

Kinematic distributions in top decay

A probe of new physics and top-polarization

at

TOP09 : Top quark Physics

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by

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in collaboration with

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top-quark : A looking glass

The mass of the top-quark is very large ($m_t \sim 175 \text{ GeV}$)

- its decay width ($\Gamma_t \sim 1.5 \text{ GeV}$) is much larger than the typical scale of hadronization, i.e. it decays before getting hadronized. The spin information of top-quark is translated to the decay distribution.

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We have a clean looking glass for new physics.

Anomalous t -decay

Anomalous tbW vertex :

$$\Gamma^\mu = \frac{g}{\sqrt{2}} \left[\gamma^\mu (f_{1L} P_L + f_{1R} P_R) - \frac{i\sigma^{\mu\nu}}{m_W} (p_t - p_b)_\nu (f_{2L} P_L + f_{2R} P_R) \right]$$

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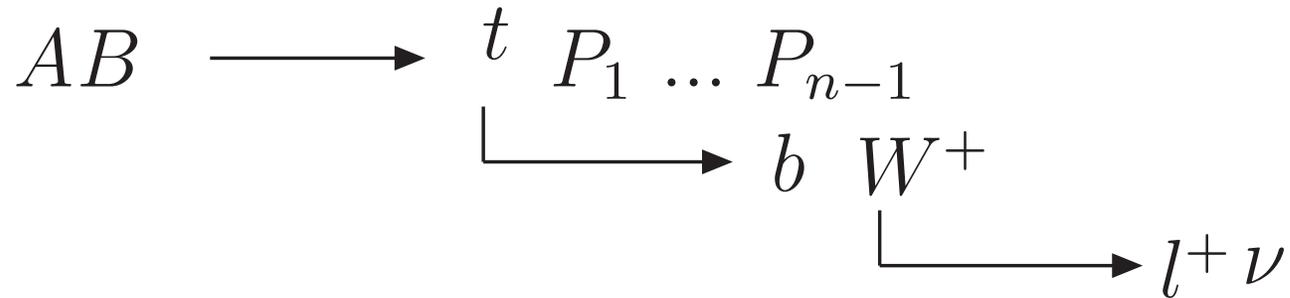
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$$\frac{1}{\Gamma_t} \frac{d\Gamma_t}{d \cos \theta_f} = \frac{1}{2} \left(1 + \alpha_f P_t \cos \theta_f \right)$$

$$\alpha_l = 1 - \mathcal{O}(f_i^2)$$

$$\alpha_b = - \left[\frac{m_t^2 - 2m_W^2}{m_t^2 + 2m_W^2} \right] + \Re(f_{2R}) \left[\frac{8m_t m_W (m_t^2 - m_W^2)}{(m_t^2 + 2m_W^2)^2} \right] + \mathcal{O} \left(\frac{m_b}{m_W}, f_i^2 \right)$$

Lepton distribution



Lepton distribution is independent of anomalous tbW coupling if

- t -quark is on-shell; narrow-width approximation for t -quark,
- anomalous couplings f_{1R} , f_{2R} and f_{2L} are small,
- narrow-width approximation for W -boson,
- b -quark is mass-less,
- $t \rightarrow bW(\ell\nu_\ell)$ is the only decay channel for t -quark.

Decay distribution

Narrow-width approximation for t -quark \Rightarrow

$$|\overline{\mathcal{M}}|^2 = \frac{\pi \delta(p_t^2 - m_t^2)}{\Gamma_t m_t} \sum_{\lambda, \lambda'} \rho(\lambda, \lambda') \Gamma(\lambda, \lambda')$$

where,

$$\rho(\lambda, \lambda') = M_\rho(\lambda) M_\rho^*(\lambda') \quad \text{and} \quad \Gamma(\lambda, \lambda') = M_\Gamma(\lambda) M_\Gamma^*(\lambda').$$

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$$\rho(\lambda, \lambda') = M_\rho(\lambda) M_\rho^*(\lambda') \quad \text{and} \quad \Gamma(\lambda, \lambda') = M_\Gamma(\lambda) M_\Gamma^*(\lambda').$$

$$d\sigma = \sum_{\lambda, \lambda'} \left[\frac{(2\pi)^4}{2I} \rho(\lambda, \lambda') \delta^4(k_A + k_B - p_t - \sum_i^{n-1} p_i) \frac{d^3 p_t}{2E_t (2\pi)^3} \prod_i^{n-1} \frac{d^3 p_i}{2E_i (2\pi)^3} \right] \\ \times \left[\frac{1}{\Gamma_t} \left(\frac{(2\pi)^4}{2m_t} \Gamma(\lambda, \lambda') \delta^4(p_t - p_b - p_\nu - p_\ell) \frac{d^3 p_b}{2E_b (2\pi)^3} \frac{d^3 p_\nu}{2E_\nu (2\pi)^3} \right) \frac{d^3 p_\ell}{2E_\ell (2\pi)^3} \right].$$

Decay distribution

Production part ($\phi_t = 0$):

$$\int \frac{d^3 p_t}{2E_t (2\pi)^3} \prod_i^{n-1} \frac{d^3 p_i}{2E_i (2\pi)^3} \frac{(2\pi)^4}{2I} \rho(\lambda, \lambda') \delta^4 \left(k_A + k_B - p_t - \left(\sum_i^{n-1} p_i \right) \right)$$
$$= d\sigma_{2 \rightarrow n}(\lambda, \lambda') dE_t d\cos\theta_t.$$

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Decay part (in rest rest frame of t -quark) :

$$\frac{1}{\Gamma_t} \frac{(2\pi)^4}{2m_t} \int \frac{d^3 p_\ell}{2E_\ell(2\pi)^3} \frac{d^3 p_b}{2E_b(2\pi)^3} \frac{d^3 p_\nu}{2E_\nu(2\pi)^3} \Gamma(\lambda, \lambda') \delta^4(p_t - p_b - p_\nu - p_\ell)$$

$$= \frac{1}{32\Gamma_t m_t} \frac{E_\ell}{(2\pi)^4} \frac{\langle \Gamma(\lambda, \lambda') \rangle}{m_t E_\ell} dE_\ell d\Omega_\ell dp_W^2.$$

Angular brackets stands for averaging over ϕ_b about \vec{p}_l .

Decay density matrix

In the rest frame of t -quark, we have

$$\begin{aligned}\langle \Gamma(\pm, \pm) \rangle &= g^4 m_t |\Delta_W(p_W^2)|^2 (1 \pm \cos \theta_l) \times E_\ell^0 F(E_\ell^0), \\ \langle \Gamma(\pm, \mp) \rangle &= g^4 m_t |\Delta_W(p_W^2)|^2 (\sin \theta_l e^{\pm i \phi_l}) \times E_\ell^0 F(E_\ell^0).\end{aligned}$$

where $\Delta_W(p_W^2) = \frac{1}{p_W^2 - m_W^2 + i \Gamma_W m_W}$

$$\begin{aligned}F(E_\ell^0) &= \left[(m_t^2 - m_b^2 - 2p_t \cdot p_l) \left(|f_{1L}|^2 + \Re(f_{1L} f_{2R}^*) \frac{m_t}{m_W} \frac{p_W^2}{p_t \cdot p_l} \right) \right. \\ &\quad \left. - 2\Re(f_{1L} f_{2L}^*) \frac{m_b}{m_W} p_W^2 - \Re(f_{1L} f_{1R}^*) \frac{m_b m_t}{p_t \cdot p_l} p_W^2 \right]\end{aligned}$$

In general,

$$\langle \Gamma(\lambda, \lambda') \rangle = (m_t E_\ell^0) |\Delta(p_W^2)|^2 g^4 A(\lambda, \lambda') F(E_\ell^0)$$

Angular distribution of lepton

Combining production and decay part, we have

$$\begin{aligned} d\sigma &= \frac{1}{32 \Gamma_t m_t (2\pi)^4} \left[\sum_{\lambda, \lambda'} d\sigma_{2 \rightarrow n}(\lambda, \lambda') \times g^4 A^{c.m.}(\lambda, \lambda') \right] \\ &\times dE_t d\cos\theta_t d\cos\theta_\ell d\phi_\ell \\ &\times E_\ell F(E_\ell) dE_\ell dp_W^2 \end{aligned}$$

and

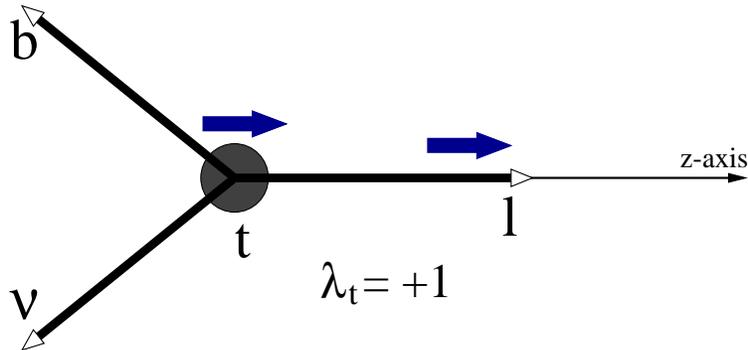
$$\Gamma_t \propto \int E_\ell F(E_\ell) dE_\ell dp_W^2$$

Contribution from anomalous tbW couplings cancels between numerator and denominator, if $t \rightarrow bW$ is the only decay channel.

\Rightarrow Lepton angular distribution is independent of anomalous tbW interactions.

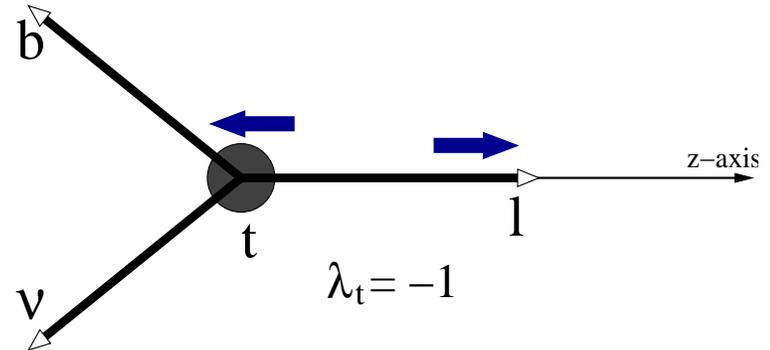
Angular distribution of lepton

(Thanks to Peskin)



$$\mathcal{M}(t_{\uparrow}; \phi_b = 0) = C_{SM} + \mathcal{O}(f_i)$$

$$\mathcal{M}(t_{\uparrow}; \phi_b) = \mathcal{M}(t_{\uparrow}; \phi_b = 0) e^{-i(0)\phi_b}$$

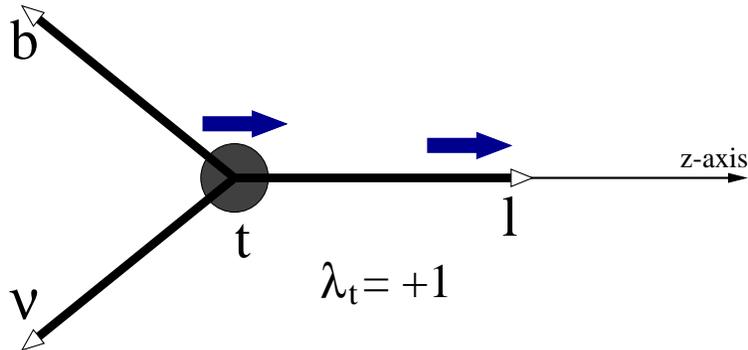


$$\mathcal{M}(t_{\downarrow}; \phi_b = 0) = 0 + \mathcal{O}(f_i)$$

$$\mathcal{M}(t_{\downarrow}; \phi_b) = \mathcal{M}(t_{\downarrow}; \phi_b = 0) e^{-i(-1)\phi_b}$$

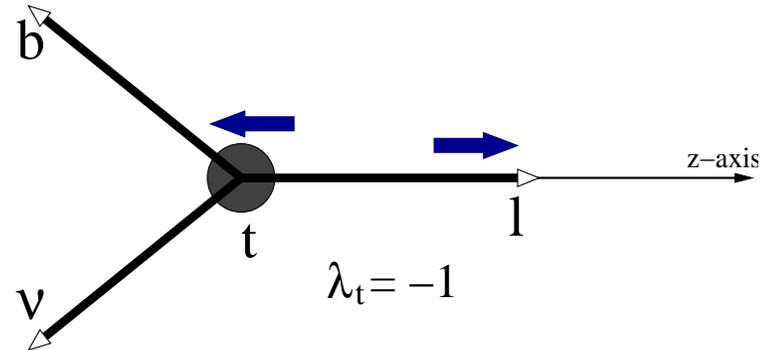
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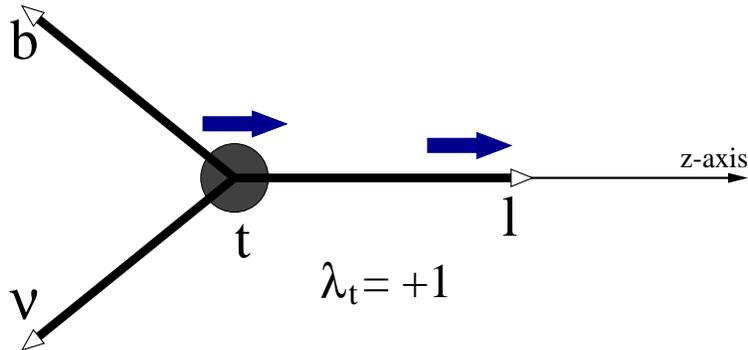
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The density matrix of the configuration is:

$$\Gamma_t \propto \begin{bmatrix} 1 + \mathcal{O}(f_i) + \mathcal{O}(f_i^2) & (\) e^{-i\phi_b} \\ (\) e^{i\phi_b} & \mathcal{O}(f_i^2) \end{bmatrix}$$

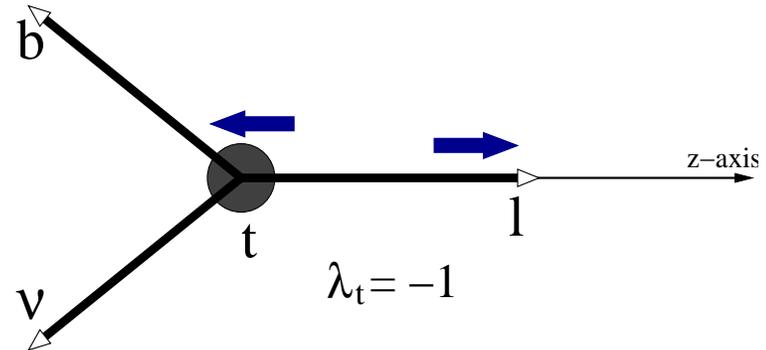
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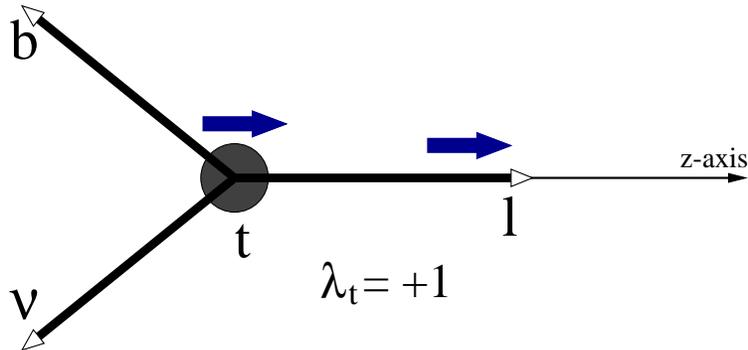
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Integration over ϕ_b gives:

$$\langle \Gamma_t \rangle \propto \begin{bmatrix} 1 + \mathcal{O}(f_i) + \mathcal{O}(f_i^2) & 0 \\ 0 & \mathcal{O}(f_i^2) \end{bmatrix}.$$

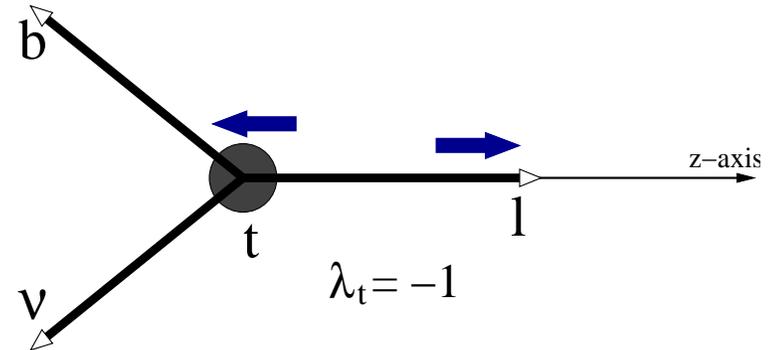
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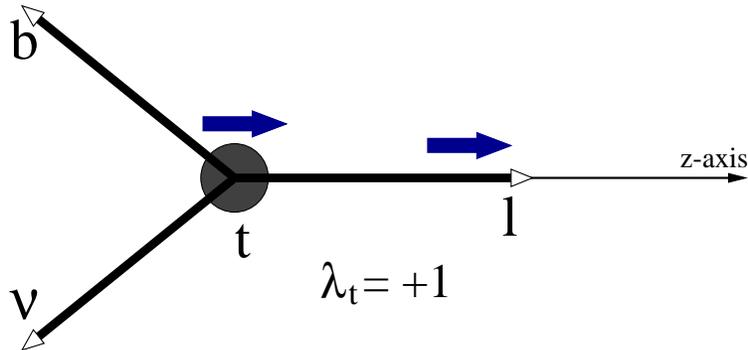
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Neglecting the terms of $\mathcal{O}(f_i^2)$ we get:

$$\langle \Gamma_t \rangle \propto (1 + \mathcal{O}(f_i)) \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},$$

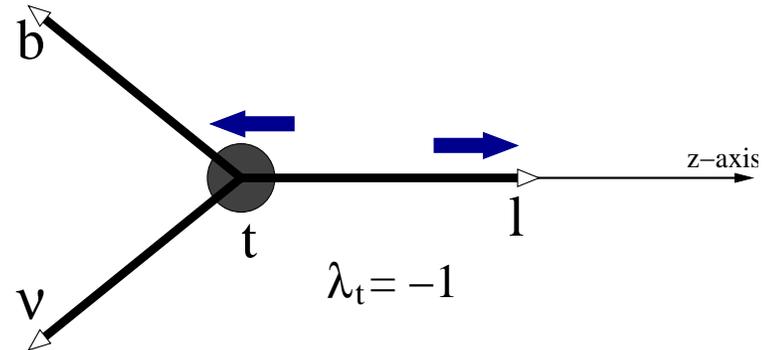
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Rotation of \vec{p}_l to (θ_l, ϕ_l) gives:

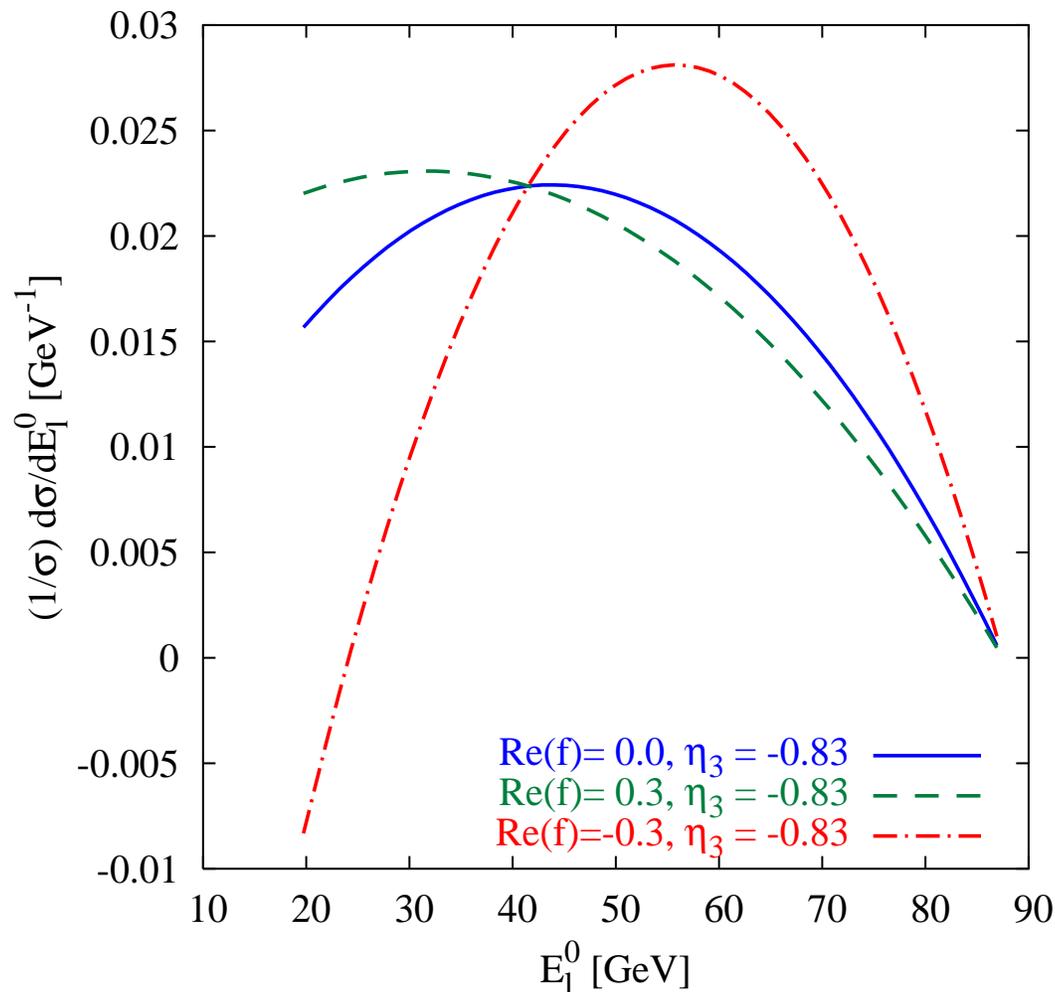
$$\langle \Gamma_t \rangle \propto (1 + \mathcal{O}(f_i)) \begin{bmatrix} 1 + \cos \theta_l & \sin \theta_l e^{i\phi_l} \\ \sin \theta_l e^{-i\phi_l} & 1 - \cos \theta_l \end{bmatrix},$$

Lepton's angular distribution remains unchanged.

E_l distribution is changed.

Energy distribution of lepton

The E_ℓ^0 distribution (in the top-rest-frame) depends only on the possible **new physics in $t \rightarrow bW$ decay**.



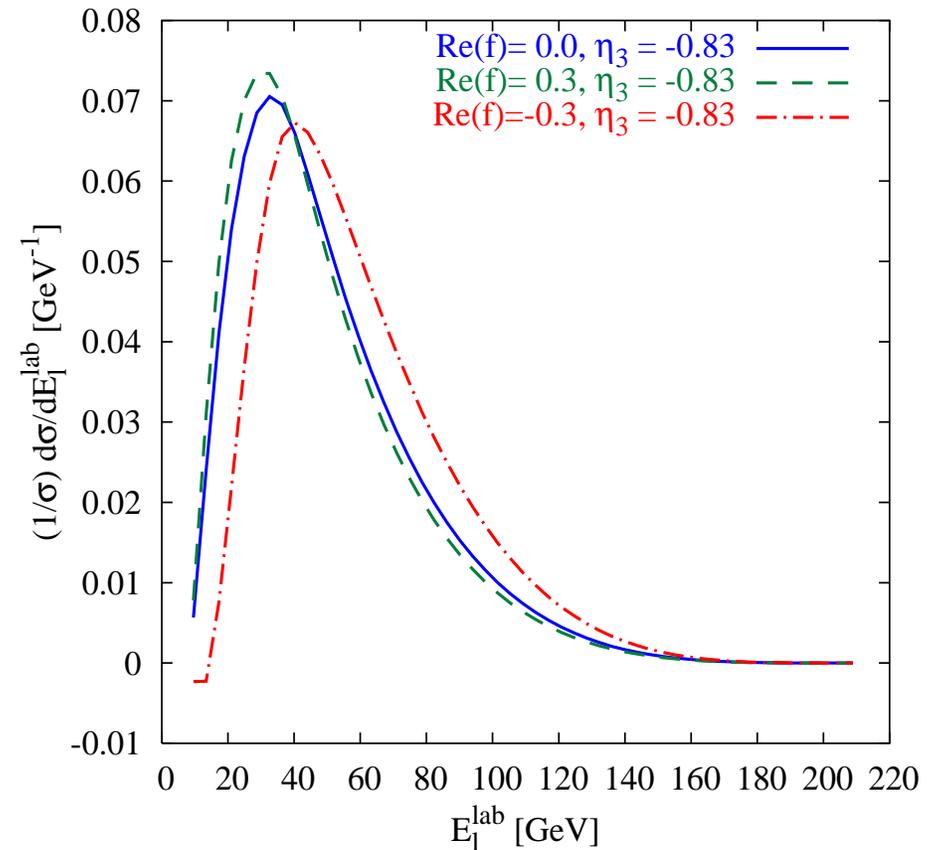
$$\frac{d\sigma}{dE_\ell^0} \propto \int E_\ell^0 F(E_\ell^0) dp_W^2$$

**Independent of
production mechanism
of t -quark !!**

Energy distribution of lepton

The E_ℓ distribution (in the lab frame) depends on

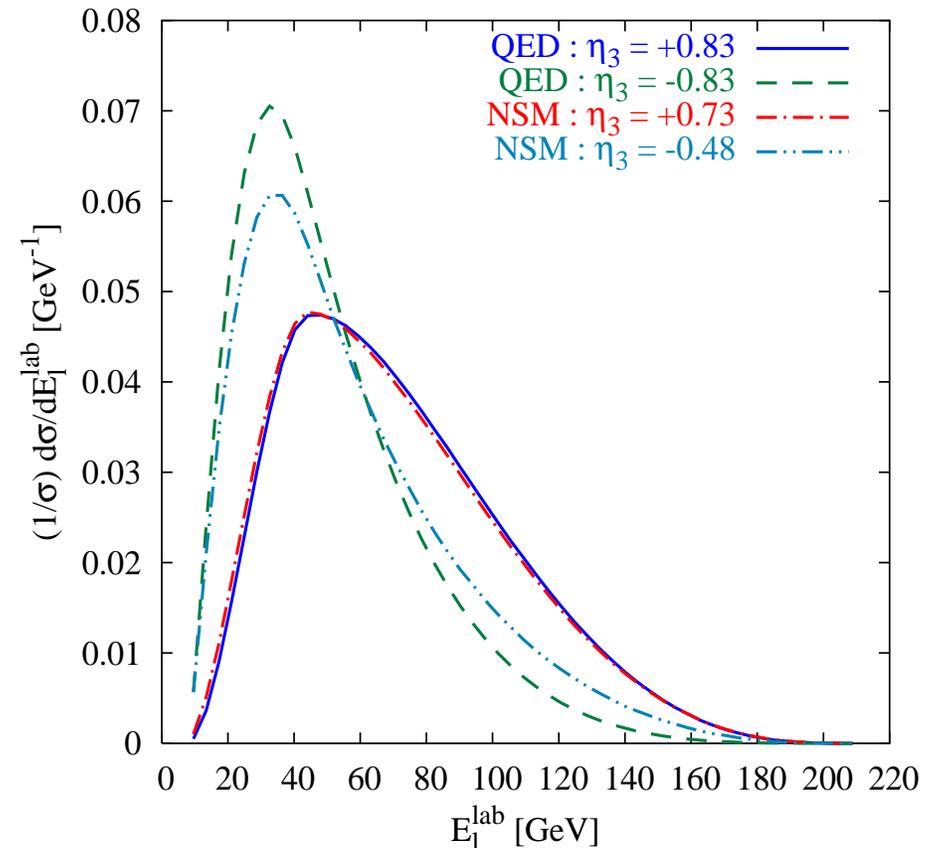
- the anomalous couplings in the tbW vertex
⇒ **new physics in t -decay**



Energy distribution of lepton

The E_ℓ distribution (in the lab frame) depends on

- the anomalous couplings in the tbW vertex
⇒ **new physics in t -decay**
- the energy distribution and the polarization of t -quark ⇒ **dynamics of t -production**



Polarization of t -quark

Polarized cross-sections :

$$\int \frac{d^3 p_t}{2E_t (2\pi)^3} \left(\prod_{i=1}^{n-1} \frac{d^3 p_i}{2E_i (2\pi)^3} \right) \frac{(2\pi)^4}{2I} \rho(\lambda, \lambda') \delta^4 \left(k_A + k_B - p_t - \left(\sum_{i=1}^{n-1} p_i \right) \right) = \sigma(\lambda, \lambda').$$

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Total cross-section :

$$\sigma_{tot} = \sigma(+, +) + \sigma(-, -)$$

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Polarization density matrix :

$$P_t = \frac{1}{2} \begin{pmatrix} 1 + \eta_3 & \eta_1 - i\eta_2 \\ \eta_1 + i\eta_2 & 1 - \eta_3 \end{pmatrix},$$

$$\eta_3 = (\sigma(+, +) - \sigma(-, -)) / \sigma_{tot}$$

$$\eta_1 = (\sigma(+, -) + \sigma(-, +)) / \sigma_{tot}$$

$$i \eta_2 = (\sigma(+, -) - \sigma(-, +)) / \sigma_{tot}$$

Polarization of t -quark

Polarization through leptonic decay of t -quark :

$$\frac{\eta_3}{2} = \frac{\sigma(p_\ell \cdot s_3 < 0) - \sigma(p_\ell \cdot s_3 > 0)}{\sigma(p_\ell \cdot s_3 < 0) + \sigma(p_\ell \cdot s_3 > 0)}$$

$$\frac{\eta_2}{2} = \frac{\sigma(p_\ell \cdot s_2 < 0) - \sigma(p_\ell \cdot s_2 > 0)}{\sigma(p_\ell \cdot s_2 < 0) + \sigma(p_\ell \cdot s_2 > 0)}$$

$$\frac{\eta_1}{2} = \frac{\sigma(p_\ell \cdot s_1 < 0) - \sigma(p_\ell \cdot s_1 > 0)}{\sigma(p_\ell \cdot s_1 < 0) + \sigma(p_\ell \cdot s_1 > 0)}$$

$$s_i \cdot s_j = -\delta_{ij} \quad p_t \cdot s_i = 0$$

For $p_t^\mu = E_t(1, \beta_t \sin \theta_t, 0, \beta_t \cos \theta_t)$, we have

$$s_1^\mu = (0, -\cos \theta_t, 0, \sin \theta_t), \quad s_2^\mu = (0, 0, 1, 0), \quad s_3^\mu = E_t(\beta_t, \sin \theta_t, 0, \cos \theta_t)/m_t.$$

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η_2 : transverse polarization normal to the production plane.

Simplest quantity to measure;

requires reconstruction of t -production plane;

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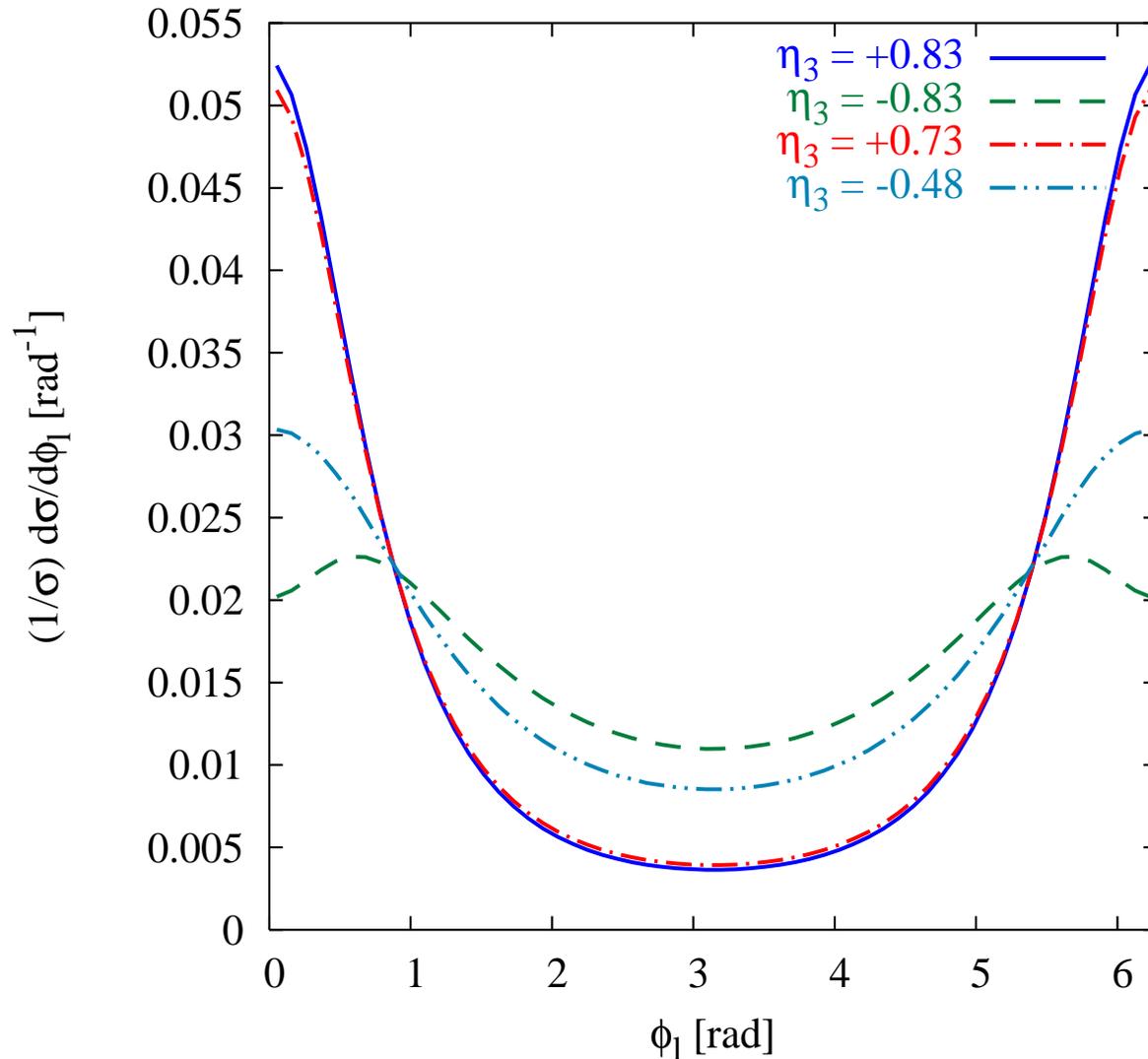
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Energy and angular distribution of leptons in lab frame can be used as a measure of the t -polarization.

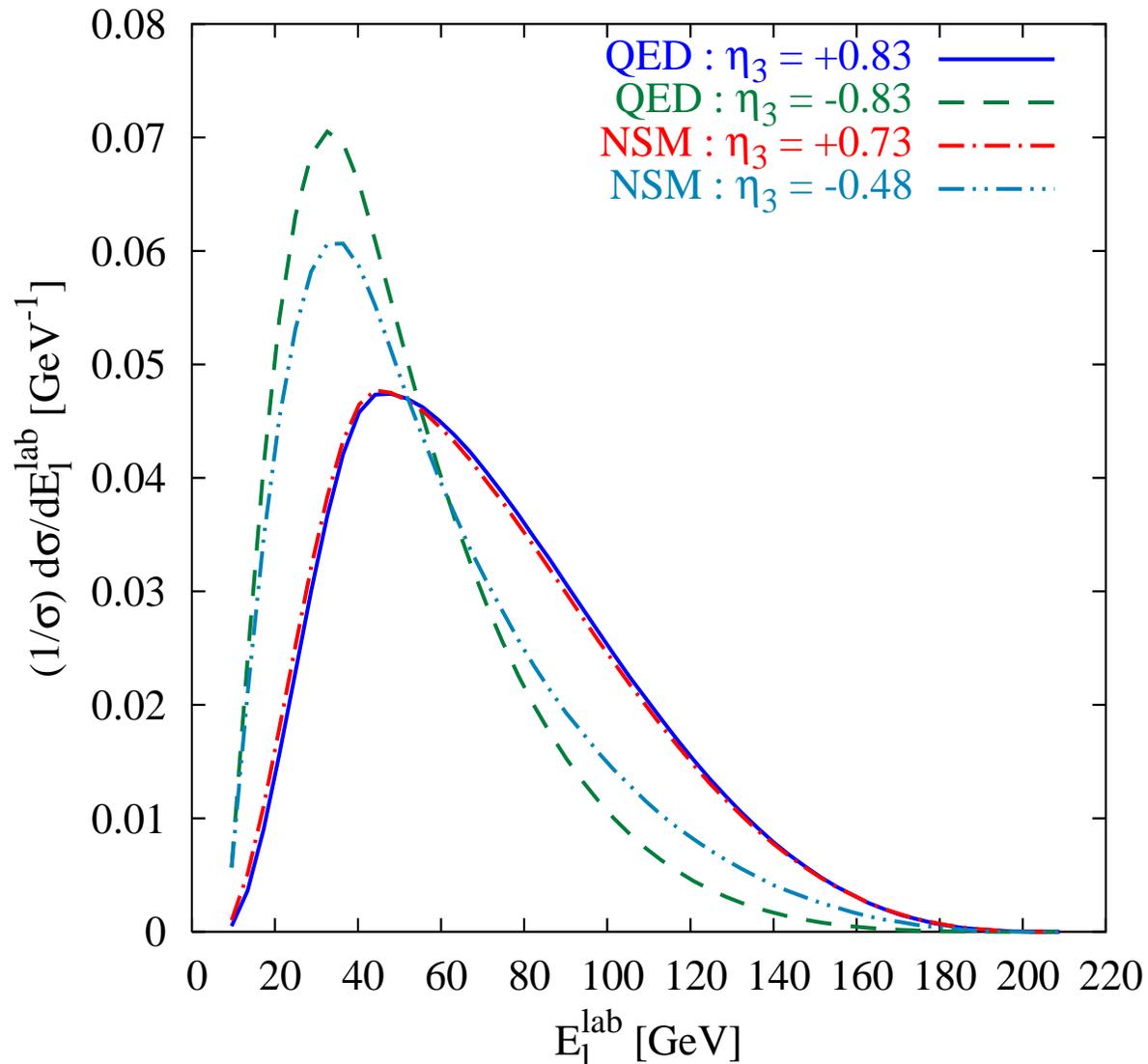
Polarization estimation

Lab frame azimuthal distribution of leptons:



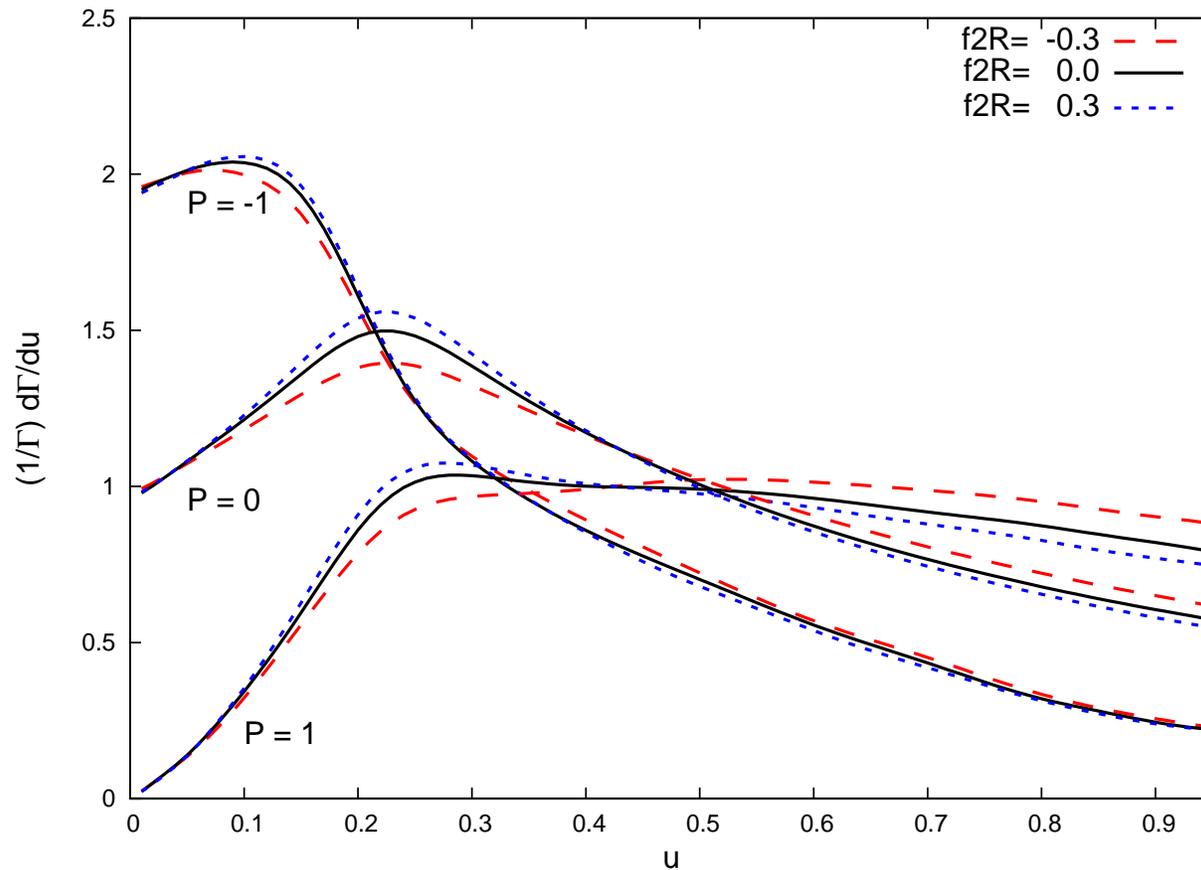
Polarization estimation

Lab frame energy distribution of leptons:



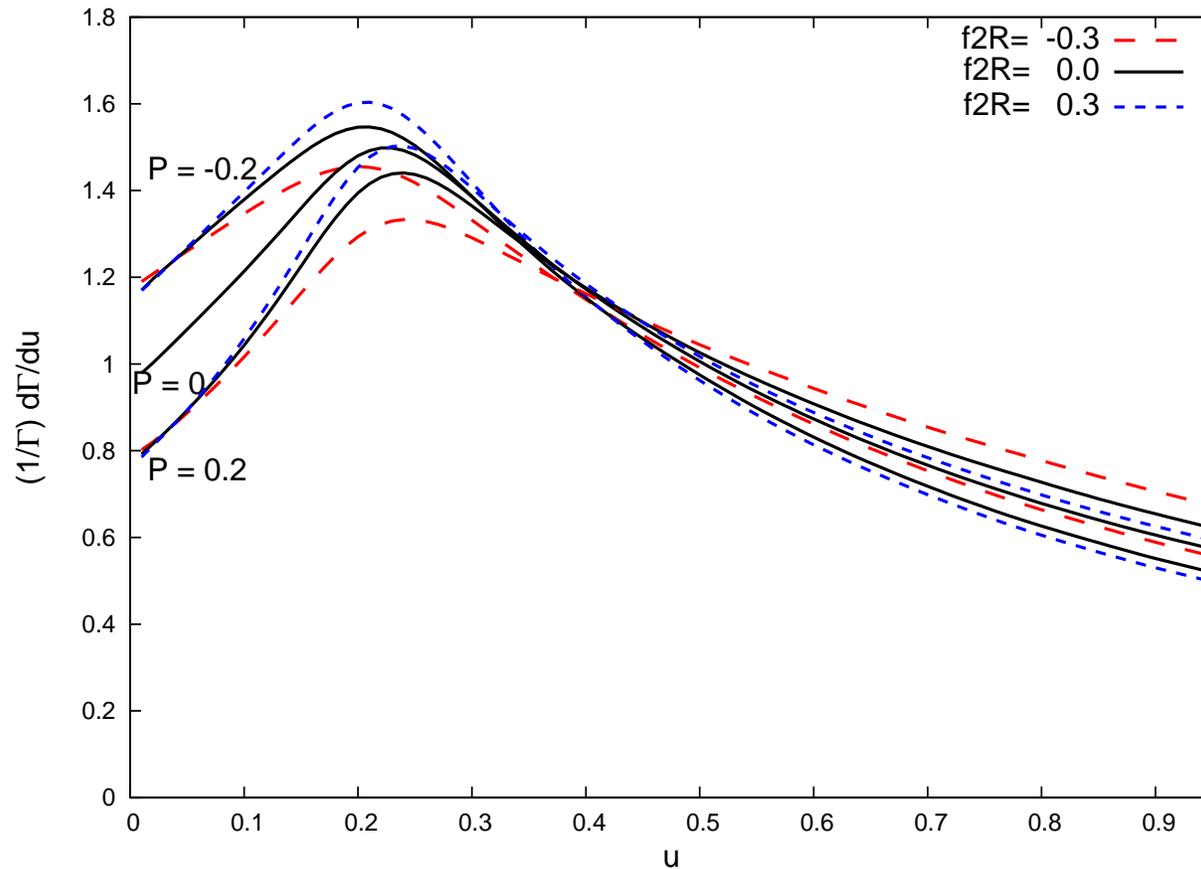
Polarization estimation

Lepton energy fraction in lab frame $u = E_l / (E_l + E_b)$ (Shelton)
(From Rohini's talk)



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- Lepton energy distribution, in the t -rest-frame, is a pure probe of possible new physics in $t \rightarrow bW$ decay independent of top production mechanism.
- Polarization of t -quark can be measured (quantitatively) through angular asymmetries (w.r.t. $p_l \cdot s_i$) of decay leptons.

Conclusions

- Lepton angular distribution is a **pure** probe of possible new physics in **any** process of t -quark production, independent of possible new physics in $t \rightarrow bW$ decay.
- Lepton energy distribution, in the t -rest-frame, is a pure probe of possible new physics in $t \rightarrow bW$ decay independent of top production mechanism.
- Polarization of t -quark can be measured (quantitatively) through angular asymmetries (w.r.t. $p_l \cdot s_i$) of decay leptons.
- Azimuthal and energy distribution of decay lepton in the lab-frame is a good probe of t -polarization.