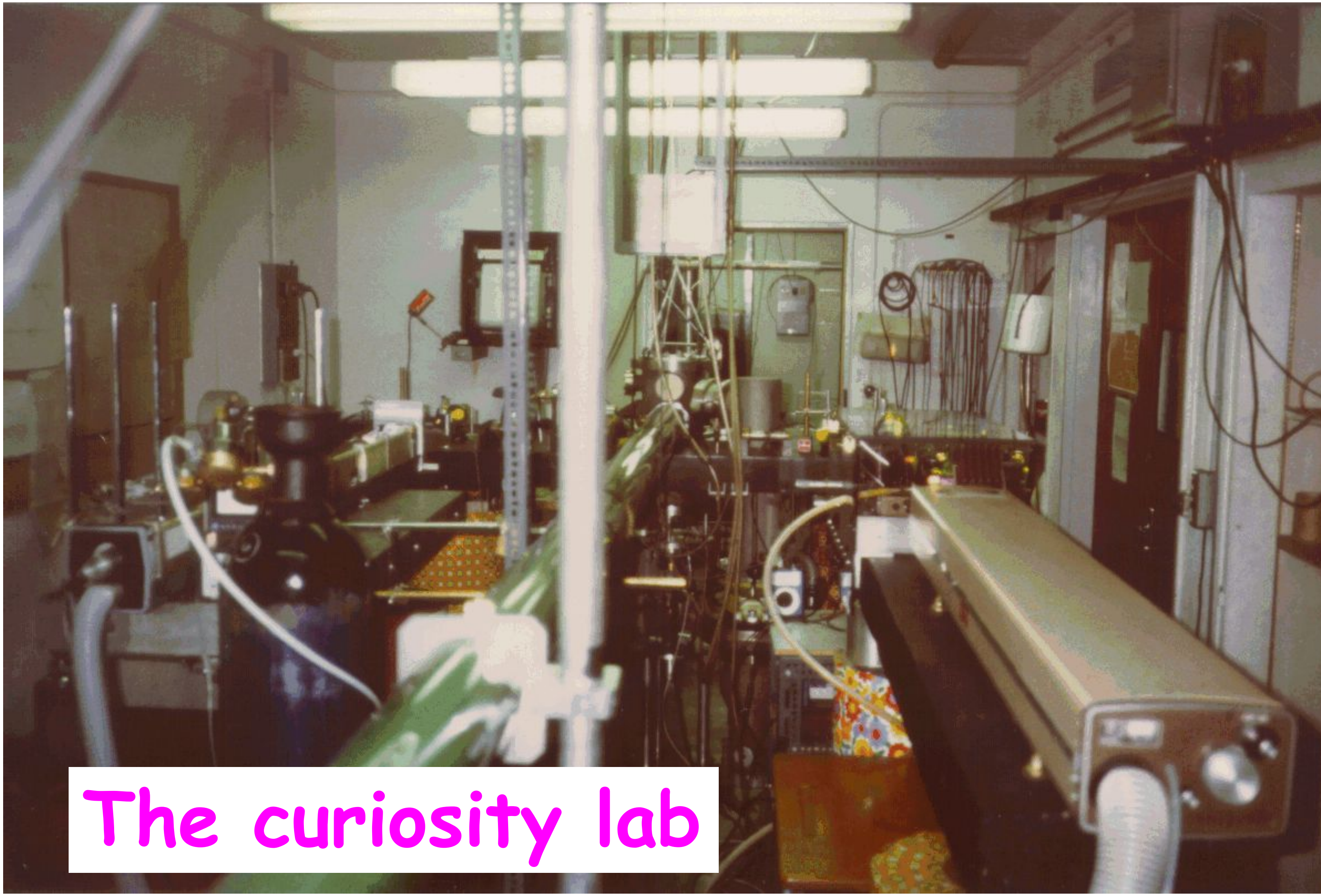
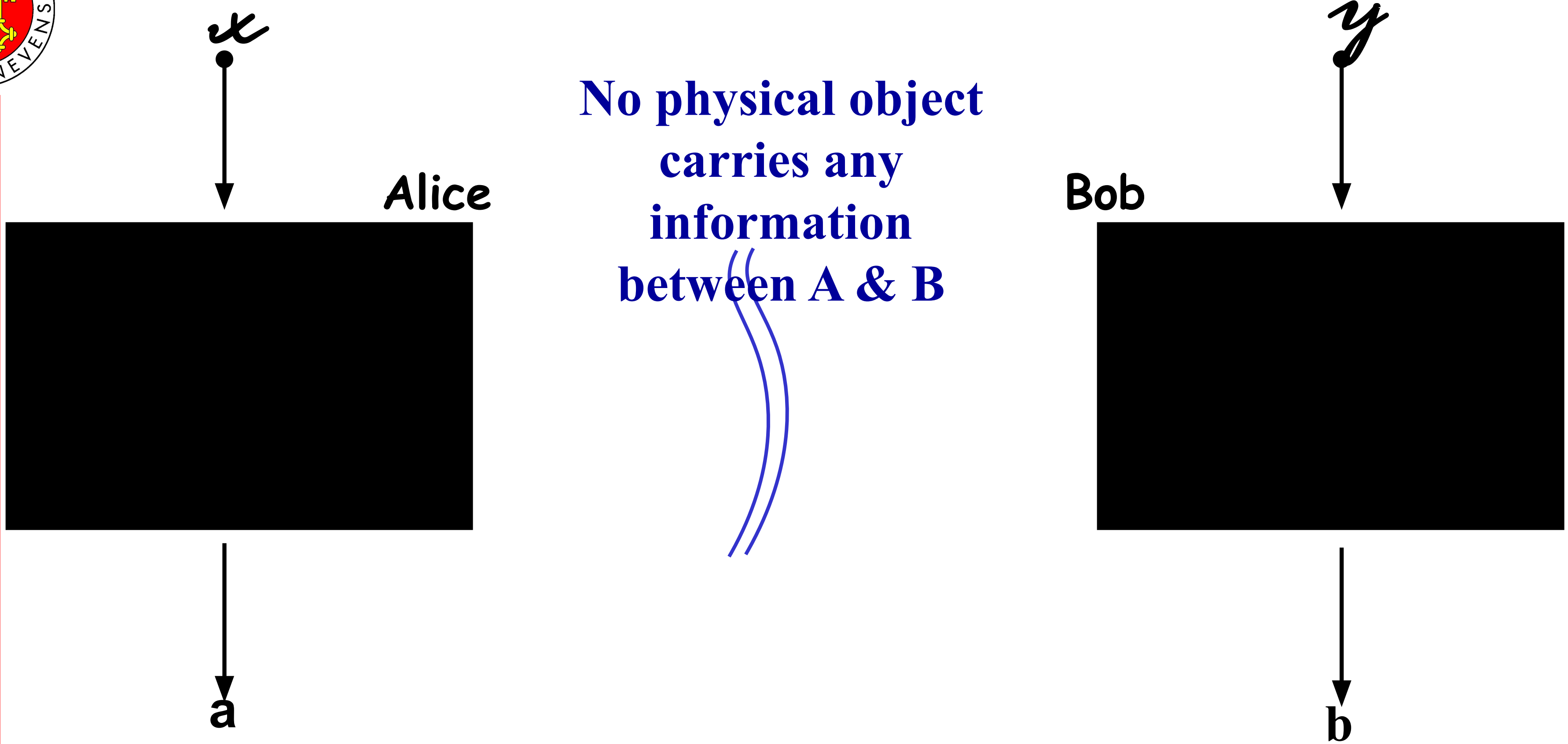


# Bell tests




$$\sum_{a,b,x,y=0,1} (-1)^{a+b-x\cdot y} \; P(a,b \mid x,y) \leq 2$$

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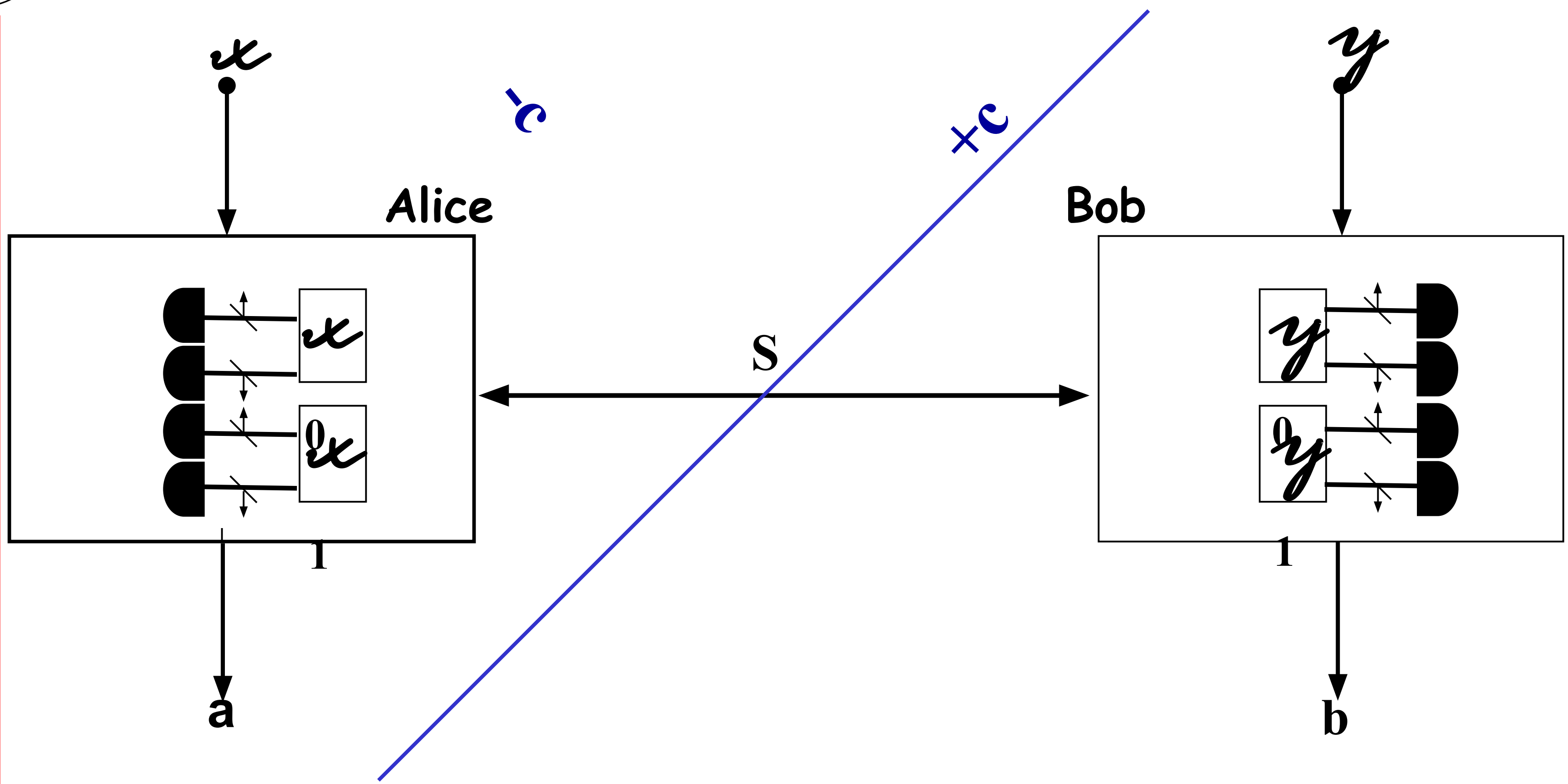
# Bell tests: add a source and open the boxes

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# Bell tests: add a source and open the boxes

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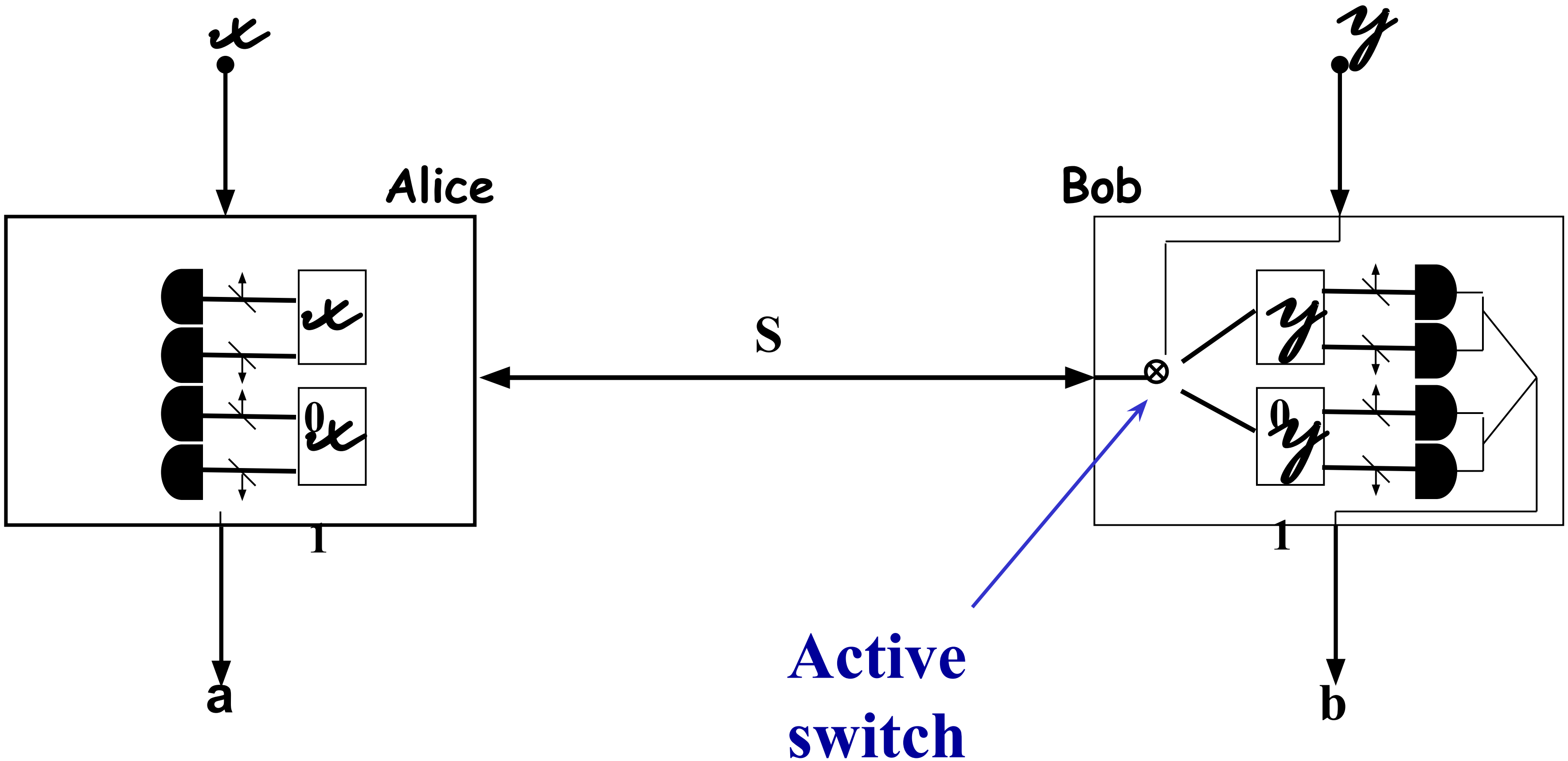

# Bell tests: add a source and open the boxes

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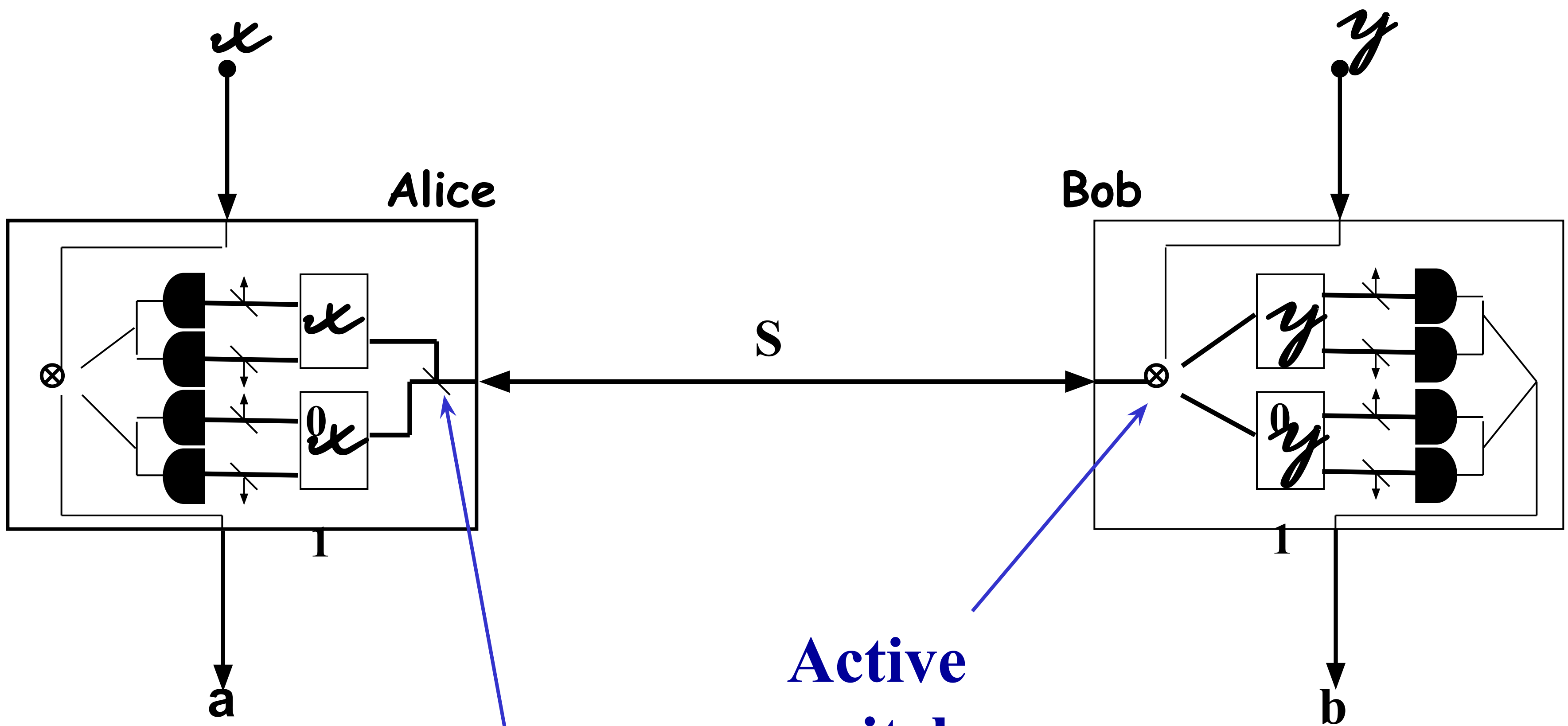


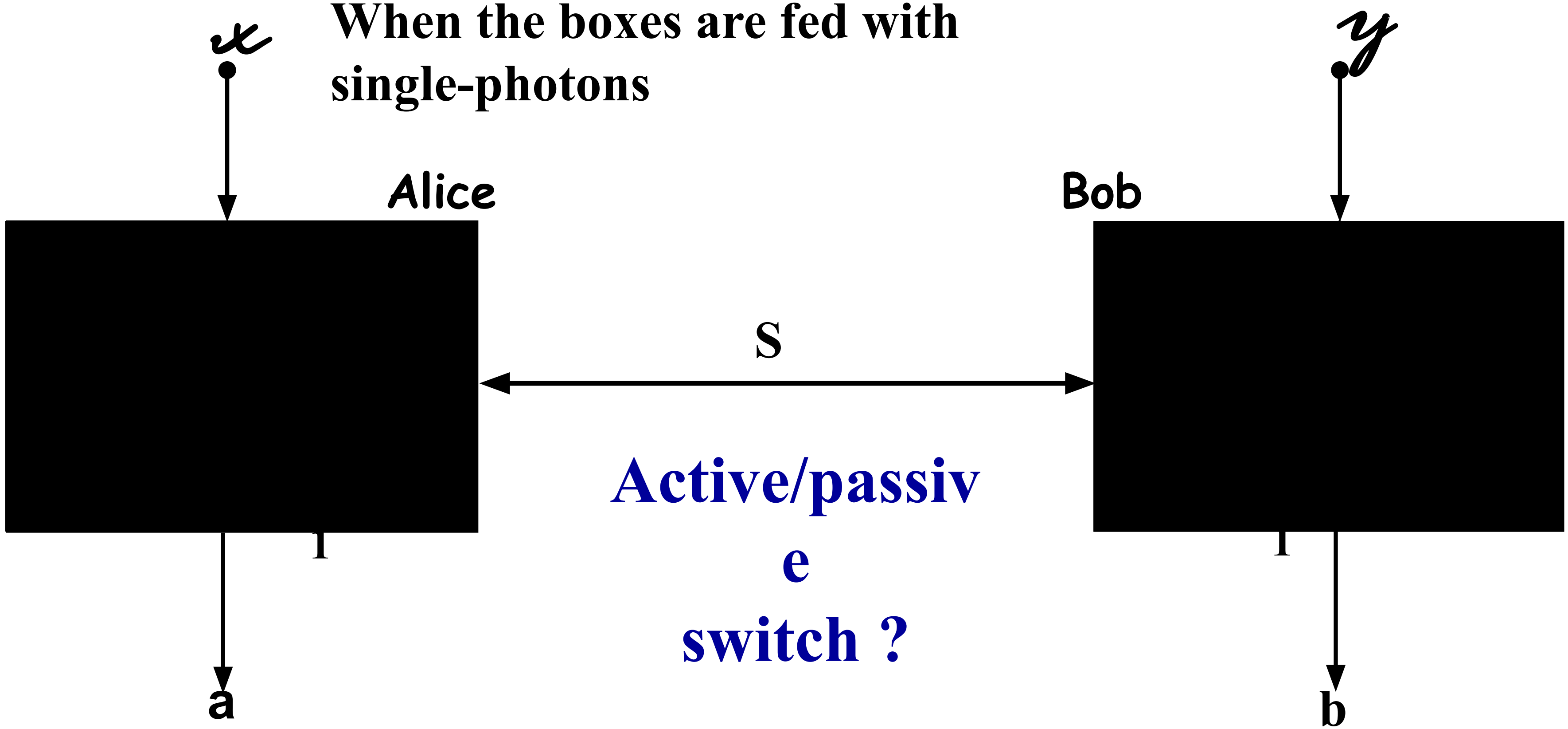

**Active switch**

**Passive switch (Zbinden switch)**

# Bell tests: add a source and open the boxes

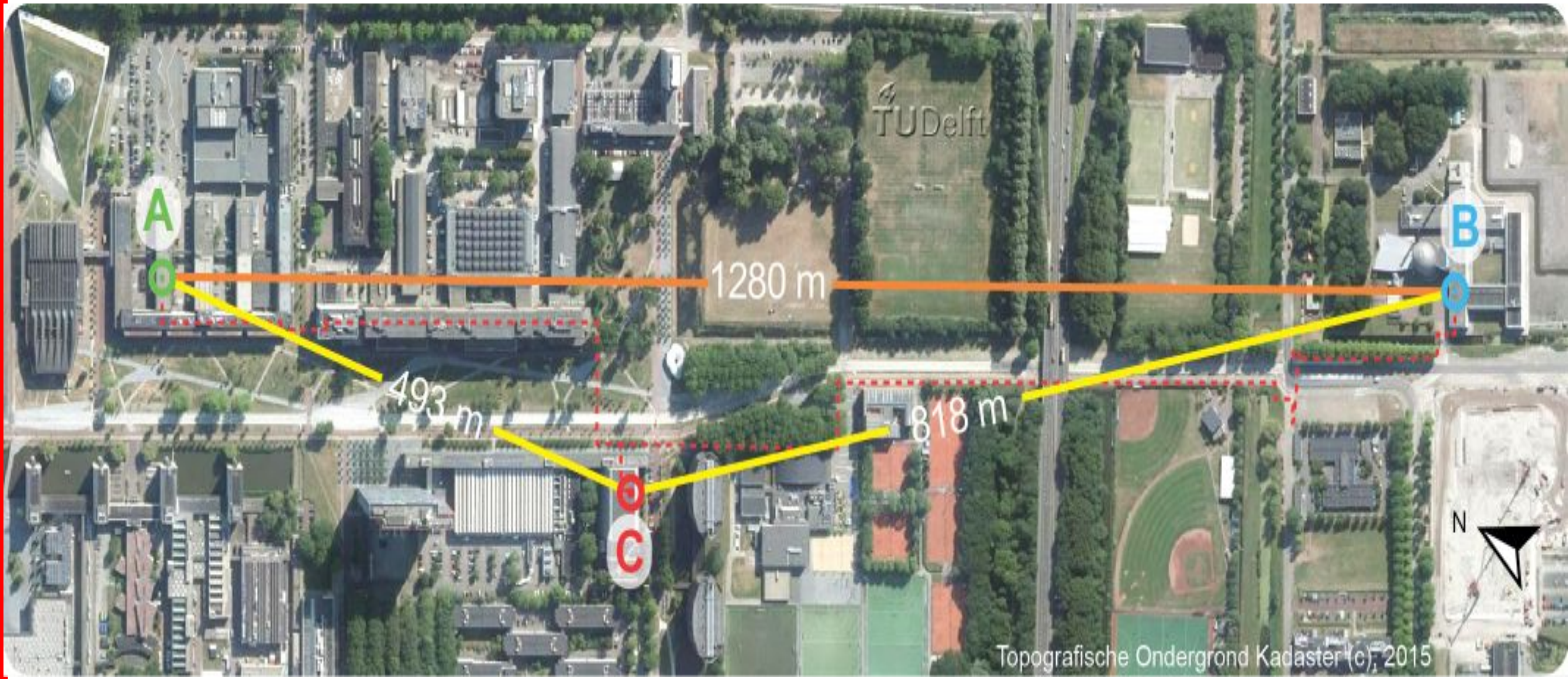
**From a black box perspective, as long as the detection efficiencies are smaller than 50%, it is impossible to distinguish the active and passive switches !**

When the boxes are fed with single-photons




**Active/passiv
e
switch ?**

# Loophole-free Bell tests:
# Delft - Vienna – NIST - Munich

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# Nature is Nonlocal

- J. S. Bell Epistemol. Lett. 2 (1975) But if a hidden variable theory is local it will not agree with quantum mechanics, and if it agrees with quantum mechanics it will not be local. This is what the theorem says.
- Physics: Amazing ! How can these two locations out there in space-time know about each other ?
- According to Quantum Physics **Quantum correlation just happen, somehow from outside space-time :**
  **there is no story in space-time**
  **that can tell us how it happens.**
- **Computer Science: What can one do with nonlocal non-signaling (quantum) correlations ?**

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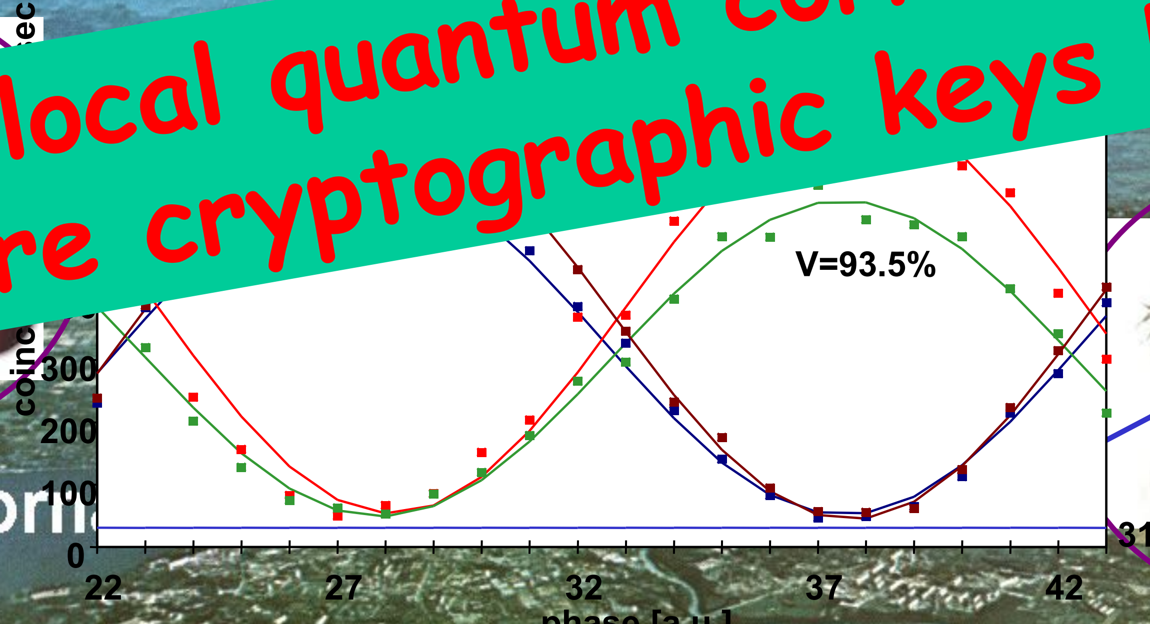
# Bell inequality violation over 10 km, Geneva, 1997

**Non-local quantum correlations are cryptographic keys !**

Eker
t
1991





Bellevue

**Intellectually fascinating. How does nature do it?**

**What can one do with these entirely new kinds of correlations?**

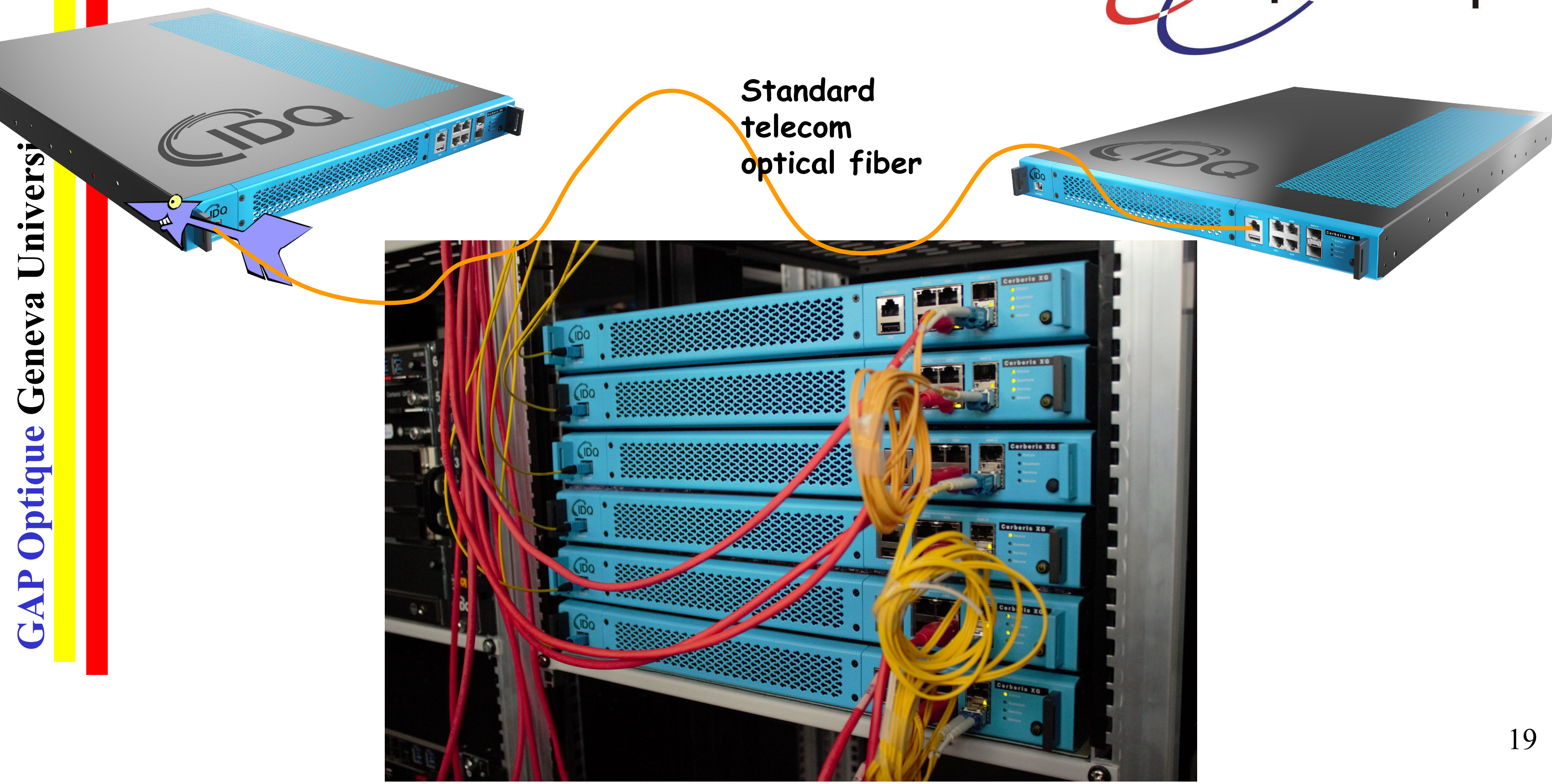
swisscom

**The real-world lab**

# Quantum Key Distribution



- Encode a quantum state into a photon and send the latter from Alice to Bob:



# From «no action at a distance» to quantum randomness

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- Assume no action at a distance (i.e. no-signaling).
- Assume there are independent systems.

x

**Promise: $a \oplus b \approx x \cdot y$**

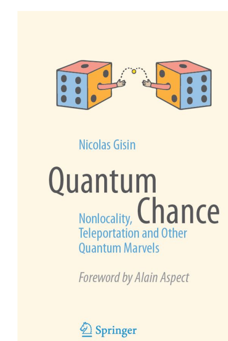
y


distanc
e


*a*

b

**Theorem:**
**a and b are truly random**

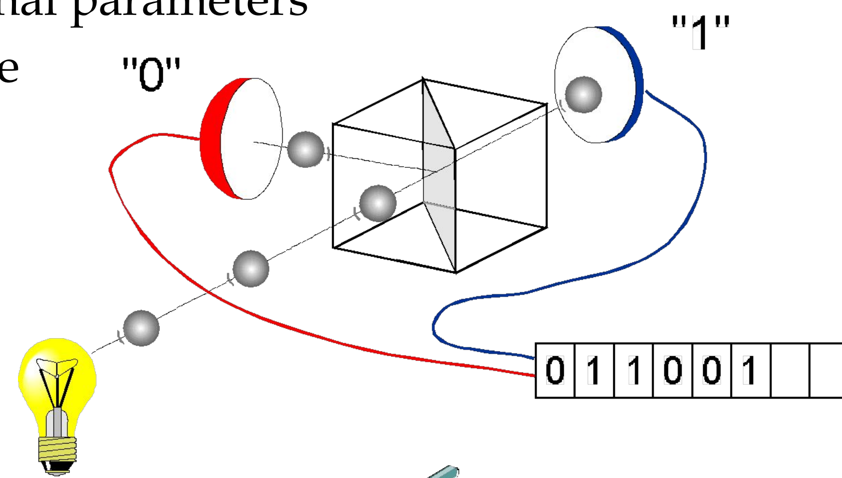



N. Gisin
Quantum
Chance
Springer 2014

S. Pironio et al., Nature 464, 1021 (2010)

# From Physics to Technology

- Quantum physics is fundamentally random
  - Cannot be influenced by any external parameters
  - Output is completely unpredictable
- High bit rate
  - 4 or 16 Mbits/s
- Markets
  - Cryptography
  - Gaming and Lotteries
  - Scientific




Key generation


4 cm

**Certified by Swiss Federal Office of Metrology**

Schweizerische Eidgenossenschaft
Confédération suisse
Confederazione Svizzera
Confederaziun svizra

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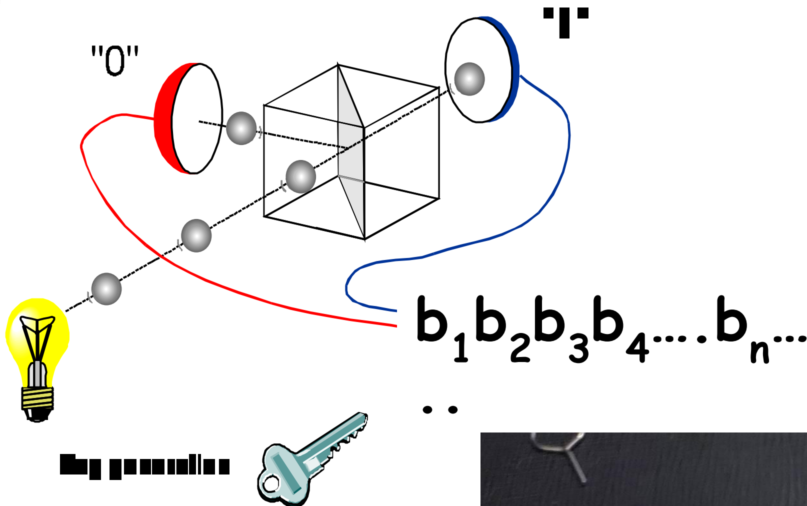
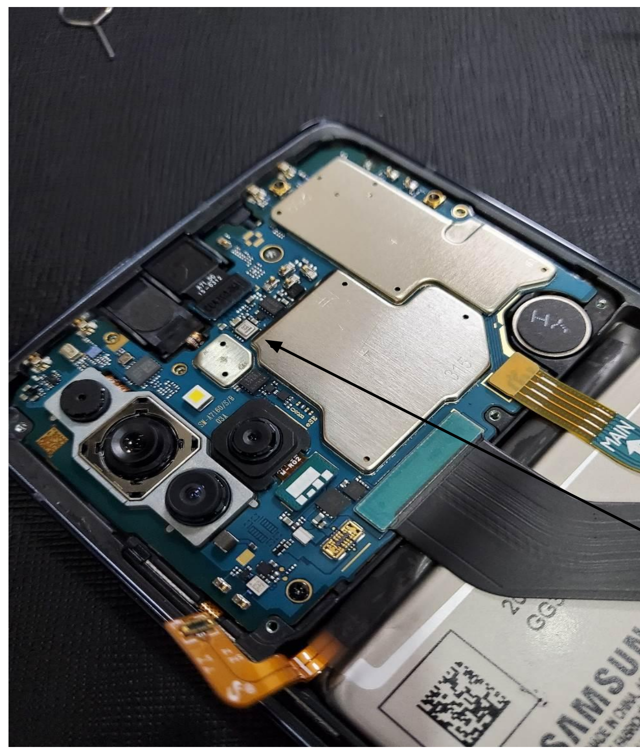
# Quantum Random number generator




New information gets created as time passes

First mass application of Q technologies


mm size
μw
power

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# How does Nature perform the trick ?

- How can these two locations out there in space-time know about each other ?
- How does an event *A* know that it is nonlocally correlated to another event B ?
- Who keeps track of who is entangled with whom ?

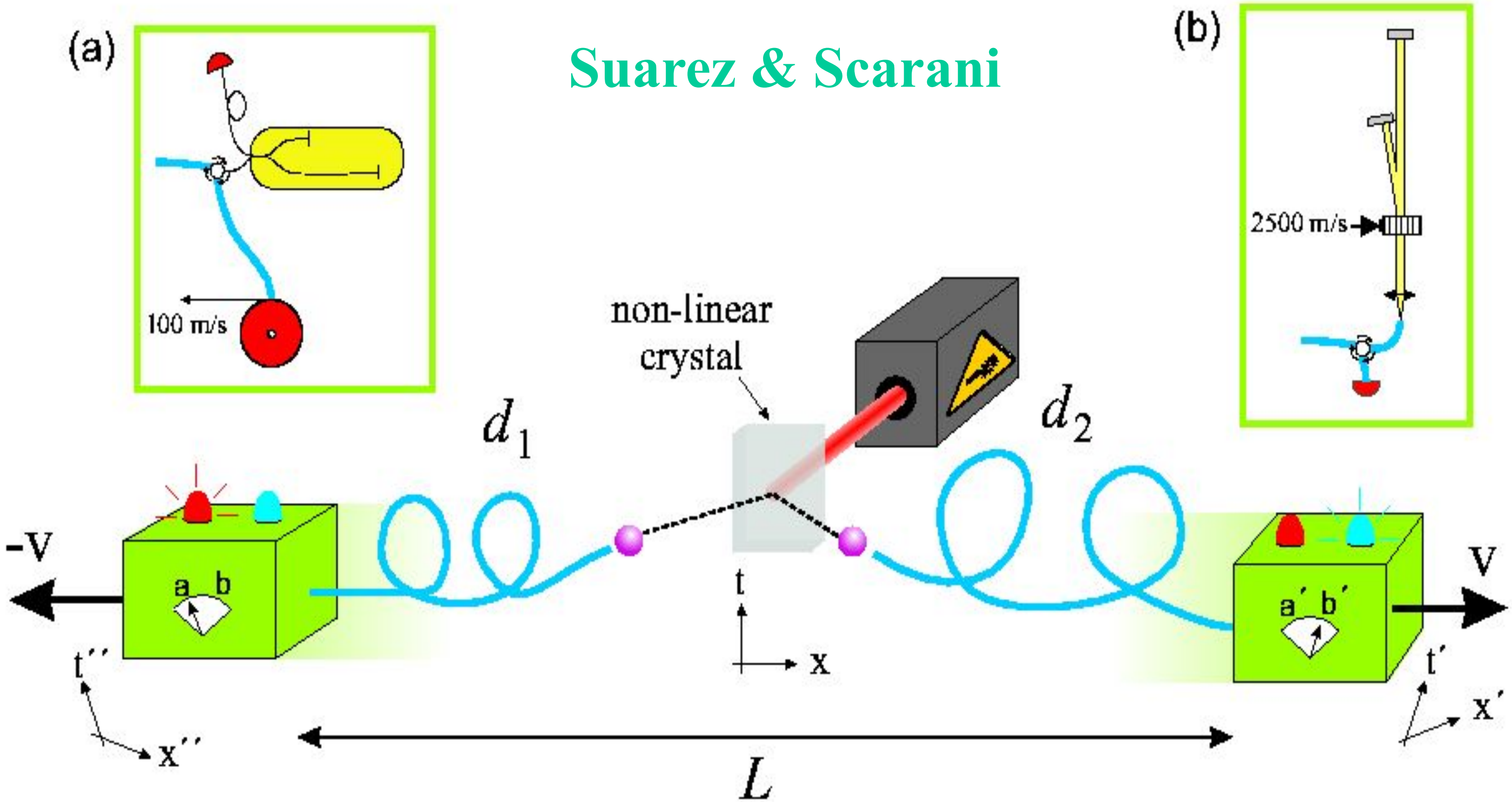
Quantum nonlocality : how does nature do it ?, Science, 326, 1357, 2009; arXiv:0912.1475

# Further experiments: before-before configurations

**NG, Sundays in a quantum engineer's life, quant-ph/0104140**

**in *Quantum [Un]speakable*, pp 199-208, ed. R.A. Bertlmann and A. Zeilinger, Springer 2002**

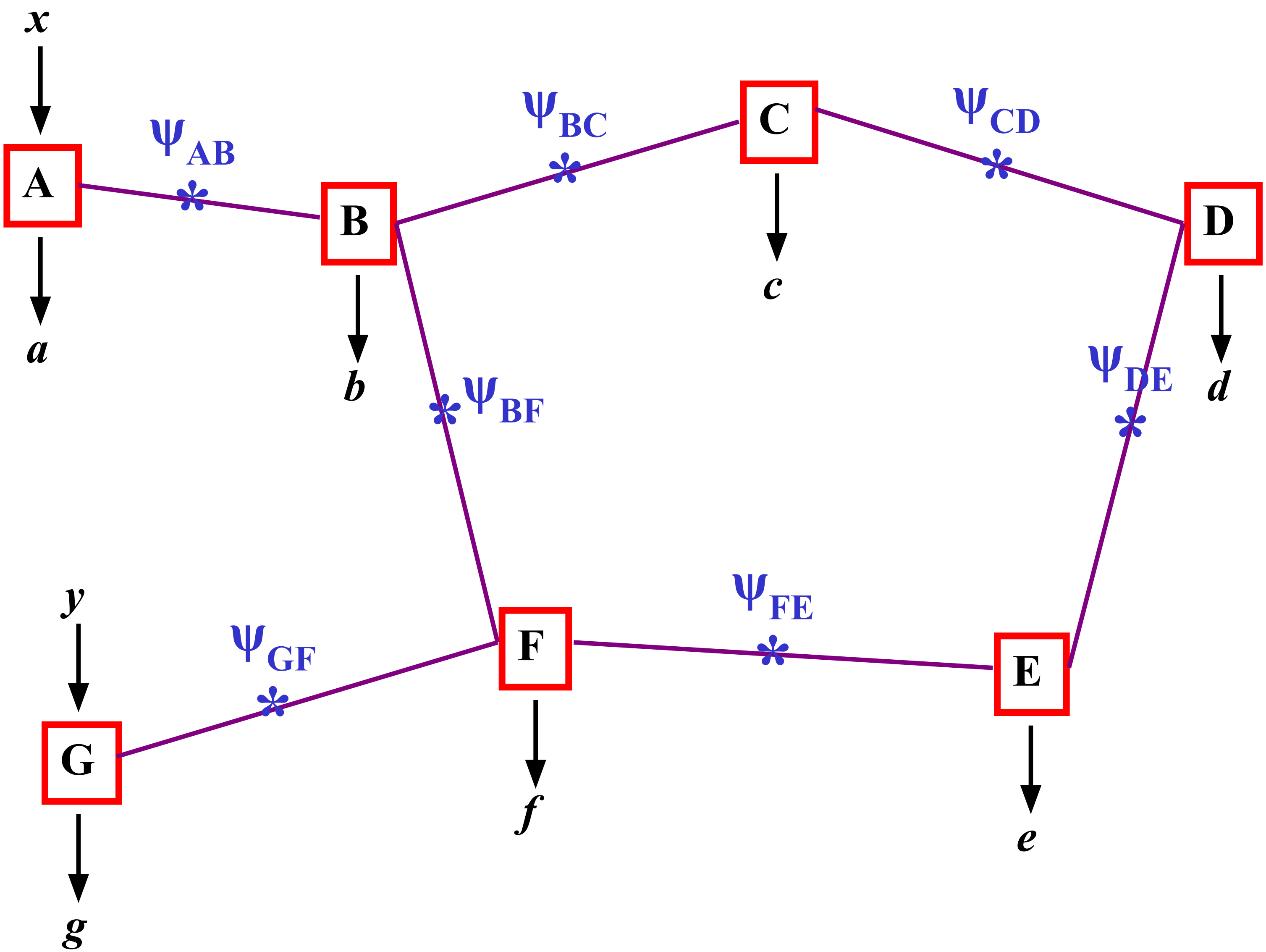
**Suarez & Scarani**




**PRL 88,120404,2002; J.Phys.A 34,7103,2001; Phys.Lett.A 276,1,2000**

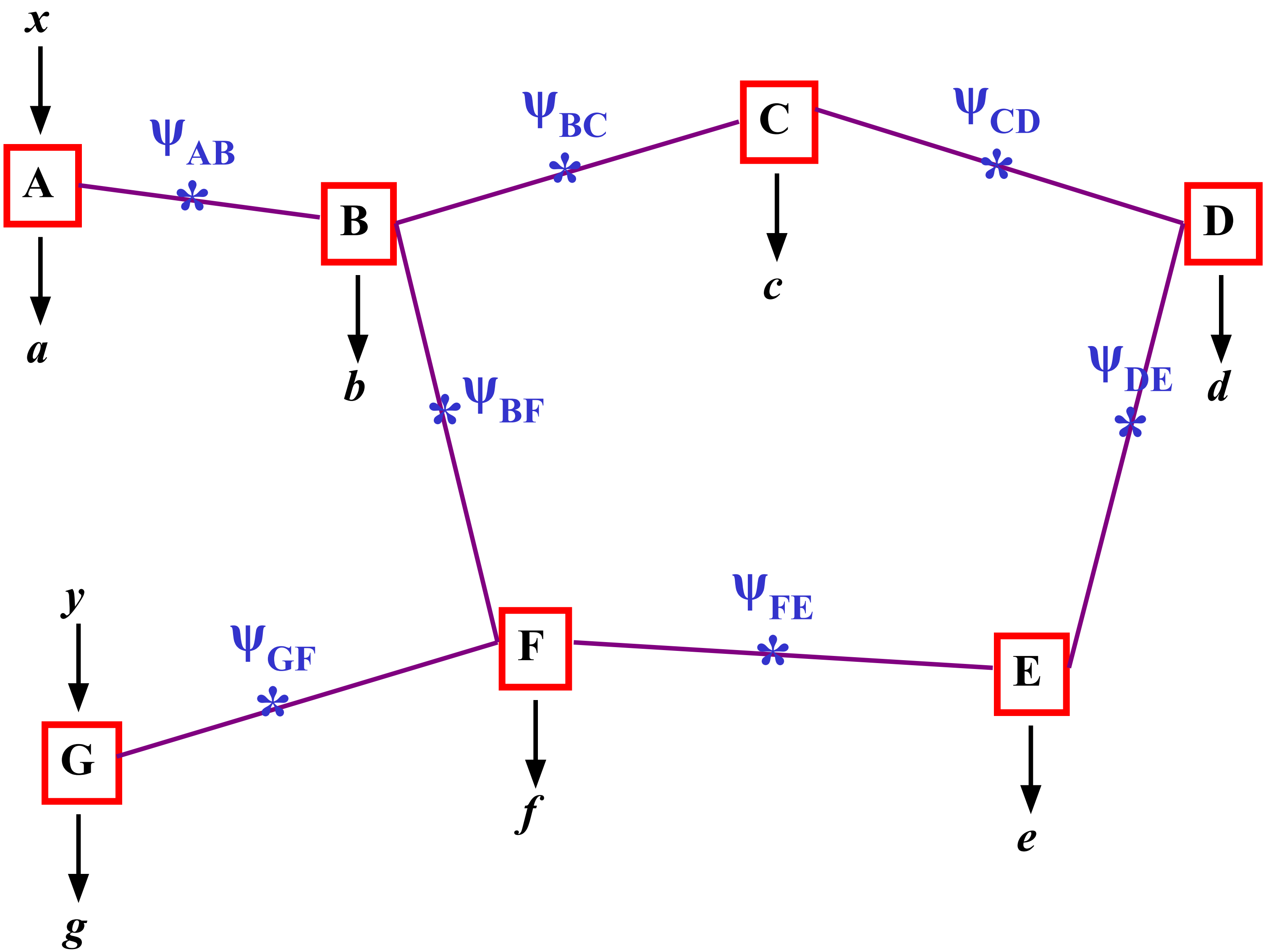
# Quantum Networks

## With <u>independent</u> quantum states $\psi_{ij}$



# How to certify the quantumness of a network?

**With <u>independent</u> quantum states $\psi_{ij}$**

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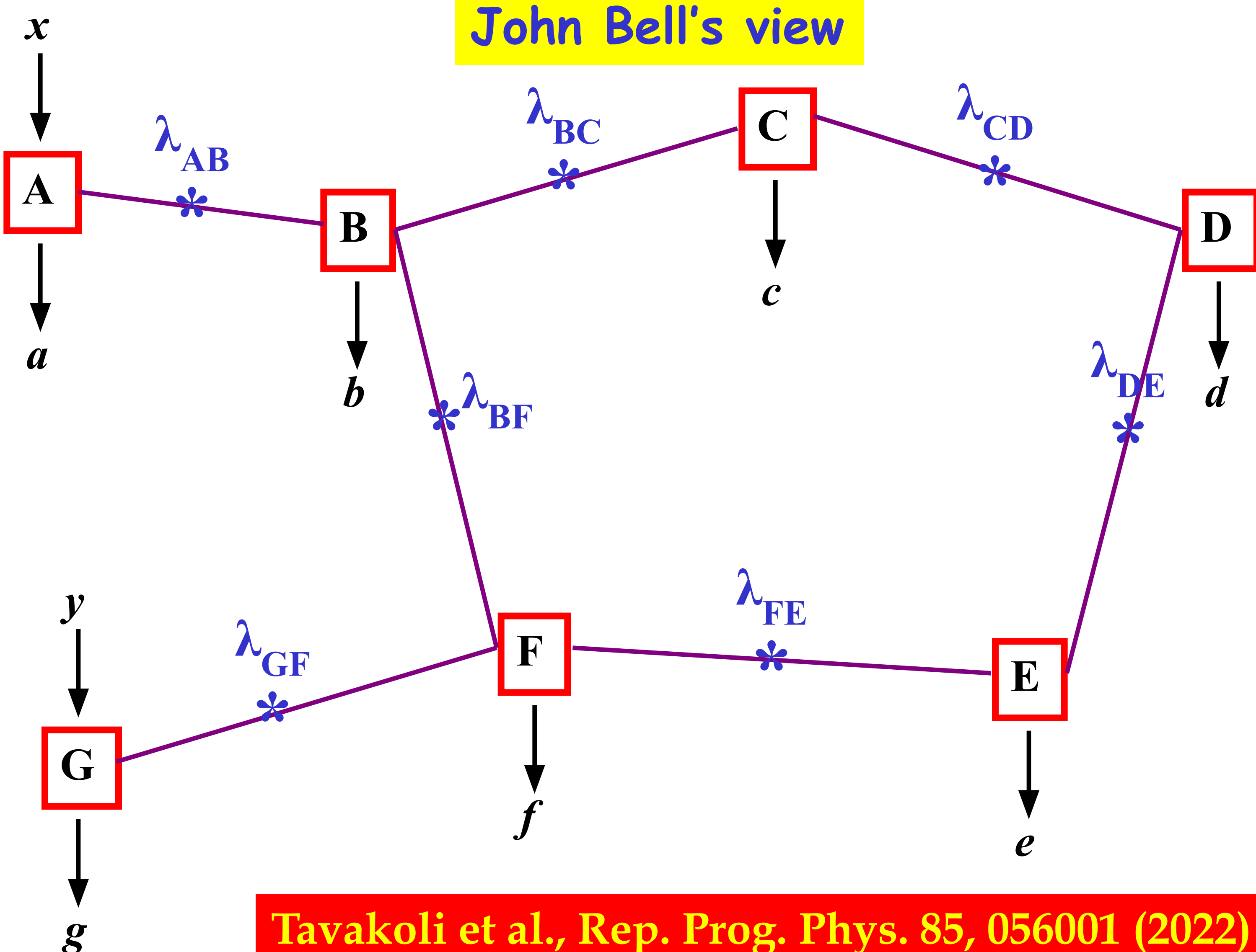
# Classical Networks

**With <u>independent</u> shared randomness $\lambda_{ij}$**

**John Bell's view**




**Tavakoli et al., Rep. Prog. Phys. 85, 056001 (2022)**

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# Two views on (non-)locality

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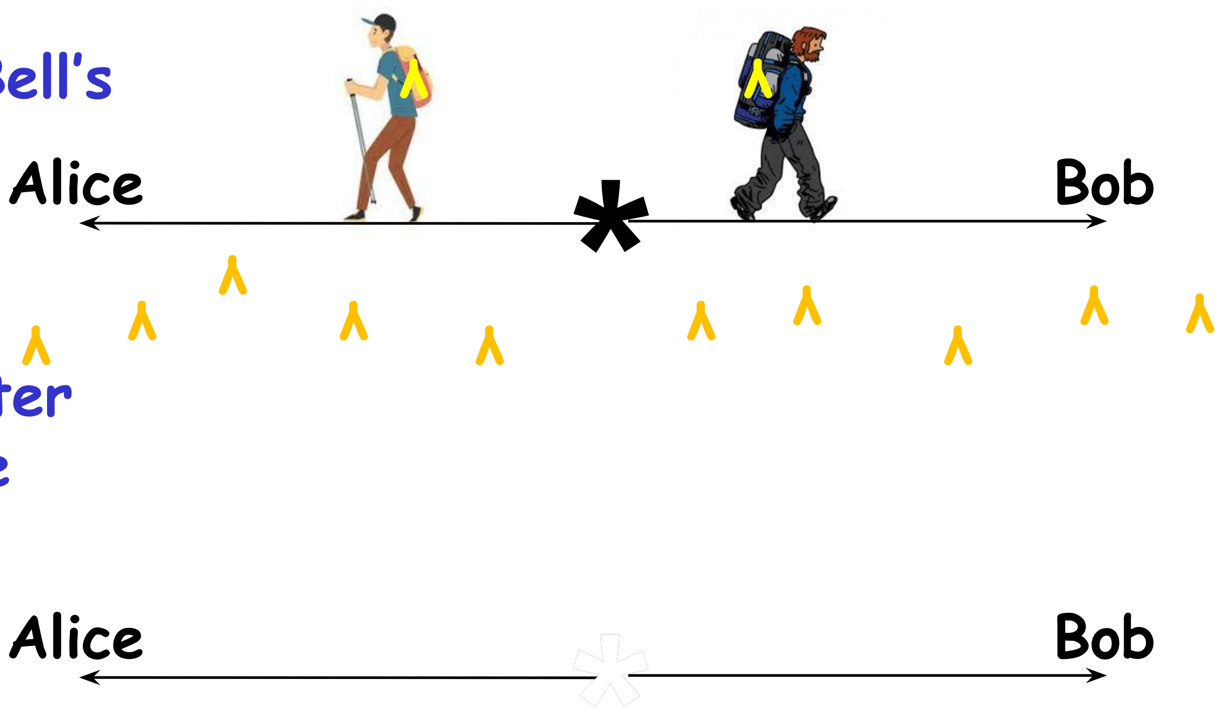
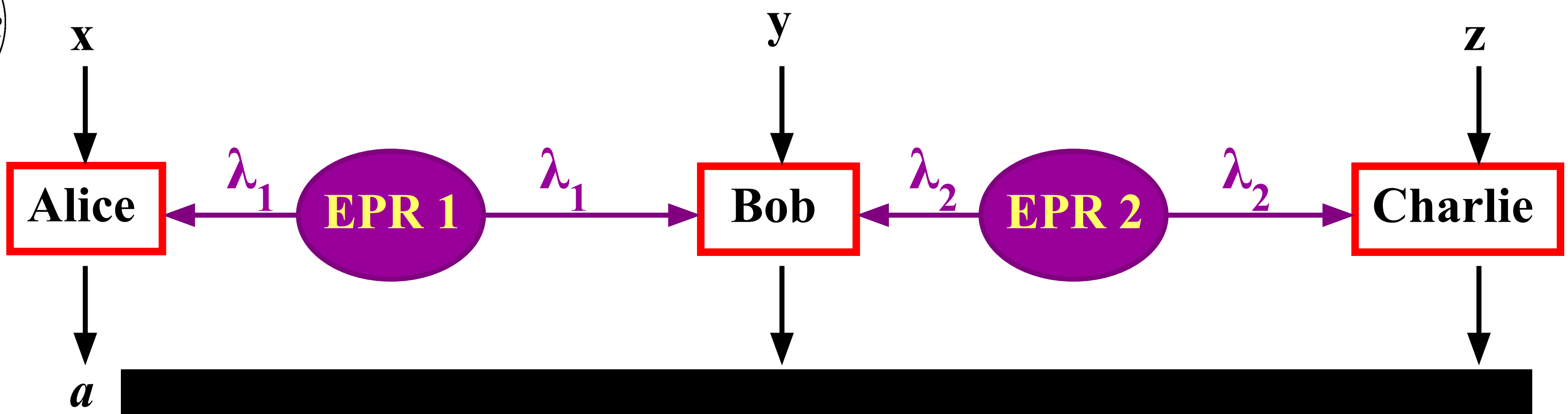
John Bell's view:

Computer science view:



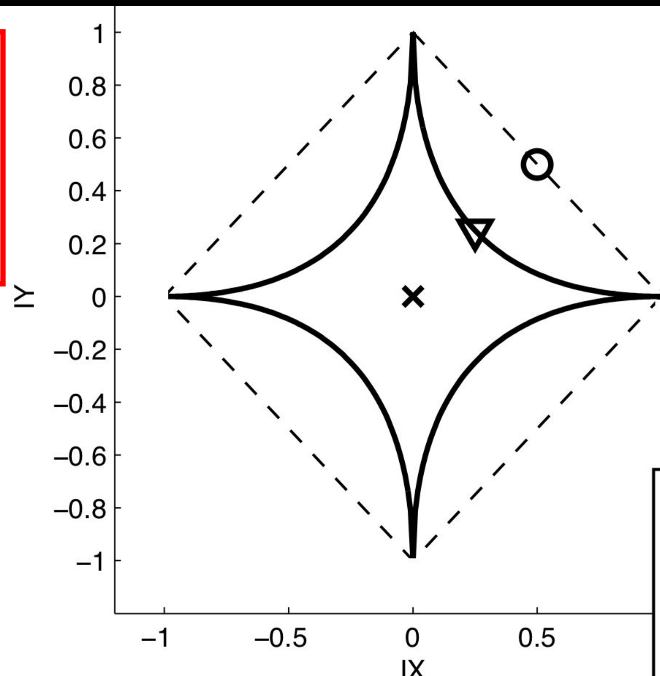
# Bi-locality

Branciard, NG, Pironio
PRL 104, 170401-1/4, 2010




[redacted]

The set of bilocal $p_{biloc}(a,b,c|x,y,z)$ is not convex:

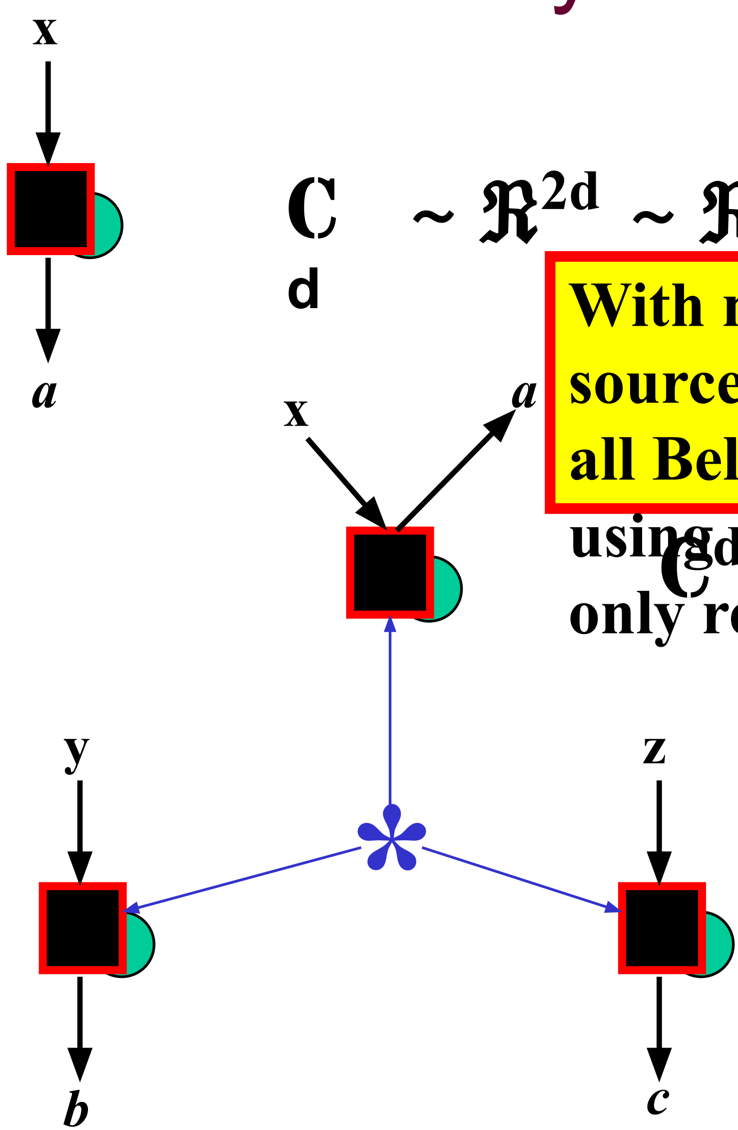



**Bilocal set**

$$\sqrt{|I_X|} + \sqrt{|I_Y|} \leqslant 1$$

Tavakoli et al., Rep. Prog. Phys. 85, 056001 (2022)

# Quantum Physics needs Complex Nbs




$\mathbb{C}^d \sim \Re^{2d} \sim \Re^{d} \otimes \Re^{2}$ add one qubit

**With n parties connected to <u>one</u> source all Bell scenarios can be described using only real numbers.** PRL 102, 020505 (2009).

$$\mathbb{C}^{d1} \otimes \mathbb{C}^{d2} \otimes \mathbb{C}^{d3} \sim \Re^{d1+d2+d3} \otimes \Re^{2}$$

add one global logical qubit

$|\underline{0}\rangle = |000\rangle + |011\rangle + |101\rangle + |110\rangle$

$|\underline{1}\rangle = |111\rangle + |100\rangle + |010\rangle + |001\rangle$

M.Renou et al., arXiv:2101.10873, Nature 600, 625 (2021)

# Quantum Physics needs Complex Nbs

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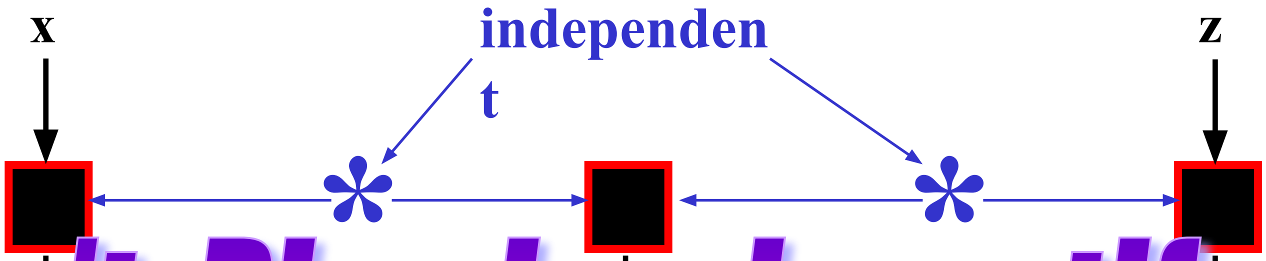


**Isn't Physics beautiful ?**

**<u>Theorem</u>: In the entanglement swapping scenario with x=1,2,3 and z=1,2,3,4,5,6 some Bell inequality ($CHSH_3$) achieves a maximal value with "complex Q theory" of $6\sqrt{2} \approx 8.49$, while "real Q theory" is limited below 7.66**

**Networks with independent sources can't be described using only real Hilbert spaces !**

**Experiments (all in China): PRL 128, 040402 (2022); PRL 128, 040403 (2022); arXiv:2201.04177**

**Fewer settings: A.Bednorz & J. Batle, arXiv:2206.02212**

M.Renou et al., arXiv:2101.10873, Nature 600, 625 (2021)

# How does Nature perform the trick ?

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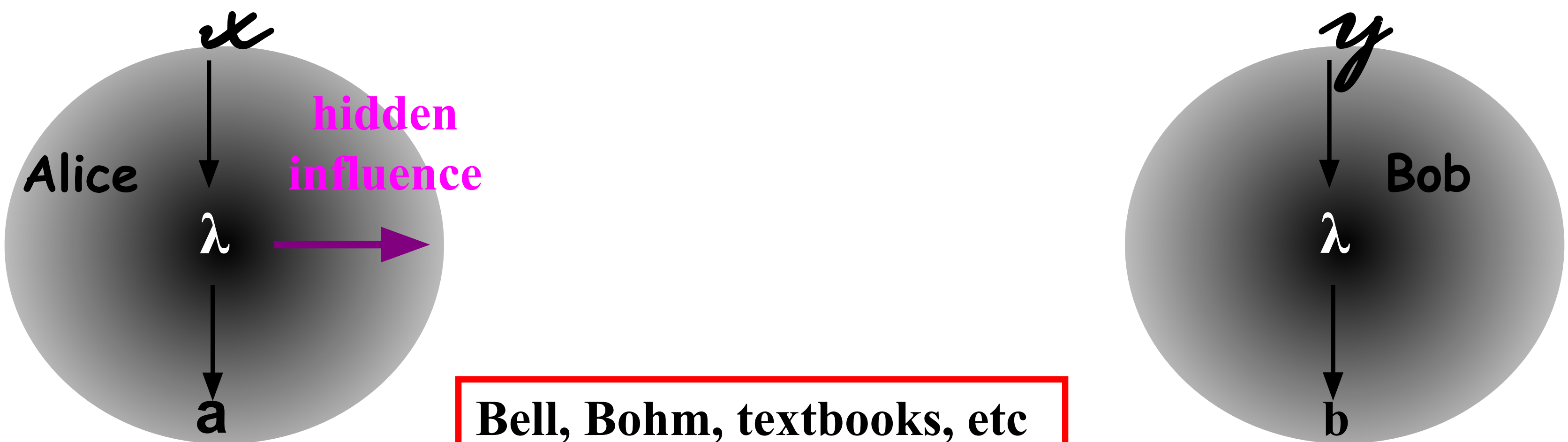
**Assume a real influence propagates from A to B, but with finite speed**




Bell, Bohm, textbooks, etc

# How does Nature perform the trick ?



**Assume a real influence propagating faster than light but with finite speed**

$$p(a,b|x,y,\lambda) = p(a|x,\lambda) \cdot p(b|y,\lambda)$$

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**In which frame should the events be simultaneous ?**

Salart et al., Nature 454, 861, 2008
Cocciaro et al., PLA 375, 379, 2011
J-W Pan's group, PRL 110, 260407, 2013
L. Santamaria et al., scientific report 13.8201

# Let's test these hypothetical preferred reference frame


Alice and Bob,
east-west orientation,
perfect synchronization
with respect to earth
⇒ perfect synchronization
w.r.t any frame
moving
perpendicular to the
A-B axis
⇒ in 12 hours all hypothetical privileged frame
are scanned.

Ph. Eberhard, private communication

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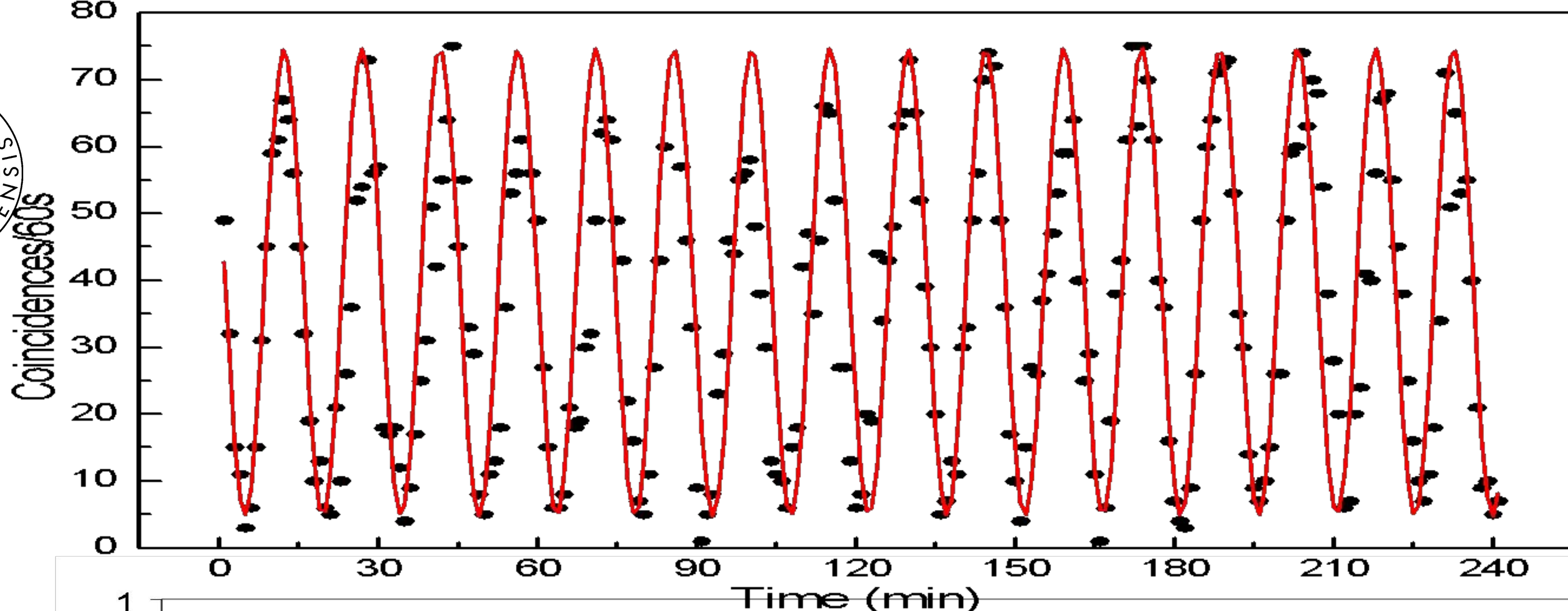


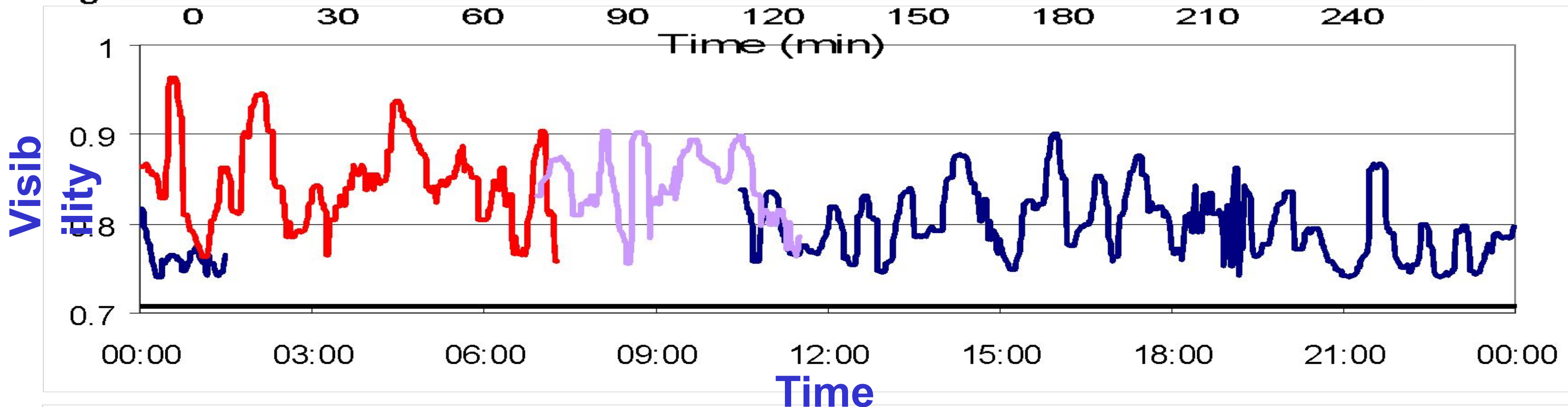


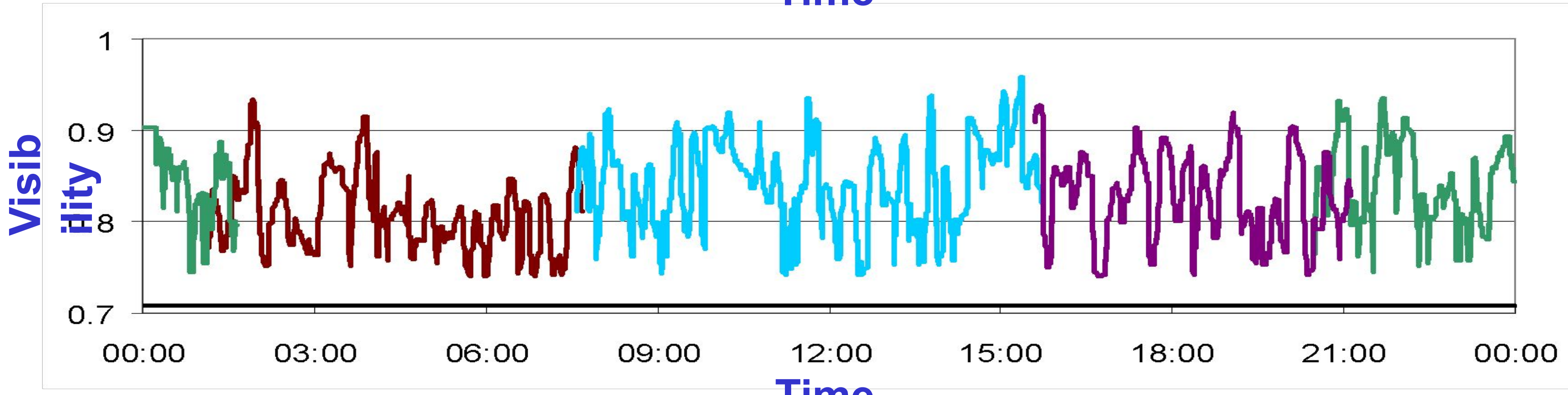

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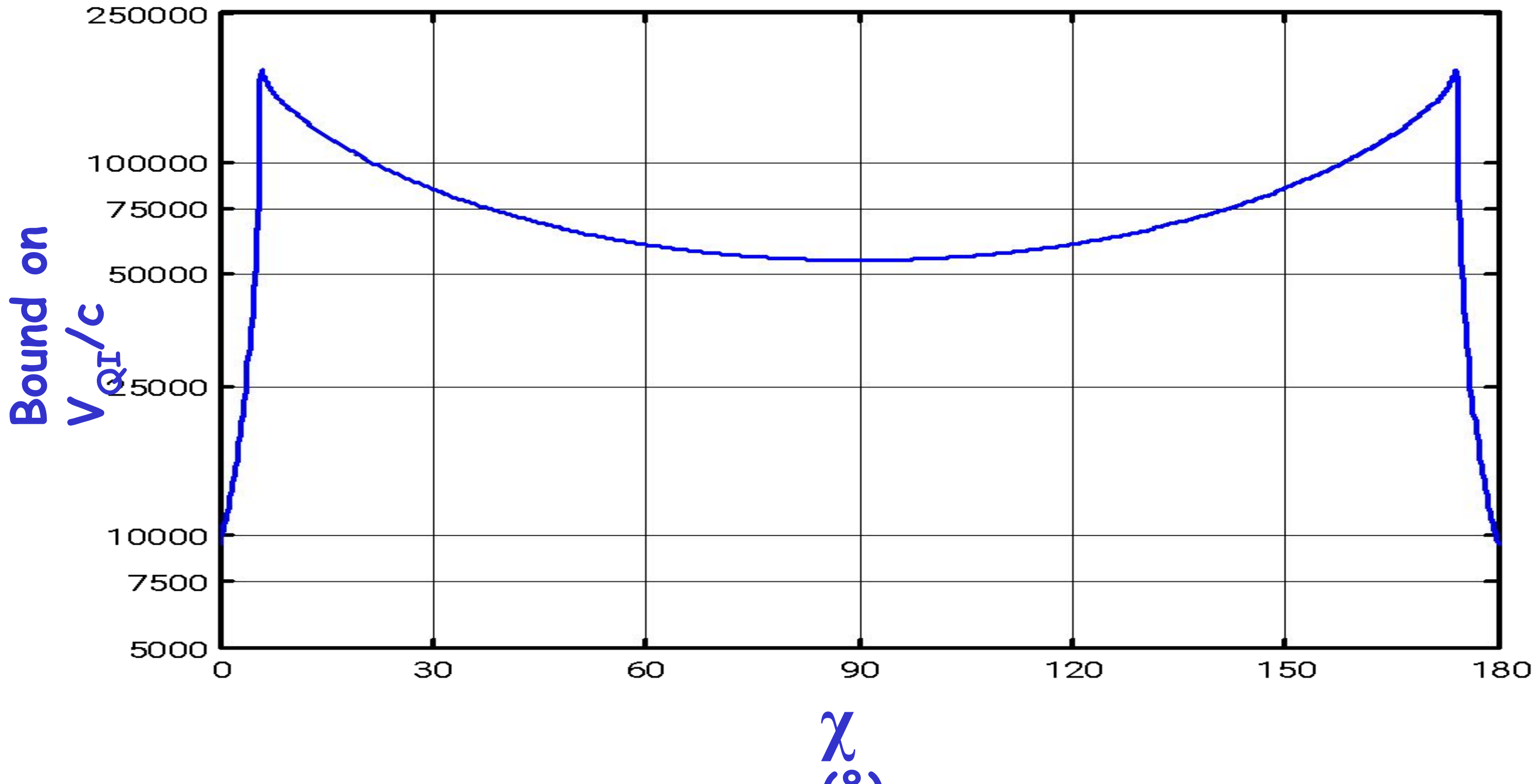
## Bound assuming the Earth's speed is ≤ 300 km/s




**Conclusion: quantum correlations are indeed truly nonlocal.**

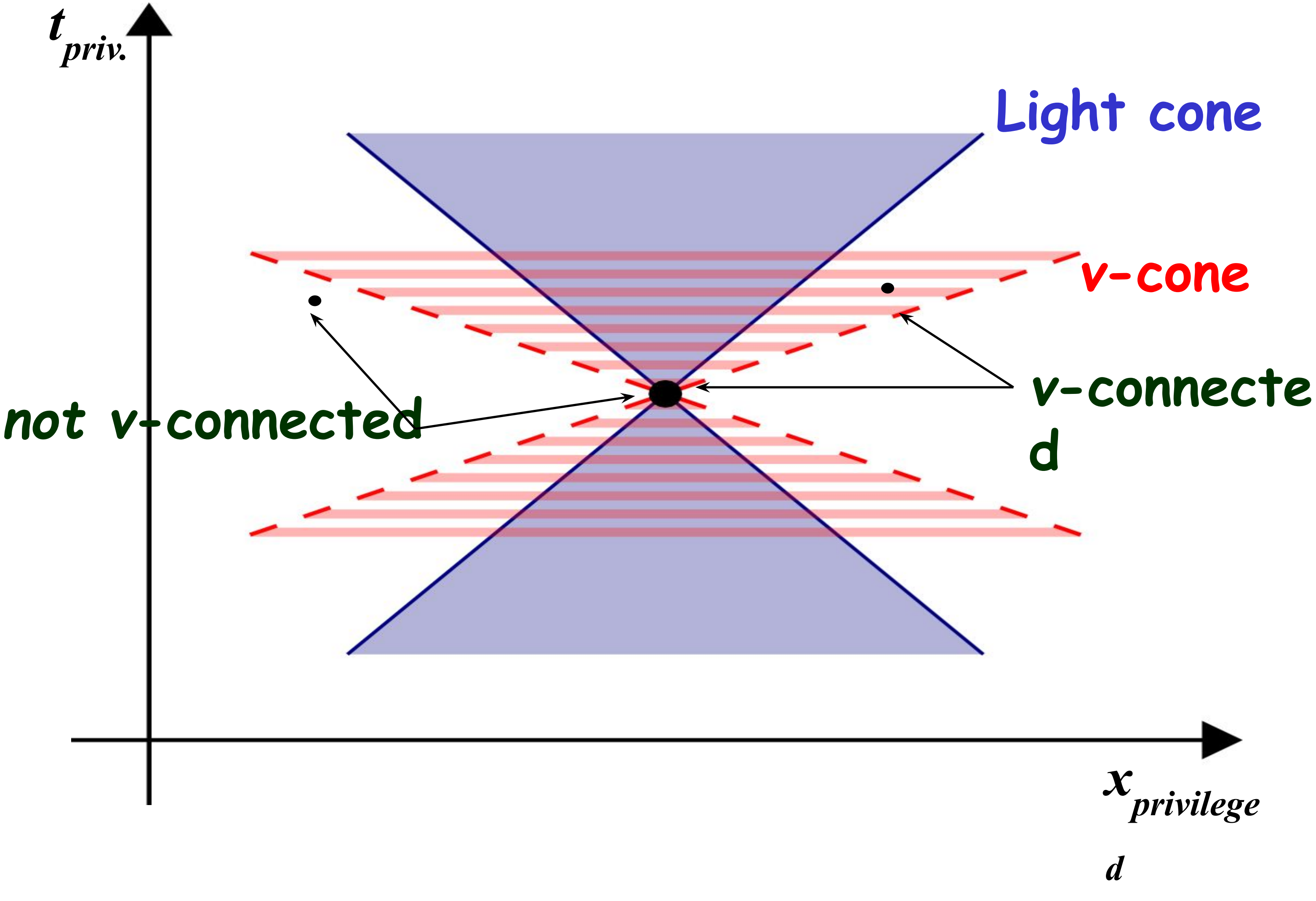
**Indeed, to maintain a description based on spooky action at a distance, one would have to assume speeds even larger than the bound obtained in our experiment**

PRL 88,120404,2002; J.Phys.A 34,7103,2001; Phys.Lett.A

# And so ?

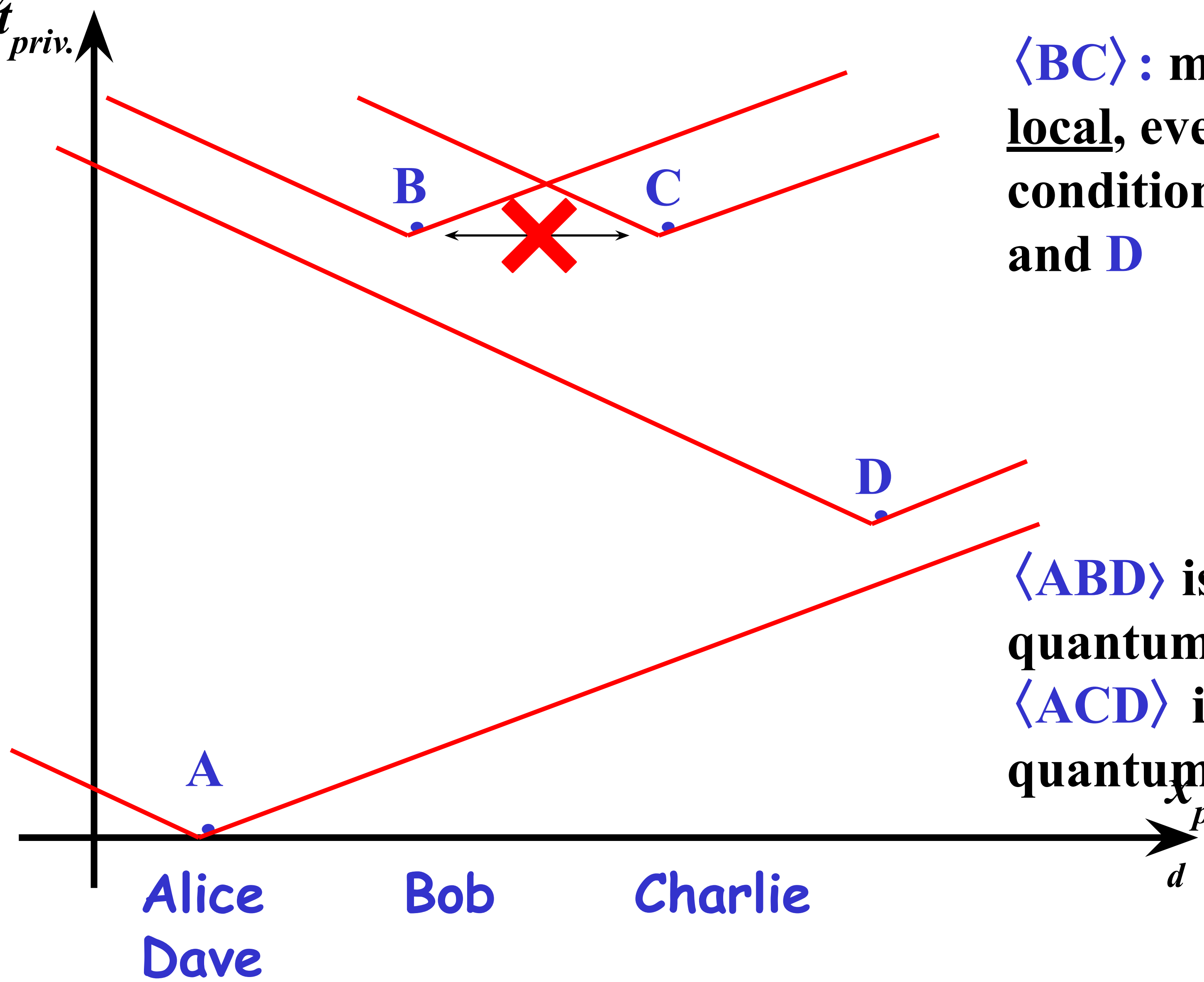
- The influence may merely propagate faster,
- or may not exist at all.
- 2-party experiments will never be able to exclude hidden influences, only set lower bounds on its speed.
- **With only 2 parties, the hypothetical hidden influence could remain hidden for ever.**

# *v*-causality



$t_{priv.}$
Light cone
v-cone
v-connecte
d
not v-connected
$x_{privilege}$
d

# *V*-causality leads to signalling (1)




⟨BC⟩ : must be <u>local</u>, even if conditioned on A and D

⟨ABD⟩ is quantum

⟨ACD⟩ is quantum

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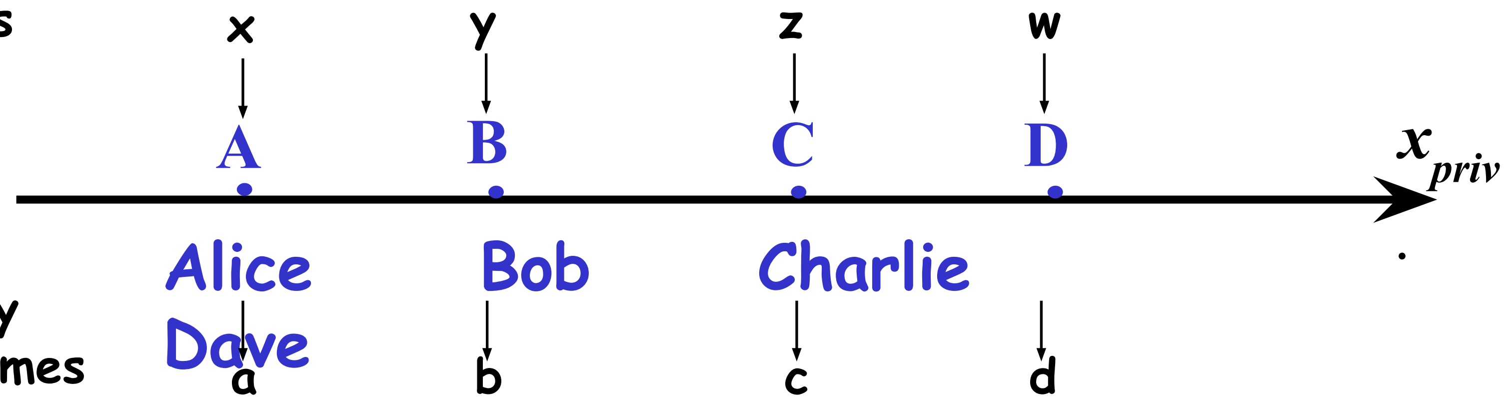
# $\mathcal{V}$-causality leads to signalling (2)

binary inputs 0/1




binary outcomes ±1

<u>Theorem</u>: If $p(a,b,c,d|x,y,z,w)$ is formally non-signalling and $p(b,c|y,z, a,x,d,w)$ is local for all $a,x,d,w$, then

$$J \leq 7$$

NO BC

**Where** $J = -3\langle A_1\rangle - \langle B_0\rangle - \langle B_1\rangle - \langle C_0\rangle - 3\langle D_0\rangle - \langle A_1B_0\rangle - \langle A_1B_1\rangle + \langle A_0C_0\rangle$
$+ 2\langle A_1C_0\rangle + \langle A_0D_0\rangle + \langle B_0D_1\rangle - \langle B_1D_1\rangle - \langle C_0D_0\rangle - 2\langle C_1D_1\rangle$
$+ \langle A_0B_0D_0\rangle + \langle A_0B_0D_1\rangle + \langle A_0B_1D_0\rangle - \langle A_0B_1D_1\rangle - \langle A_1B_0D_0\rangle$

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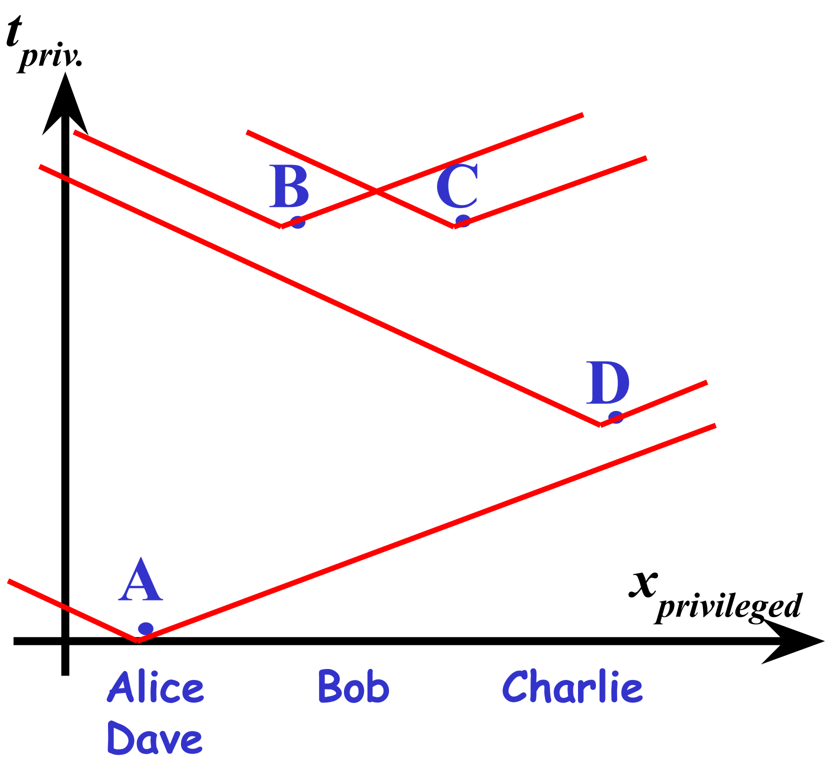
$$J = -3\langle A_0\rangle - \langle B_0\rangle - \langle B_1\rangle - \langle C_0\rangle - 3\langle D_0\rangle - \langle A_1B_0\rangle - \langle A_1B_1\rangle + \langle A_0C_0\rangle$$
$$+ 2\langle A_1C_0\rangle + \langle A_0D_0\rangle + \langle B_0D_1\rangle - \langle B_1D_1\rangle - \langle C_0D_0\rangle - 2\langle C_1D_1\rangle$$
$$+ \langle A_0B_0D_0\rangle + \langle A_0B_0D_1\rangle + \langle A_0B_1D_0\rangle - \langle A_0B_1D_1\rangle -$$
$$- \langle A_1B_1D_0\rangle + \langle A_0C_0D_0\rangle + 2\langle A_1C_0D_0\rangle + 2\langle A_0C_1D_1\rangle$$

NO BC

**Any v-causal model predicts the same value for J as**




*v-causal predictions differ from Q theory, but since J doesn't contain any term involving B and C, the*

Moreover, in an experiment B and C do not need to be measured in the same run.
⇒ <u>No B-C timing issue</u> !

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# *v*-causality leads to signalling

**Fact**: there are quantum states and measurements predicting J>7

**Theorem**: If p(a,b,c,d|x,y,z,w) is formally non-signalling and p(b,c|y,z, a,x,d,w) is local for all a,x,d,w, then

$$J \leq 7$$

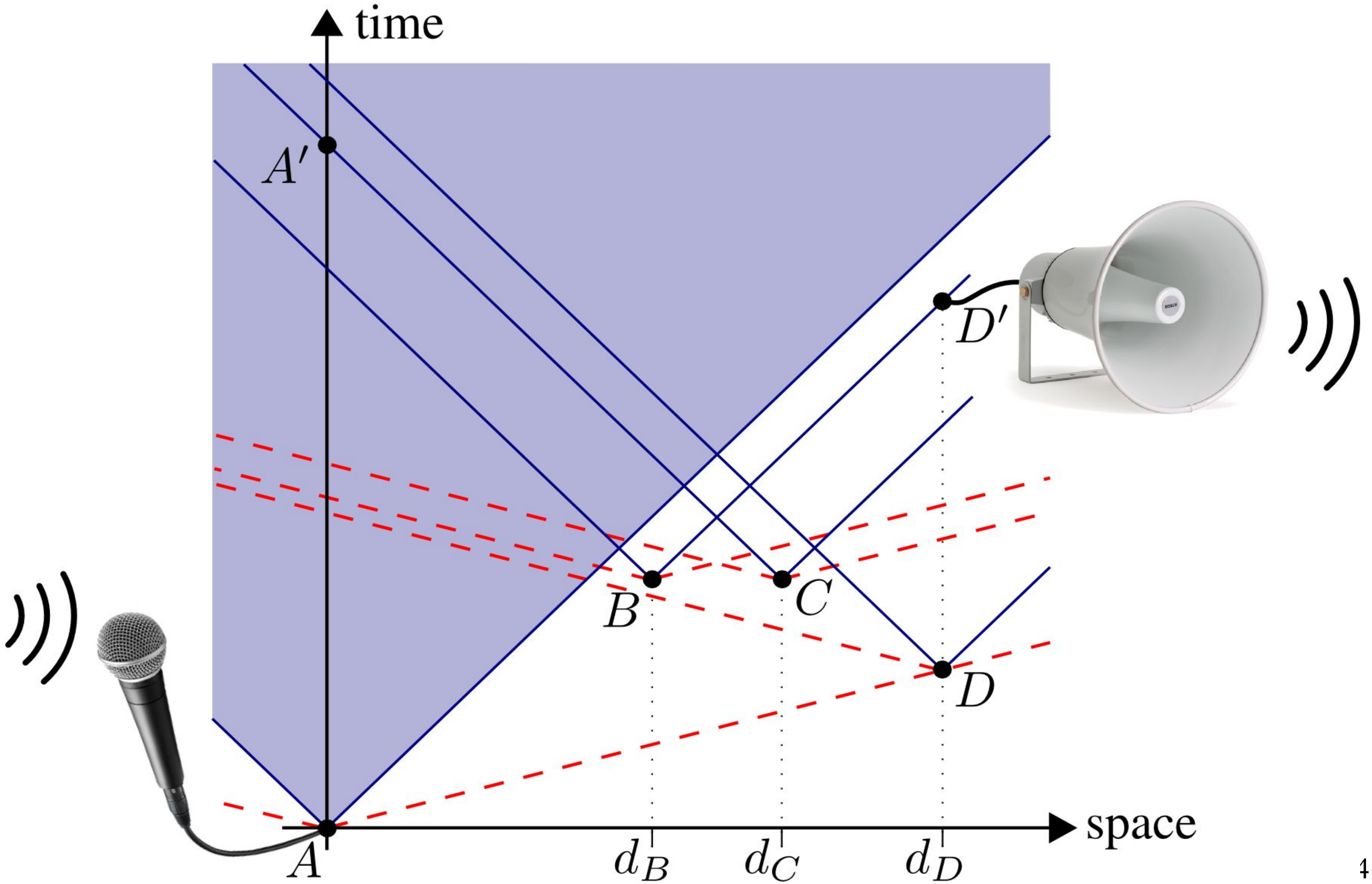
**Consequence**: Since any *v*-causal model predicts that p(b,c|y,z, a,x,d,w) is local, p(a,b,c,d|x,y,z,w) must be formally signalling.

**Note**: in *v*-causal models, the hidden influence is carrying the information; hence – here – signalling is not "non-physical communication".

A similar inequality involving only 3 parties: T. Barnea et al., PRA 88, 022123 (2013)

# *v*-causality leads to supraluminal communication

at the level of classical inputs and outputs



These experiments:

Salart et al., Nature 454, 861, 2008
Cocciaro et al., PLA 375, 379, 2011
J-W Pan's group, PRL 110, 260407, 2013
L. Santamaria et al., scientific report 13.8201 (2023)

should be improved

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Admittedly, such experiments might only set lower bounds on v (*or revolutionaries physics*)

Which goal should the community set?

All the above exps find $v \geq$ 50 to 100 thousand times c

c/speed sound ≈ $10^6$

next generation exp should aim at $v \approx 10^7$ c

2 orders improvement is feasible:

x10 thanks to snspd

x10 thanks to longer distances

+ faster data acquisition

# Understanding Entanglement
## Joint Quantum Measurements

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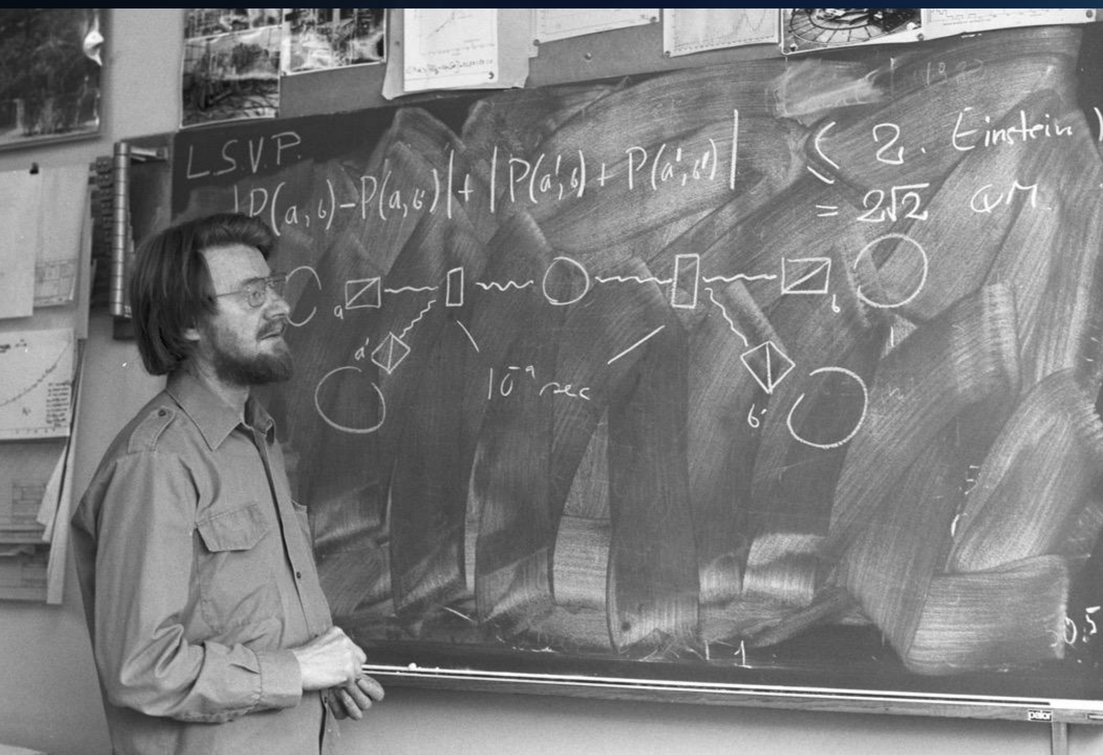
1. After more than 30 years of Q info, we know a lot about entanglement btw 2 (or more) parties, our friends Alice & Bob.
   But what do we know about joint measurements, measurements whose eigenstates (or POVM elements) are entangled?
2. Most physicists – even within our community - would not be able to mention any joint measurement beyond the famous BSM.

The Virtuous Circle of Knowledge and Innovation
Speaker: **Michele Grossi**

# Bell inequalities




# Conclusion

- Quantum Nonlocality presents us with a tremendous challenge to tell stories about how things happen in nature.

**To tell stories about how nature does it one needs some "nonlocal story-tool", e.g. nonlocal randomness: a random event that is able to manifest itself at several locations.**