Quantum Foundations

- A Gateway to Quantum Gravity and Unification -

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Foundational Problems of Quantum Theory

- Collapse of the wave function and absence of macroscopic superpositions.
- Randomness of measurement outcomes and origin of Born probability rule.
- The puzzle of quantum non-locality.
- Super-quantum non-local correlations allowed by special relativity.
- The problem of time in quantum theory:

  one can assume classical spacetime in quantum theory only if the universe is dominated by macroscopic classical objects.

Addressing these problems also guides us to a theory of quantum gravity, and of unification.

Plan of the Talk

- I. Dynamical model for collapse of the wave function
- II. A theory underlying dynamical collapse : Trace Dynamics
- III. Incorporating quantum gravity : Generalised Trace Dynamics
- IV. Removing classical time: Octonions, and Unification of Forces

I. Collapse of the Wave Function

- Assume that lifetime of a superposition is not infinite, but finite.
- A superposition of position states of a particle spontaneously localises to a width r_c once in $T = 1/\lambda$ sec
- Collapse models: $\lambda \sim 10^{-17}\ \mathrm{s}^{-1}, \quad r_c \sim 10^{-5}$ cm
- For a quantum system with N entangled particles, collapse rate is

$$N\lambda \sim 10^6 \text{ for } N \sim 10^{23}$$

- A stochastic, non-linear, non-unitary but norm-preserving modification of the Schrodinger equation.
- Being tested by experiments.
- Collapse models are non-relativistic. Relativistic model must include collapse in time.
- What is the fundamental origin of spontaneous collapse?

Consider a generalisation of Schrodinger evolution

- Such that the generalised evolution is deterministic, non-unitary, but norm-preserving:

$$H = \hbar\omega H_0 + i\gamma A$$

- H is not self-adjoint; evolution of state-vector $|\phi\rangle$ is non-unitary and does not preserve norm.

- Insist on norm-preservation of physical state $|\psi\rangle = |\phi\rangle/\sqrt{\langle\phi|\phi\rangle}$ obeying

$$\frac{d|\psi\rangle}{dt} = \left[-i\omega H_0 + \gamma\left(A - \langle A\rangle\right)\right]|\psi\rangle$$

$$\frac{d\rho}{dt} = -i\omega[H_0, \rho] + \gamma\{A, \rho\} - 2\gamma Tr(\rho A)\rho$$

What is the nature of such evolution?

- Consider a two-level qubit system, with H_0, ρ, A as 2×2 matrices.
- Write these matrices in the eigenbasis of A, having real eigenvalues λ_0, λ_1 with $\lambda_0 > \lambda_1$

$$H_0 = \begin{pmatrix} a_0 & b_{0r} + ib_{0i} \\ b_{0r} - ib_{0i} & d_0 \end{pmatrix} \qquad \rho = \frac{1}{2}\begin{pmatrix} 1+z & x-iy \\ x+iy & 1-z \end{pmatrix} \qquad A = \begin{pmatrix} \lambda_0 & 0 \\ 0 & \lambda_1 \end{pmatrix}$$

- Here, $\vec{v} = (x, y, z)$ is the Bloch vector.

$$\dot{x} = -\omega\left[(a_0 - d_0)y + 2b_{0i}z + \frac{\gamma}{\omega}(\lambda_0 - \lambda_1)xz\right]$$

$$\dot{y} = \omega\left[(a_0 - d_0)x - 2b_{0r}z - \frac{\gamma}{\omega}(\lambda_0 - \lambda_1)yz\right]$$

$$\dot{z} = \omega\left[2b_{0i}x + 2b_{0r}y - \frac{\gamma}{\omega}(\lambda_0 - \lambda_1)(z^2 - 1)\right]$$

Breakdown of superposition for large coupling

- Consider $\gamma/\omega \gg 1, \quad \gamma/\omega \ll -1$

$$\dot{x} = -\frac{\gamma}{\omega}(\lambda_0 - \lambda_1)xz \ , \qquad \dot{y} = -\frac{\gamma}{\omega}(\lambda_0 - \lambda_1)yz \ , \qquad \dot{z} = -\frac{\gamma}{\omega}(\lambda_0 - \lambda_1)(z^2 - 1)$$

$$\text{Solution:} \quad z = \frac{1 - \exp[-2\gamma(\lambda_0 - \lambda_1)t/\omega]}{1 + \exp[-2\gamma(\lambda_0 - \lambda_1)t/\omega]}$$

- $\gamma/\omega \gg 1, \quad z \to 1, \quad x, y \to 0$ system is driven to eigenstate $|0\rangle$ of A
- $\gamma/\omega \ll -1, \quad z \to -1, \quad x, y \to 0$ system is driven to eigenstate $|1\rangle$ of A

- Deterministic, non-unitary, norm-preserving evolution can lead to breakdown of superposition when non-unitarity is significant.

The origin of randomness is coarse-graining

- Assume the non-unitary deterministic theory to hold at some time resolution τ at which quantum theory has not yet been tested.
- We ask what is the emergent theory when coarse-graining is done to $t \gg \tau$
- If the anti-self-adjoint coupling γ is negligible, the emergent theory is quantum mech
- If the anti-self-adjoint coupling γ is significant, interpret γ as a measure of coupling with measuring apparatus.
- Randomness in γ arises because time-of-arrival is not measured to resolution τ
- Therefore, because of coarse-graining in time, outcomes appear random, even though the outcomes are deterministic in the underlying theory.
- How to derive Continuous Spontaneous Localisation (CSL) ?

CSL from coarse-graining a deterministic non-unitary dynamics

- Recall: $\dfrac{d|\psi\rangle}{dt} = \left[-i\omega H_0 + \gamma\left(A - \langle A\rangle\right)\right]|\psi\rangle$

- Let $\gamma \to \sqrt{\Gamma}W(t)$ and let $X = A - \langle A\rangle$ and assume

$$X \to \sqrt{\Gamma}XW(t) - \Gamma\left(X^2 - \left\langle X^2\right\rangle\right)$$: leads to CSL

- $d|\psi_t\rangle = \left[-i\omega H_0 dt - \dfrac{\Gamma}{2}(A - \langle A\rangle)^2 dt + \sqrt{\Gamma}(A - \langle A\rangle)dW\right]|\psi_t\rangle$
- The corresponding master equation is of the Lindblad form.
- Spontaneous localisation: role of measuring apparatus played by `noise field'.
- Noise field = `atoms of space-time-matter': interaction with system is deterministic but it appears random because of coarse-graining.

Conclusion

Quantum theory is an emergent approximation to an underlying deterministic theory which is non-unitary but norm-preserving.

Randomness is because of coarse-graining over the time-of-arrival.

II. Adler's theory of Trace Dynamics

- Conventional quantisation in the Heisenberg picture: given a Hamiltonian dynamics, raise classical dynamical variables to the status of matrices/operators.
- Replace Poisson brackets by Heisenberg commutation relations.
- Replace Hamilton's equations of motion by Heisenberg equations of motion.

———————

- Adler's trace dynamics: do raise classical dynamical variables to matrices, but do not impose Heisenberg commutation relations on these matrices. Instead:
- Construct Lagrangian / Hamiltonian dynamics with matrix-valued dynamical variables.

Trace dynamics

$$S = \sum_i \int d\tau \, \frac{1}{2} m_i \left(\frac{dq_i}{d\tau} \right)^2$$

From real numbered dynamical variables to matrices: $q \to \mathbf{q}$

$$S = \sum_i \int d\tau \, Tr \left[\frac{1}{2} \frac{L_p^2}{L^2} \left(\frac{d\mathbf{q}_i}{d\tau} \right)^2 \right]$$

Novel conserved charge: $\tilde{C} = \sum_i \left[\mathbf{q}_{Bi}, \mathbf{p}_{Bi}\right] - \left\{\mathbf{q}_{Fi}, \mathbf{p}_{Fi}\right\}$: Pre-quantum theory

(Adler-Millard charge)

Trace dynamics —> Quantum (field) theory as an emergent phenomenon

- TD is a Lorentz-invariant matrix-valued Lagrangian / Hamiltonian dynamics.
  It is deterministic, non-unitary but norm-preserving.

- Assume trace dynamics to hold at some time resolution τ_P

- Ask: what is the emergent dynamics at time resolution $\tau \gg \tau_P$? : Two possibilities:

(i) In the approximation that H is self-adjoint and evolution is unitary:

- Statistical thermodynamics is used to show that the novel charge is equipartitioned.

$$[q_B, p_B] = i\hbar \quad ; \quad \{q_F, p_F\} = i\hbar$$

- Heisenberg equations of motion, and QFT, emerge at thermodynamic equilibrium.

Trace dynamics

- (ii) Second possibility:

- If the anti-self-adjoint part of the Hamiltonian is significant, evolution breaks quantum linear superposition.

- Coarse-graining the dynamics introduces an apparent randomness in the evolution, while obeying Born rule : spontaneous localisation.

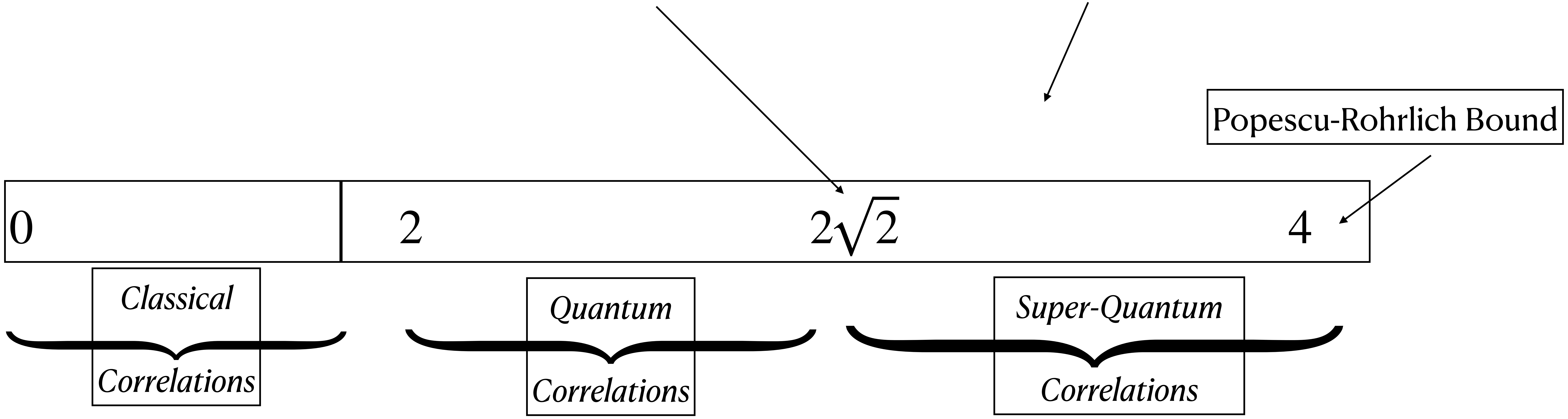
- The underlying theory of trace dynamics is deterministic and non-unitary.

- If non-unitarity is significant, linear superposition is broken.

- Upon coarse-graining, the emergent theory is either quantum theory, or it is classical dynamics.

Trace dynamics admits super-quantum nonlocal correlations

i.e. violates the Tsirelson bound in CHSH inequality



Trace Dynamics when novel charge is not equipartitioned

Suggests that quantum mechanics is approximate, not exact

Trace Dynamics

- Trace dynamics does not specify any fundamental Lagrangian.

- Also, it is a pre-quantum theory, not a pre-spacetime theory.

- Spacetime is assumed classical, and flat (no gravitation). But this is an intermediate step towards developing a pre-spacetime, pre-quantum theory.

- We now propose to include matrix-valued gravitation, and a non-commutative pre-spacetime, and also propose a fundamental Lagrangian.

- This leads to a candidate theory for quantum gravity, and for unification of fundamental forces.

- Generalised trace dynamics.

III. Generalised trace dynamics, and a Lagrangian

- The eigenvalues of the Dirac operator D on a curved space-time manifold:

$$Tr[L_P^2 D^2] \sim L_P^{-2} \int d^4x \sqrt{g}\, R + \mathcal{O}(L_P^0) = \sum_i \lambda_i^2$$

- An `atom' of space-time: $\lambda_i \to \hat{\lambda}_i \equiv \dot{q}_{Bi}$ $\quad \lambda_i^2 \to \int d\tau\, Tr[\dot{q}_{Bi}^2]$ Connes Time τ

- An atom of space-time-matter (an `aikyon') :

$$\frac{S}{\hbar} = \int \frac{d\tau}{\tau_{Pl}}\, Tr\Big\{ \frac{L_P^2}{L^2} \left[\dot{q}_B^\dagger + \frac{L_P^2}{L^2}\beta_1 \dot{q}_F^\dagger\right] \times \left[\dot{q}_B + \frac{L_P^2}{L^2}\beta_2 \dot{q}_F\right] \Big\}$$

- Including Yang-Mills fields [Spectral action principle of Chamseddine and Connes]

$$\frac{S}{\hbar} = \int \frac{d\tau}{\tau_{Pl}}\, Tr\Big\{ \frac{L_P^2}{L^2} \left[\left(\dot{q}_B^\dagger + i\frac{\alpha}{L} q_B^\dagger\right) + \frac{L_P^2}{L^2}\beta_1 \left(\dot{q}_F^\dagger + i\frac{\alpha}{L} q_F^\dagger\right)\right] \times \left[\left(\dot{q}_B + i\frac{\alpha}{L} q_B^\dagger\right) + \frac{L_P^2}{L^2}\beta_2 \left(\dot{q}_F + i\frac{\alpha}{L} q_F\right)\right] \Big\}$$

Generalised trace dynamics: Equations of Motion

- Equations of Motion: $q_1 = q_B + \beta_1 \frac{L_P^2}{L^2} q_F$; $\qquad q_2 = q_B + \beta_2 \frac{L_P^2}{L^2} q_F$

$$\ddot{q}_1 = -\frac{\alpha^2}{L^2} q_1 \ ; \qquad \ddot{q}_2 = -\frac{\alpha^2}{L^2} q_2$$

- Define:

$$\dot{Q}_B = \frac{1}{L}(i\alpha q_B + L\dot{q}_B); \qquad \dot{Q}_F = \frac{1}{L}(i\alpha q_F + L\dot{q}_F);$$

$$\dot{Q}_1^\dagger = \dot{Q}_B^\dagger + \frac{L_p^2}{L^2}\beta_1 \dot{Q}_F^\dagger; \qquad \dot{Q}_2 = \dot{Q}_B + \frac{L_p^2}{L^2}\beta_2 \dot{Q}_F$$

$$\frac{S}{\hbar} = \frac{1}{2}\int \frac{d\tau}{\tau_{Pl}}\ Tr\left[\frac{L_p^2}{L^2}\dot{Q}_1^\dagger\ \dot{Q}_2\right]$$

- This is the form prior to left-right symmetry breaking

Spontaneous localisation gives rise to the classical world.

IV. Pre-spacetime and Octonions

What do numbers have to do with elementary particles?

Making physical space with numbers other than real numbers.

There are only four kinds of number systems

Which permit

Addition, Subtraction, Multiplication, Division

Real Numbers

- Natural Numbers 0, 1, 2, 3, …
- Integers …, −3, −2, −1, 0, 1, 2, 3, …
- Rational numbers .., −1, .., −1/2,.. 0, ..,1/2,.., 1,..
- Real numbers = rationals + irrationals (e.g. $\sqrt{2}$)

Complex Numbers

- Which is the number whose square is -1?
- Define the imaginary number (i) such that

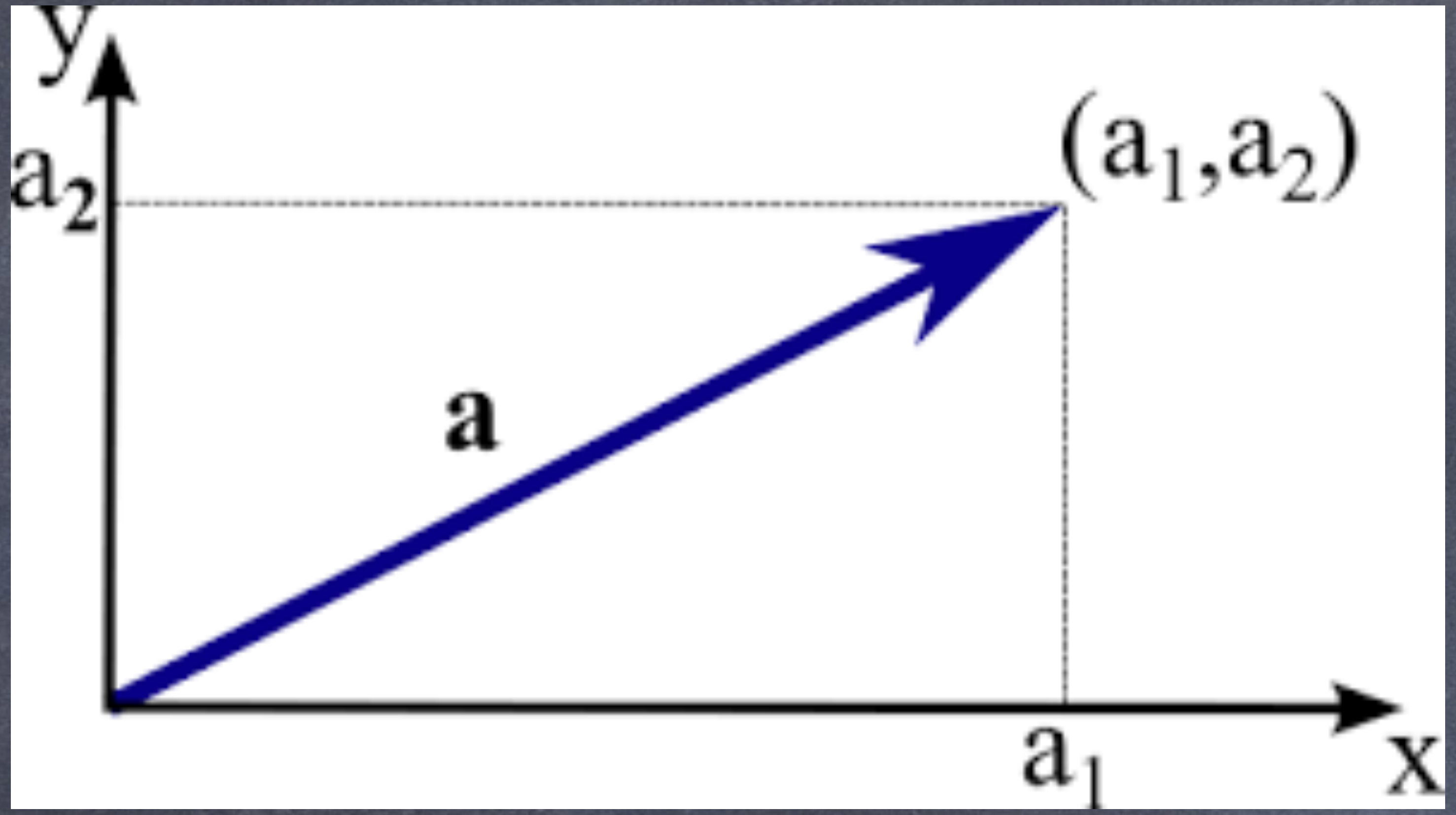
$$i^2 = -1 \qquad i \equiv \sqrt{-1}$$

- Complex Numbers: $a + ib \qquad e.g. \quad 3 + 8i$

- Rotation of a vector in a plane

The vector can be expressed as a complex number.

Complex numbers and vectors in a plane



i

Real part

$$a = a_1 + a_2 i$$

Rotate a clockwise by thirty degrees:

$$b = (\cos 30^o + i \sin 30^o)a = (\cos 30^o + i \sin 30^o) \times (a_1 + a_2 i)$$

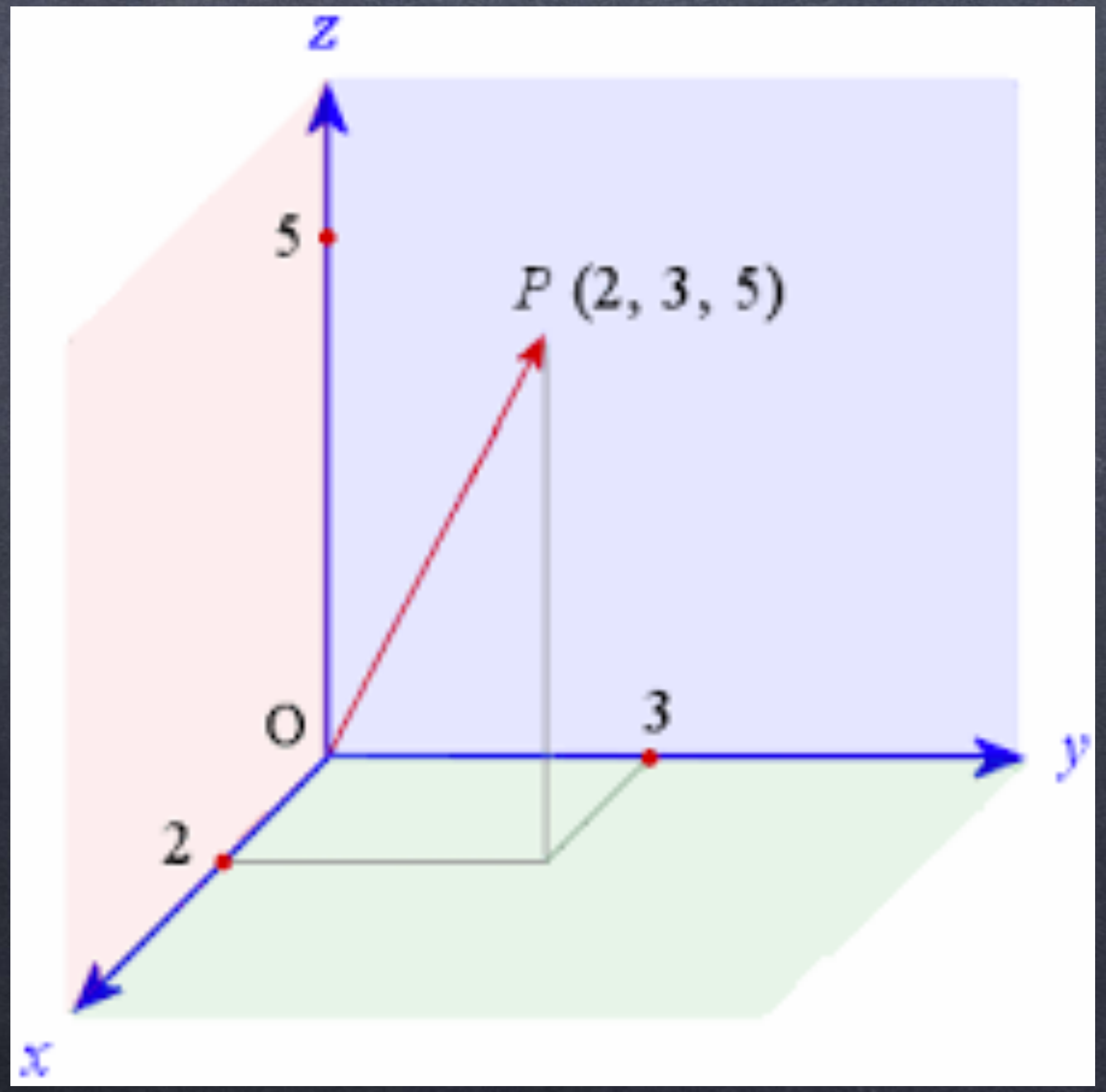
How to describe rotations in three dimensions?

Quaternions

$$q = a_0 + a_1\hat{i} + a_2\hat{j} + a_3\hat{k} \qquad \hat{i}^2 = \hat{j}^2 = \hat{k}^2 = -1$$

$$\hat{i}\hat{j} = -\hat{j}\hat{i} = \hat{k} \qquad \hat{j}\hat{k} = -\hat{k}\hat{j} = \hat{i} \qquad \hat{k}\hat{i} = -\hat{i}\hat{k} = \hat{j}$$

Rotation in three dimensions



$$P = 2\hat{i} + 3\hat{j} + 5\hat{k}$$

Rotate by thirty degrees around $\tilde{q} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$$Q = \cos 30^o + \tilde{q}\sin 30^o$$

$$P' = QPQ^{-1}$$

Quaternionic multiplication and 3-vectors

$$\vec{\mathbf{v}}_1 = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} \qquad \vec{\mathbf{v}}_2 = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}$$

$$(w_1 + \underbrace{x_1 i + y_1 j + z_1 k}_{\vec{\mathbf{v}}_1})(w_2 + \underbrace{x_2 i + y_2 j + z_2 k}_{\vec{\mathbf{v}}_2}) = (w_1, \vec{\mathbf{v}}_1)(w_2, \vec{\mathbf{v}}_2) =$$

$$(w_1 w_2 - \vec{\mathbf{v}}_1 \cdot \vec{\mathbf{v}}_2,\ w_1 \vec{\mathbf{v}}_2 + w_2 \vec{\mathbf{v}}_1 + \vec{\mathbf{v}}_1 \times \vec{\mathbf{v}}_2)$$

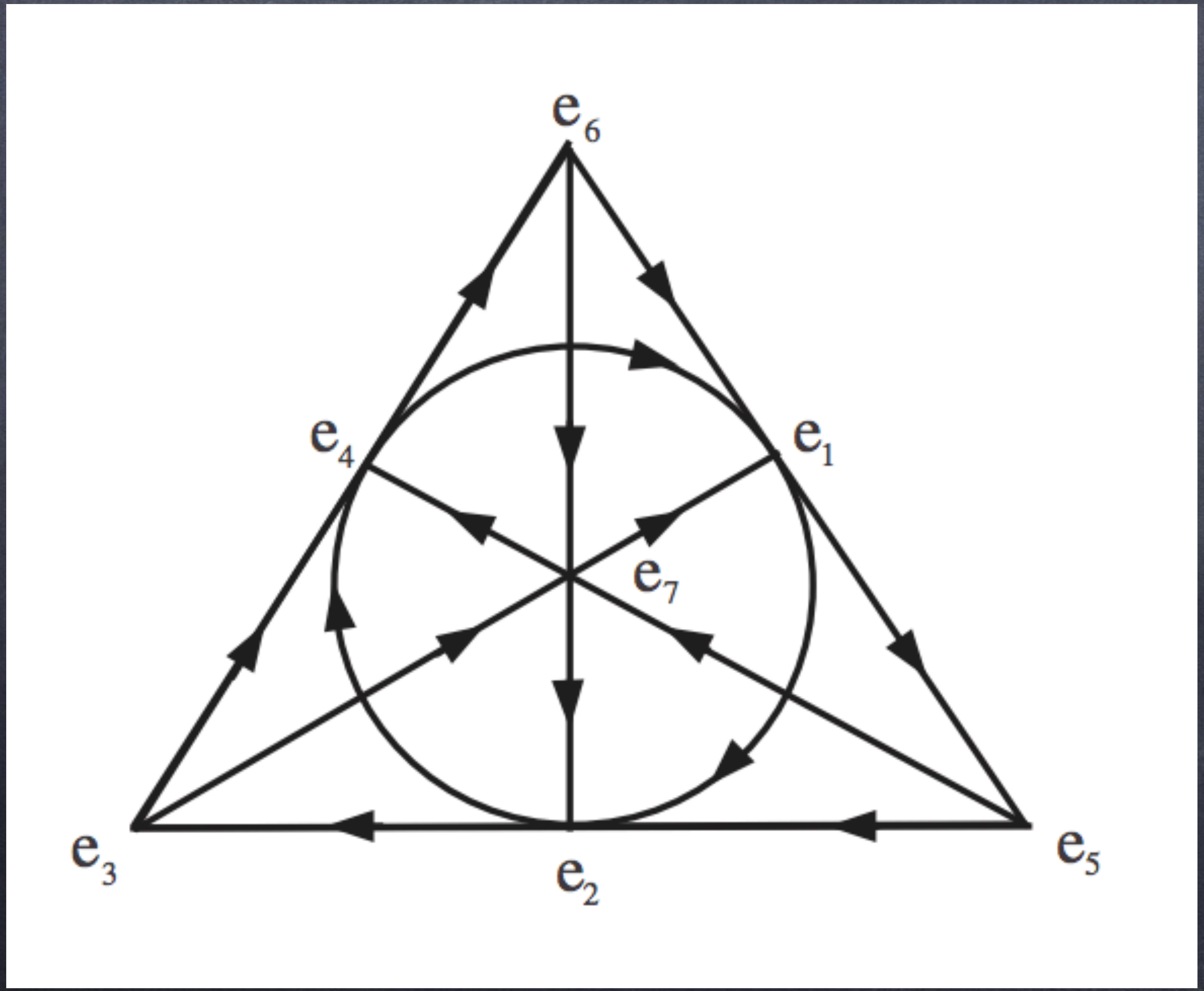
There's also a rather elegant form of this multiplication rule written in terms of the dot product and the cross product,

Octonions

$$O = a_0 + a_1e_1 + a_2e_2 + a_3e_3 + a_4e_4 + a_5e_5 + a_6e_6 + a_7e_7$$

$$e_i^2 = -1 \qquad e_ie_j = -e_je_i$$

Multiplication: Fano Plane



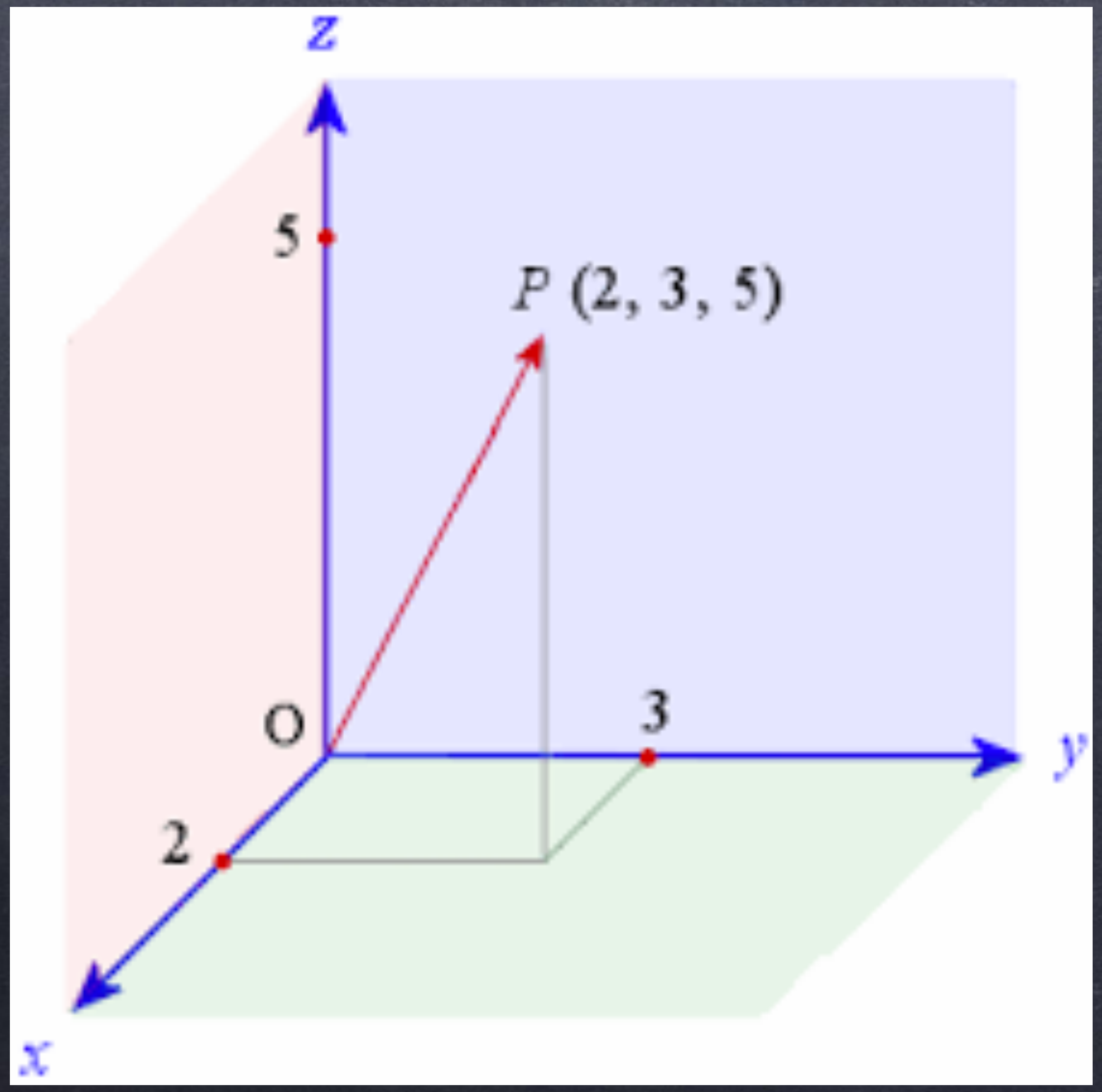
Multiplication
Is
Non-commutative
And
Non-associative

$$O_1O_2 \neq O_2O_1$$

$$O_1(O_2O_3) \neq (O_1O_2)O_3$$

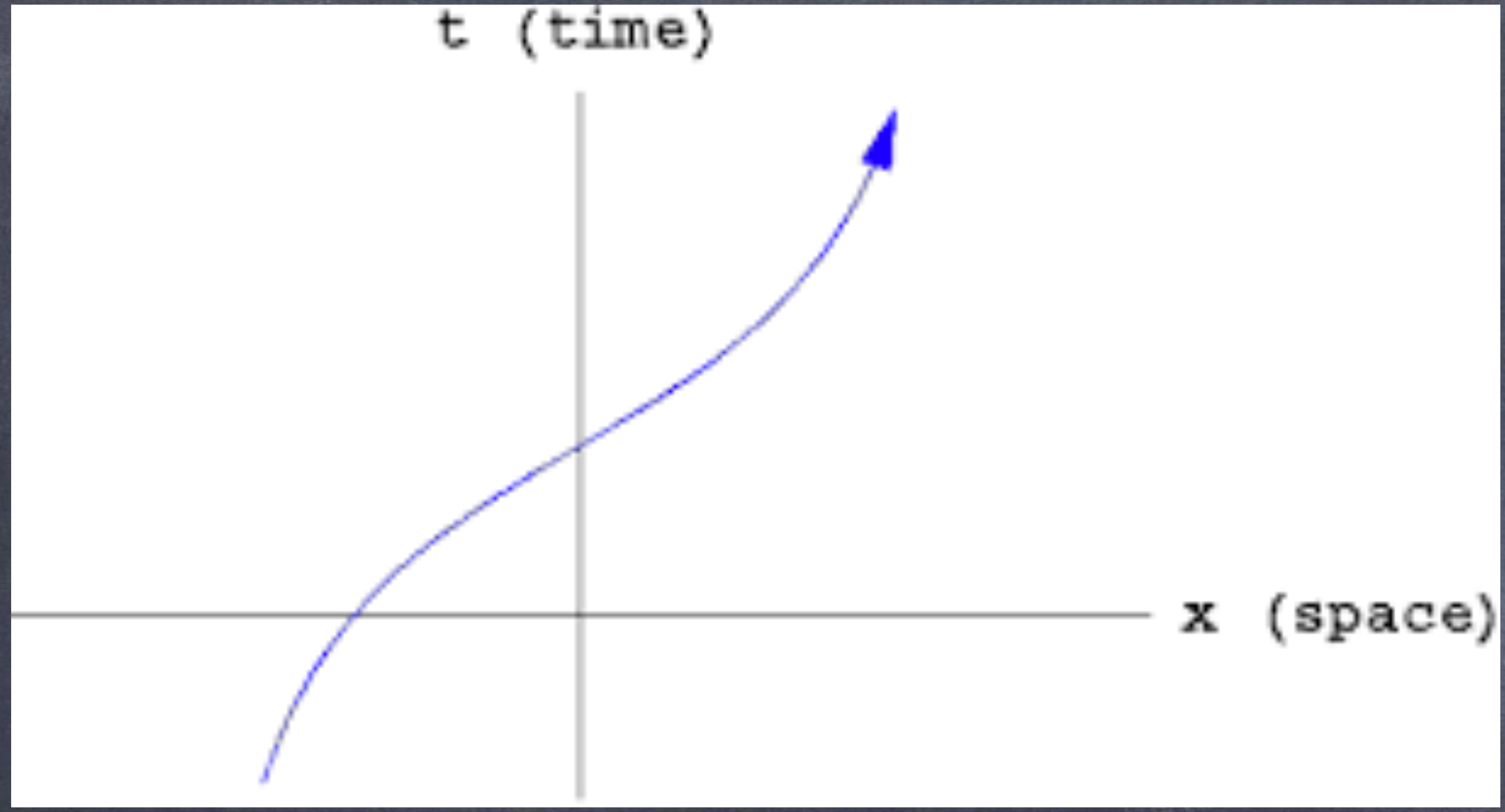
Numbers describe Physical Space

Real Line



- Three-dimensional space
- Particles move in this space.

Quaternions describe 4-D spacetime



$$
\begin{aligned}
\mathrm{SO}(n) &= \{x \in \mathbb{R}[n]\colon\ xx^* = 1,\ \det(x) = 1\},\\
\mathrm{SU}(n) &= \{x \in \mathbb{C}[n]\colon\ xx^* = 1,\ \det(x) = 1\},\\
\mathrm{Sp}(n) &= \{x \in \mathbb{H}[n]\colon\ xx^* = 1\}.
\end{aligned}
$$

- The space-time of Special Relativity Is Quaternionic
- But no restriction On properties such As charge and mass.

Octonions describe unified 'spacetime and internal gauge symmetry space'

The symmetry groups of the octonions

The five exceptional Lie groups denoted:

$$G_2, F_4, E_6, E_7, E_8$$

- G_2 is a 14-dim group : automorphism group of the octonions.
- Clifford algebras made from complex octonions are used to construct spinors.

Dirac Operator

$$D = \hat{i}\frac{\partial i}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{j}\frac{\partial}{\partial z}$$

$$D^2 = -\nabla^2$$

Generalised Trace Dynamics

Matrix dynamics over non-commuting coordinates

- For an atom of space-time-matter, e.g. :

$$S = \int d\tau \; Tr \; [\dot{q}_B + q_B]^2$$

$$\dot{q}_B \equiv D = \dot{q}_{Bx} \hat{i} \frac{\partial}{\partial x} + ...$$

$$q_B = q_{Bx} \hat{i} + \ldots$$

- Assume the Lagrangian to have an $E_8 \times E_8$ symmetry and

$$E_8 \rightarrow SU(3) \times E_6 \rightarrow SU(3) \times SU(3) \times SU(3) \times SU(3)$$

reveals the standard model, general relativity, and two new forces.

Space-time from

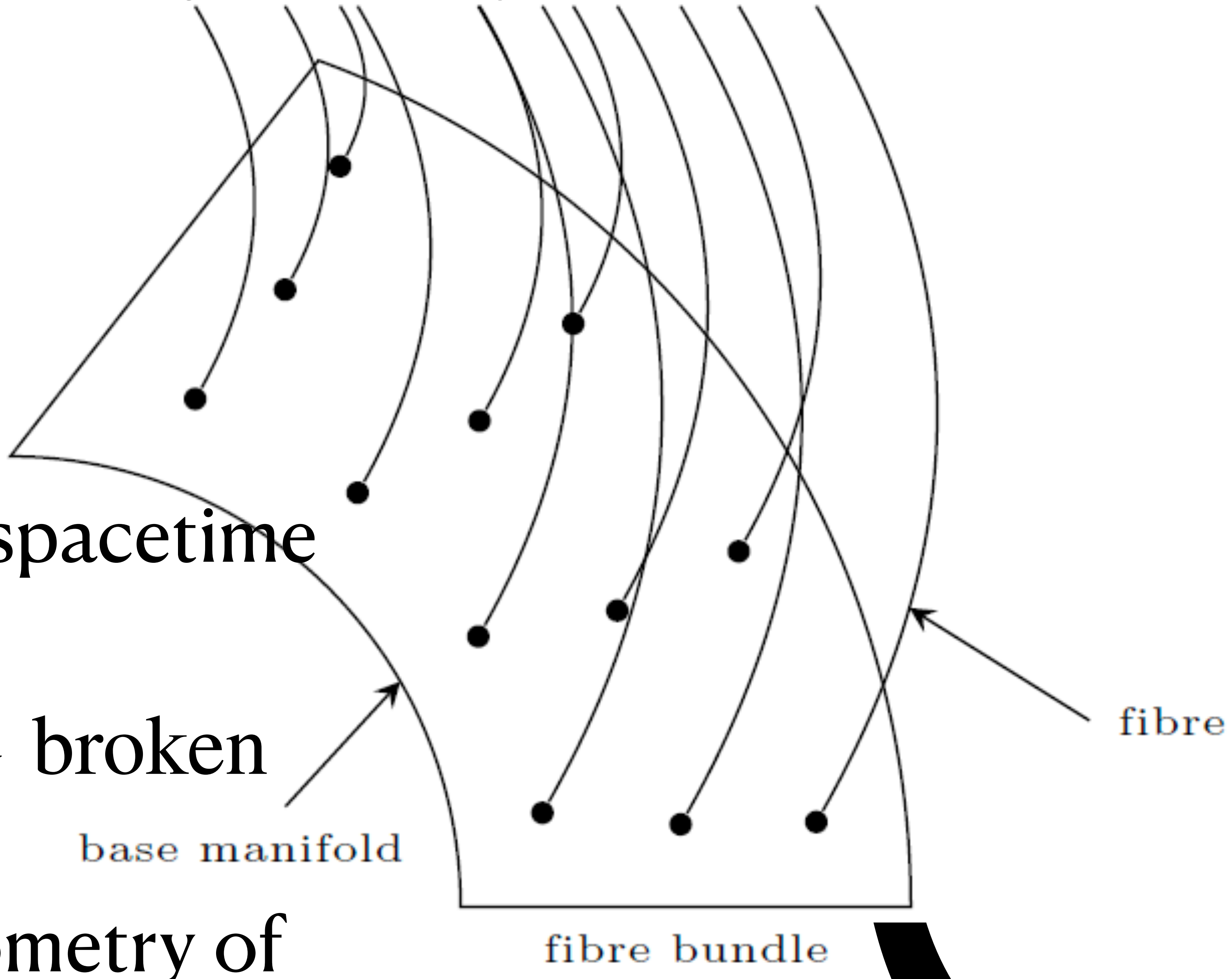
Collapse of the wave function

Big Bang ——> Atoms of space-time-matter

$E_8 \otimes E_8$ symmetry

Electro-weak symmetry breaking ... also a chiral symmetry breaking

$SU(3)_c \otimes U(1)_{\text{em}}$



Second 4D (anti-)spacetime

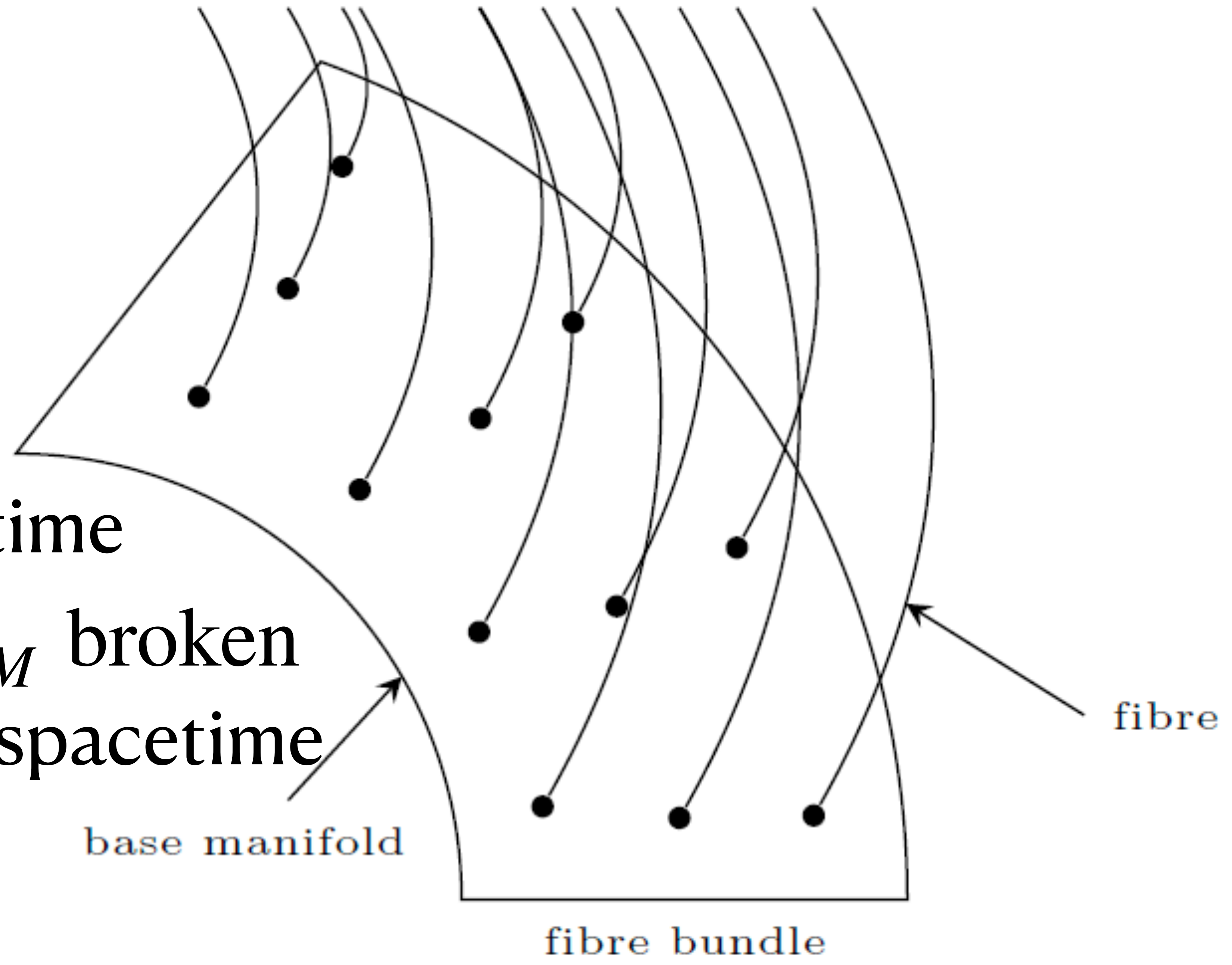
$SU(2)_L \otimes U(1)_Y$ broken

Weak force = geometry of 2nd (anti-)spacetime

$(-,-,-,+)$

Two Higgs

$SU(3)_{\text{grav}} \otimes U(1)_{DEM}$



Our 4D spacetime

$SU(2)_R \otimes U(1)_{YDEM}$ broken

GR=geometry of 1st spacetime

$(-,+,+,+)$

Our Universe

6D spacetime (3, 3)

Locality

Are there any testable predictions?

- No smoking gun prediction yet.
- Pre-gravitation is mediated by spin one gauge bosons. Spin-2 graviton?
- Insight into values of dimensionless fundamental constants, e.g. mass ratios.
- There are three right-handed sterile neutrinos.
- There is an additional, charged, Higgs boson.
- Electron and positron repel under $U(1)_{DEM}$
- Holographic length uncertainty relations.
- Superquantum nonlocal correlations.
- An explanation for strong CP problem, and origin of matter-antimatter asymmetry.
- General relativity modified by $U(1)_{DEM}$. MOND?

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- The problem of time in quantum theory:

  one can assume classical spacetime in quantum theory only if the universe is dominated by macroscopic classical objects.

Addressing these problems guides us to a theory of quantum gravity, and of unification.

CONCLUSION : *Gravitation and quantum theory are emergent phenomena*

- Newtonian mechanics ——> Special Relativity ——-> General Relativity
- Newtonian mechanics ——-> Special Relativity ——-> Quantum Theory

- Merger ?
- Newtonian mechanics ———> Special Relativity ———> `Matrix dynamics on non-commuting pre-spacetime'

  Dirac operator: D = Octonionic gradient operator ——-> unification

  Lagrangian = Trace D^2

- Spontaneous collapse gives rise to classical spacetime and to classical fields
- Uncollapsed degrees of freedom obey quantum theory / Gen. Trace Dynamics