

Images and spectra of time-dependent two-component advective flow in presence of outflows

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ABSTRACT

Two-component advective flow (TCAF) successfully explains the spectral and temporal properties of outbursting or persistent sources. Images of static TCAF with Compton cloud or centrifugal pressure supported boundary layer due to gravitational bending of photons have been studied before. In this paper, we study time-dependent images of advective flows around a Schwarzschild black hole, which include cooling effects due to Comptonization of soft photons from Keplerian discs as well as the self-consistently produced jets and outflows. We show the overall image of the disc-jet system after convolving with a typical beamwidth. A long exposure image with time-dependent system need not to show the black hole horizon conspicuously, unless one is looking at a soft state with no jet or the system along the jet axis. Assuming these disc-jet configurations are relevant to radio emitting systems also, our results would be useful to look for event horizons in high accretion rate supermassive black holes in Seyfert galaxies, RL Quasars.

Key words: accretion, accretion discs – black hole physics – hydrodynamics – radiative transfer – relativistic processes.

1 INTRODUCTION

Spectral and temporal properties of galactic and extragalactic black holes suggest time dependence of various degrees. Imaging a black hole accretion disc would therefore not be accurate without inclusion of the time dependence of various disc components. There is a class of stellar mass black hole candidates, known as the outbursting sources, which totally changes its disc configurations in a matter of few days, when it goes through transitions of its spectral states from hard to hard intermediate to soft intermediate and then soft states during their rising phase. Recently, through fits of data with two-component advective flow (TCAF) solution as prescribed by Chakrabarti & Titarchuk (1995, hereafter CT95), the accretion rates of the halo and disc components, the sizes of the hot electron cloud (‘Compton cloud’) etc. were all shown to be highly variable. The Compton cloud in this solution is the inner part of the advective halo, puffed up due to slowing down by the centrifugal barrier, which not only inverse Comptonizes the soft photons to produce high-energy X-rays, but also produces outflowing matter just as a normal boundary layer. The disc component on the equatorial plane supplies soft photons. In the absence of the centrifugal pressure supported boundary layer (CENBOL) as in the soft state, the flow behaves as a standard thin disc with the inner edge close to the inner

stable circular orbit or ISCO (Shakura & Sunyaev 1973, hereafter SS73) and no jets or outflows are seen. In class-variable sources, such as GRS 1915+105, temporal variation in significant count rates, and state transitions such as (burst-on and burst-off states) are seen in a matter of few seconds (Muno, Morgan & Remillard 1999). This source also exhibits strong relativistic radio jets (Fender et al. 1999). Along with the monotonic changes in the CENBOL sizes, the frequency of the quasi-periodic oscillations (QPOs) is also seen to change monotonically (Chakrabarti et al. 2005). Theoretical work which forms the basis of TCAF is in Chakrabarti (1996) and references therein.

The time-dependent simulations (Giri, Garain & Chakrabarti 2013, and references therein) show the formation of two components in the flow with the Keplerian disc on the equatorial plane and advective halo surrounding it formed the shock and outflows exactly as envisaged in CT95. Outflows are intrinsically associated with accretion mechanism (see Chakrabarti 1990b, 1996; for further details) in this solution. The explicit ratio of mass-outflow rate to the inflow rate was first computed in Chakrabarti (1999) and was found to strongly depend on the compression ratio at the shock. From fig. 4 of Giri & Chakrabarti (2012), it is clear that the ratio of the outflow to the inflow rate increases with decreasing viscosity parameter and the inviscid flow produces maximum outflows. In outbursts or class variable sources, outflows are found to be sporadic and highly variable. Thus, it is very important to have the time-dependent spectra and images of TCAF where outflows are self-consistently

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generated from the CENBOL itself. Cooling via Comptonization is an essential mechanism by which the CENBOL reduces its size making the spectrum softer. While the CENBOL lasts, rough agreement between the cooling and compressional heating time-scales causes it to oscillate and produce the so-called QPOs (Chakrabarti, Mondal & Debnath 2015, and references therein). Detailed hydrodynamic simulations in presence of Compton cooling are in Ghosh, Garain & Chakrabarti (2011) and Garain, Ghosh & Chakrabarti (2014), where effects of cooling on the spectra, shock location, and the formation of QPOs are described.

Theoretical efforts for imaging of accretion discs started with the pioneering work by Luminet (1979), who drew the image of a standard Keplerian disc around a Schwarzschild black hole (SS73). The method of image construction was done from the observers' end. This technique is very useful as photons were not lost while tracking them from the observer. Following this, Fukue & Yokohama (1988) published the coloured version of the image. With that, they also studied the occultation effect on accretion disc light curve. A few years later, Viergutz (1992) generalized the transfer functions for Kerr geometry. Marck (1995), for the first time produced the Keplerian disc image using the actual Ray-Tracing mechanism. Bromley, Melia & Liu (2001) added polarization determination. Images of relativistic thick discs around black holes are presented by Fuerst & Wu (2004). Dexter & Agol (2009) first published 'geokerr', a public code to compute images in Kerr geometry. 'GYOTO' (Vincent et al. 2011), 'YNOGK' (Yang & Wang 2013), and 'GRTRANS' (Dexter 2016) came into public domains. The focus of these works was to prepare one to interpret results from the Event Horizon Telescope (EHT) whose objective is to identify the horizon of the supermassive black hole at the centre of our Galaxy.

Armitage & Reynolds (2003) first performed MHD simulation of Keplerian disc and produced the images where the disc structure remained the same all the time. More recently, Broderick & Loeb (2009) presented results where simulated images of force free jets (Tchekhovskoy, McKinney & Narayan 2008) launched from M87 are shown for various model parameters and polarization maps of the magnetically driven jets are obtained. However, under TCAF paradigm, one does not have to introduce magnetic field to generate outflows. The hydrodynamic inflow produces time-varying outflows self-consistently. Magnetic field may be useful for collimation and acceleration purposed further out.

GRMHD simulations in the context of our Galactic Centre were mostly performed (Ohsuga, Kato & Mineshige 2005; Noble et al. 2007; Mścibrodzka et al. 2009, 2011; Mścibrodzka, Falcke & Shiokawa 2016; Dexter et al. 2010; Hilburn et al. 2010; Dexter & Fragile 2013; Dolence et al. 2012; Shcherbakov, Penna & McKinney 2012; Sądowski et al. 2013; Roelofs et al. 2017) considering a very low accretion rate as applicable. Cooling term was neglected as appropriate for Sgr A*. In fact, Dibi et al. (2012) added the cooling term and found the effect to be negligible in case of Sgr A*. Drappeau et al. (2013) constrained the mass accretion rate to be $\sim 10^{-9} M_{\odot} \text{yr}^{-1}$ with a highly spinning central black hole. This accretion rate is much lower than that of the earlier result ($\sim 10^{-5} M_{\odot} \text{yr}^{-1}$) predicted by Coker & Melia (1997). A detailed review on the simulations and observational possibilities of the black hole shadow of Sgr A* was presented by Falcke & Markoff (2013). But, most of the earlier studies were based on the RIAF model to simulate spectra and images of Sgr A* only. However, there exist a large number of SMBHs in Radio Loud Quasars, Seyfert galaxies, where the accretion rate could be substantially high (Bian & Zhao 2003) and thus formation of disc and cooling becomes important.

Another situation which might induce a sudden high-mass inflow is from tidal disruption of neighbouring object (Gillesen et al. 2012).

Chatterjee, Chakrabarti & Ghosh (2017a; hereafter CCG17a) showed the static images of TCAF with general relativistic thick discs acting as the Compton cloud and relativistic Keplerian disc (Page & Thorne 1974) acting as the source of seed photons. Images for various disc rates and inclination angle are shown with their observable spectral counterparts. The variation of optical depth of the CENBOL medium is achieved by changing the halo rate (\dot{m}_h). They have shown that by increasing the optical depth, the Keplerian disc from the other side is completely blocked by the CENBOL medium. Also, with increasing inclination angle, the spectral hardening can be seen in their work. Following the same geometry, Chatterjee, Chakrabarti & Ghosh (2017b; hereafter CCG17b) simulated the time lags of various energy bins and compare the simulated results with the observed counterparts.

In this paper, for the first time, we present time-dependent images of TCAF with Comptonization and outflows. Basic simulation process by hydrodynamic tvd code is explained in Section 2. Thermodynamic properties of CENBOL and Keplerian disc are presented in Section 3. Section 4 deals with Monte Carlo simulation of Comptonization and cooling mechanism. The spectra, time-dependent images with and without cooling effects are presented in the Results section. Convolved images of accretion disc at different spectral states are presented. We discuss our results and conclude this paper in the final section.

2 TIME-DEPENDENT SIMULATIONS OF TCAF

Hydrodynamic simulation procedure is similar to what has been earlier reported in Ryu, Chakrabarti & Molteni (1997), Giri et al. (2010), Giri & Chakrabarti (2012, 2013), and Giri, Garain & Chakrabarti (2015). Governing equations of tvd are explicitly presented by Molteni, Sponholz & Chakrabarti (1996), Ryu et al. (1997), and Giri & Chakrabarti (2012). The conservation equation of mass, momentum, and energy for an axisymmetric, inviscid, and non-magnetic flow takes the following form

$$\frac{\partial \mathbf{q}}{\partial t} + \frac{1}{r} \frac{\partial(r\mathbf{F}_1)}{\partial r} + \frac{\partial \mathbf{F}_2}{\partial r} + \frac{\partial \mathbf{G}}{\partial z} = \mathbf{S}, \quad (1)$$

where $\mathbf{q} = \begin{pmatrix} \rho \\ \rho v_r \\ \rho v_\theta \\ \rho v_z \\ E \end{pmatrix}$ is the state vector,

$$\mathbf{F}_1 = \begin{pmatrix} \rho v_r \\ \rho v_r^2 \\ \rho v_r v_\theta \\ \rho v_r v_z \\ (E + \rho)v_r \end{pmatrix}, \quad \mathbf{F}_2 = \begin{pmatrix} 0 \\ p \\ 0 \\ 0 \\ 0 \end{pmatrix},$$

$$\mathbf{G} = \begin{pmatrix} \rho v_z \\ \rho v_r v_z \\ \rho v_\theta v_z \\ \rho v_z^2 + p \\ (E + \rho)v_z \end{pmatrix} \text{ are the flux functions.}$$

The source function is defined as

$$S = \begin{pmatrix} 0 \\ \frac{\rho v_\theta^2}{r} - \frac{\rho r}{2(\sqrt{r^2+z^2}-1)^2\sqrt{r^2+z^2}} \\ -\frac{\rho v_r v_\theta}{r} \\ -\frac{\rho z}{2(\sqrt{r^2+z^2}-1)^2\sqrt{r^2+z^2}} \\ -\frac{\rho(rv_r+zv_z)}{2(\sqrt{r^2+z^2}-1)^2\sqrt{r^2+z^2}} \end{pmatrix}.$$

The energy density $E = p/(\gamma - 1) + \rho(v_r^2 + v_\theta^2 + v_z^2)/2$ is the sum of thermal and kinetic energy of the infalling matter. The equation corresponding to energy density can be extracted as

$$\begin{aligned} \frac{\partial E}{\partial t} + \frac{1}{r} \frac{\partial[r(E + \rho)v_r]}{\partial r} + \frac{\partial(E + \rho)v_z}{\partial z} \\ = -\frac{\rho(rv_r + zv_z)}{2(\sqrt{r^2+z^2}-1)^2\sqrt{r^2+z^2}}. \end{aligned}$$

During cooling, change in E directly modifies the flow configurations following equation (1).

The gravitational field, as far as the hydrodynamics is concerned, is assumed to follow the Paczyński & Wiita (1980) pseudo-Newtonian potential which is given by

$$\phi(r, Z) = -\frac{GM_{bh}}{r - r_g}, \quad (2)$$

where G is the universal gravitational constant, M_{bh} is the mass of the black hole kept at $10 M_\odot$ throughout the simulation, $r_g = \frac{2GM_{bh}}{c^2} = 1$ is the Schwarzschild radius, and $r = \sqrt{r^2 + Z^2}$. Velocity of light is denoted by c . The flow is considered to be adiabatic and axisymmetric with the black hole sitting at centre of the cylindrical coordinate system defined by (r, θ, Z) . In our current simulation, we have ignored viscosity to have the strong and hot shock and consequently highest outflow rate. Thus, the specific angular momentum λ is constant everywhere. Also, from Chakrabarti 1989 and Chakrabarti 1990b, it is known that for a steady state situation the specific energy ϵ is also constant. Matter is injected at the outer boundary with specific energy $\epsilon = 0.003$ (in units of c^2 and specific angular momentum $\lambda = 1.8$) in units of cr_g .

TVD scheme was originally developed by Harten (1983) to solve the hyperbolic equations present in the hydrodynamic conservation equations. This method works on the second-order accuracy by modifying first-order flux function to use the non-oscillatory results in the second-order solutions. The details of the code were presented in Ryu et al. (1993, 1997) where the code is modified to suit black hole accretion.

Our computational domain is defined as follows: $0 < r < 100$ and $0 < Z < 100$ in r - Z plane with a grid resolution of 512×512 . So, each grid size is $\delta r = 0.195r_g$ and $\delta Z = 0.195r_g$. The black hole absorption boundary is kept at $1.5r_g$ after which all peices of information about the matter as well as photons are lost. Before starting the simulation, entire grid is filled with a low-floor density (10^{-8} where the injected density is 1 in code unit) to avoid any numerical singularities. At the outer boundary sub-Keplerian matter is allowed to flow through. During the accretion, the matter follows the inviscid hybrid accretion mechanism (Chakrabarti 1990a) and forms a shock at a specified position. We assume axisymmetry and perform our hydro-simulation on the first quadrant of the r - Z plane. The black hole is located at the origin of the r - Z coordinate system. The initial pressure is chosen such that the sound speed as of the interior material is same as the incoming matter. Also, inside the computational domain, all the velocity components are

set to zero initially. The simulation result does not depend on the chosen initial condition as the initial matter is washed away by the incoming matter in a few dynamical time. On the right radial boundary, we use an inflow boundary condition. Incoming matter enters the computational domain through this boundary. Since the density of the incoming matter is chosen to be unity, it has 10^8 times higher pressure and it rushes towards the central black hole practically through vacuum until it hits the centrifugal barrier.

Total dynamical time-scale for this simulation is 4000, which in physical units ($\frac{2GM_{BH}}{c^3} \times f_i$, where f_i contains informations of each time) is about 40 s. In the present context, the hydrodynamical simulation has been performed for Galactic black hole candidates. However, these results will also hold for supermassive or intermediate mass black holes.

3 DENSITY AND TEMPERATURE PROFILE OF THE ELECTRON CLOUD

CENBOL is a region of hot electron cloud where inverse Comptonization of soft photons originated from the Keplerian disc occurs. The electrons in this region are ultra-relativistic. So, $\gamma = 4/3$ is a good assumption for this region. Number density (per cm^3) and temperature (in the kilo electron volt) of the electrons are supplied for each grid. The pressure and density are related via polytropic equation of state $p = K\rho^\gamma$, where p is the pressure, K the entropy constant, and ρ is the density of the flow. From this, we can write $T \propto \rho^{1/3}$, where T is the temperature of the electron cloud. In our studies, the maximum number density of electron is $\sim 10^{19} \text{ cm}^{-3}$ and temperature is $\sim 250 \text{ keV}$. Hydrodynamical simulation changes the configuration of CENBOL region for each time stamp. The variation of electron cloud geometry with number density and temperature as given in the colour bar is presented in Fig. 1.

From Fig. 1, the region of outflows can be distinctly seen as a region where temperature and electron number density drop rapidly. Over the time, the geometry of the CENBOL region changes as hydrodynamical simulation progresses. One can easily differentiate between Figs 1a and f as the amount of outflow increased substantially in the time passed. The inner isobaric contours of the CENBOL region (Figs 1a–f) behave similar to thick discs (Abramowicz, Jaroszynski & Sikora 1978; Kozłowski et al. 1978; Paczyński & Wiita 1980; Rees et al. 1982; Chakrabarti 1985) as pointed out by Molteni, Lanzafame & Chakrabarti (1994; hereafter MLC94).

The TVD method simulates a true accretion mechanism. The injected matter washes away the initial low-floor matter and attains a quasi-steady state. Inflowing matter undergoes a shock due to the centrifugal barrier. After the shock, matter starts to flow towards black hole. The mass-absorption rate through the black hole boundary ($1.5r_g$) is represented in the Fig. 2. It can be seen that the mass-absorption rate after the transient phase saturates at ~ 75 per cent of the injected mass.

First simulation of this kind was performed by MLC94 followed by Molteni, Ryu & Chakrabarti (1996) and Giri & Chakrabarti (2012, 2013). The outflows are generated out of inflowing sub-Keplerian matter forming a shock due to centrifugal barrier. The oscillation of the shock front after attaining a steady state and the consequence of that on light curves are well studied in MLC94, Giri et al. (2010), and Garain, Ghosh & Chakrabarti (2012b). This formation of shock induces a temperature rise in the post-shock region, which acts as the base of the outflows or Jets. Due to the increase of temperature in the post-shock region force due to gradient of pressure pushes matter outward along the vertical direction. This creates the outflows. In HD simulations, this is not

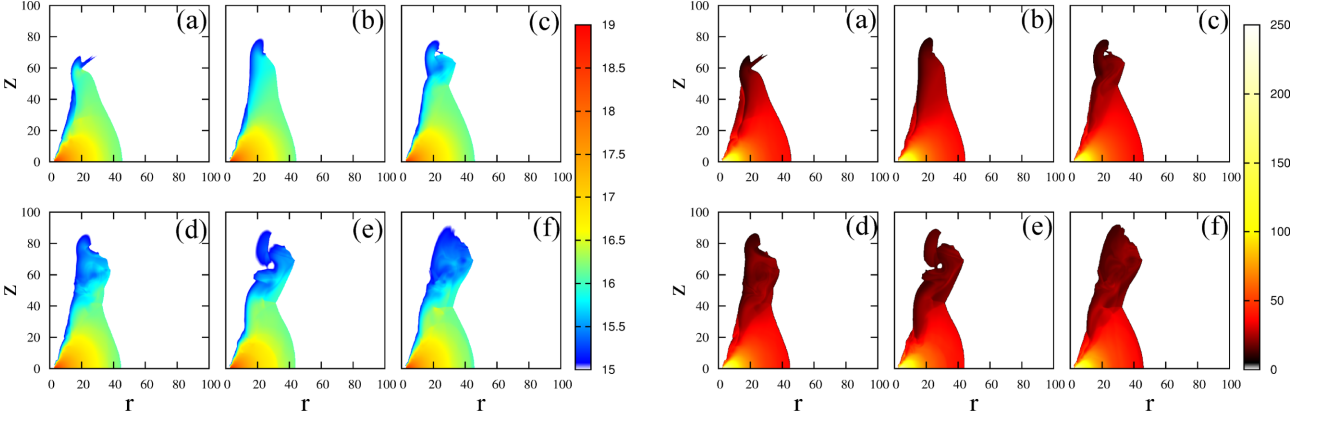


Figure 1. Electron density (in the units of number per cm^3 shown in log scale, left-hand panel) and temperature (in the units of keV, right-hand panel) distribution in the post-shock region at (a) $t = 14.25$ s, (b) $t = 14.35$ s, (c) $t = 14.45$ s, (d) $t = 14.55$ s, (e) $t = 14.65$ s, and (f) $t = 14.75$ s.

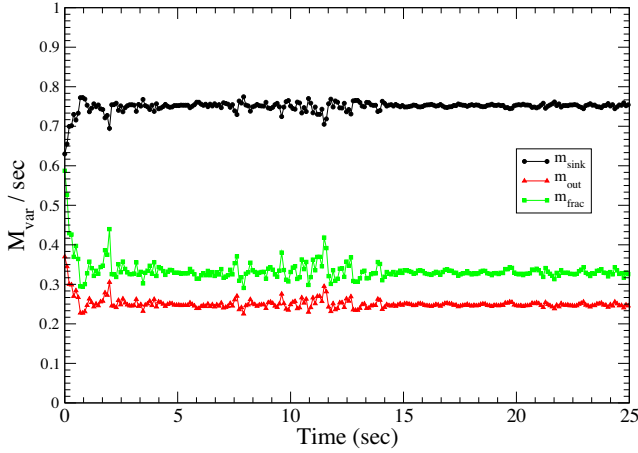


Figure 2. Fraction of mass absorption (circle-black), outflows (triangle-red), and the ratio of outflowing matter to that of absorption (square-green) through the inner boundary with time are shown. Injected halo rate (\dot{m}_h) is kept fixed at 0.1.

well collimated. One may require magnetic effects for collimation (Chakrabarti 2013). To show details, we have shown only the post-shock region. The sub-Keplerian flow of the pre-shock region has much lower density and optical depth as compared to the post-shock region (GC13). Maximum number of scatterings occurs in the post-shock region and for an observer trying to capture the image of an accretion disc in presence of the outflows, the last scattering surface of the CENBOL will appear as the shape of the Compton cloud.

3.1 Keplerian disc acting as the soft photon source

In order not to include a time dependence in the Keplerian disc, we put it on the equatorial plane, whose purpose is to supply soft photons as per Page & Thorne (1974) prescription. We employ the Monte Carlo simulations Pozdnyakov, Sobol & Sunyaev (1983) to compute the resulting spectrum which includes the original source photons from this Keplerian disc as well as those scattered from the CENBOL and the outflows. The Keplerian disc is truncated at the inner edge at the CENBOL surface and the outer edge is extended up to $100r_g$ for simulation

purpose.

$$F(r) = \frac{F_c(\dot{m}_d)}{(r - 3/2)r^{5/2}} \times \left[\sqrt{r} - \sqrt{3/2} + \frac{\sqrt{3/2}}{2} \log \left(\frac{(\sqrt{r} + \sqrt{3/2})(\sqrt{3} - \sqrt{3/2})}{(\sqrt{r} - \sqrt{3/2})(\sqrt{3} + \sqrt{3/2})} \right) \right]$$

and

$$T(r) = \left(\frac{F(r)}{\sigma} \right)^{1/4} \quad (3)$$

where, $F_c(\dot{m}_d) = \frac{3m\dot{m}_d}{8\pi r_g^3}$, \dot{m}_d is the Keplerian disc accretion rate in Eddington unit, $\sigma = \frac{2\pi^5 k_B^4}{15h^3 c^3}$ is the Stefan–Boltzmann constant.

Photon flux emitted from the Keplerian disc surface of radius r to $r + \delta r$ is written as

$$n_\gamma(r) = \left[\frac{4\pi}{c^2} \left(\frac{k_B T(r)}{h} \right)^3 \times \zeta(3) \right] \text{cm}^{-2} \text{s}^{-1}, \quad (4)$$

where, $\zeta(3) = \sum_{l=1}^{\infty} l^{-3} = 1.202$ is the Riemann zeta function. So, the rate of photons emitted from the radius r to $r + \delta r$ is given by

$$dN(r) = 4\pi r \delta r n_\gamma(r), \quad (5)$$

where the Keplerian disc height $H(r)$ is considered as 0.1 (since $\frac{H(r)}{r} \ll 1$ for Keplerian disc region).

The disc is divided into several annuli of radial width $D(r) = 0.1$ and of mean temperature $T(r)$. The exact number of photons comes to be around $\sim 10^{39} - 10^{40} \text{ s}^{-1}$ for a disc rate of $\dot{m}_d = 0.1$. With increasing disc rate this number increases rapidly. Using a weightage factor $f_W = \frac{dN(r)}{N_{\text{comp}}(r)}$, where $N_{\text{comp}}(r) = 10^9$, we bundle these photons to save computational time.

Soft photon energy is calculated using Planck distribution law for $T(r)$. Photon number density ($n_\gamma(E)$) which corresponds to an energy E is given by

$$n_\gamma(E) = \frac{1}{2\zeta(3)} b^3 E^2 (e^{bE} - 1)^{-1}. \quad (6)$$

The process is similar to the earlier works presented by Ghosh, Chakrabarti & Laurent (2009) and Chatterjee, Chakrabarti & Ghosh (2017a,b).

4 MONTE CARLO SIMULATION OF COMPTONIZATION

In the Monte Carlo process, the photons inside the CENBOL region are assumed to follow straight line trajectories in between two scatterings. This enhances the Computational efficiency without sacrificing science. This has been pointed out by Laurent & Titarchuk (1999). The Keplerian disc emits seed photons with six random numbers associated with position and velocity. In reality, the Keplerian disc should have the maximum flux along Z-axis and minimum along the equatorial plane. This has been implemented by assuming $F_v = \int I_v \cos\theta d\Omega$. Inside the CENBOL region, a critical optical depth (τ_c) is set up using random number corresponding to each scattering. The next scattering would occur if the optical depth of a particular photon crosses τ_c .

We choose Klein–Nishina scattering cross-section σ which is given by

$$\sigma = \frac{2\pi r_e^2}{x} \left[\left(1 - \frac{4}{x} - \frac{8}{x^2} \right) \ln(1+x) + \frac{1}{2} + \frac{8}{x} - \frac{1}{2(1+x)^2} \right], \quad (7)$$

where x is given by

$$x = \frac{2E}{mc^2} \gamma \left(1 - \mu \frac{v}{c} \right), \quad (8)$$

$r_e = e^2/mc^2$ is the classical electron radius and m is the mass of the electron. This yields Thomson scattering cross-section (σ_T) in low frequencies and Compton scattering cross-section (σ_C) in higher frequencies like X-rays and γ -rays. Between each pair of scatterings, the gravitational red-shift modifies the energy of the photon. This process continues until it leaves the CENBOL region or gets sucked by the black hole. The process is similar to that presented in GCL09, Ghosh et al. (2010), Ghosh et al. (2011), and CCG17a, CCG17b.

After Comptonization, the spectra at various times are presented in Fig. 3. Fig. 3a shows the Comptonized spectra of TCAF at $t = 14.25$ s, $t = 14.45$ s, and $t = 14.75$ s. The temporal variation in the thermodynamic variables of TCAF has been shown in Fig. 1. In the absence of cooling, the Compton cloud remains hot. Thus, the variations in spectra are very small. The blue-dot-dot-dashed curve in Fig. 3a represents the injected spectra. In (b), various components such as the net spectrum (red-dashed), injected spectrum (solid-black), reflected spectrum caused by rescattering of Comptonized radiation by the Keplerian disc before it reaches the observer (blue-dot-dot-dashed), and outflow spectrum (green-dot-dashed) are shown at $t = 14.45$ s. It can be clearly seen from Fig. 3b that the reflected spectrum contributes more to the net spectrum than the outflows spectrum. Fig. 3c shows the reflected spectrum at three inclinations ($0-22.5^\circ$ (black-solid), $45-67.5^\circ$ (red-dashed), and $67.5-90^\circ$ (green-dot-dashed) at time $t = 14.45$ s. The hard energy contribution in reflected spectra increases with inclination angles while the soft energy contribution decreases. This is an expected result for the reflected component as the reflection dominates at high inclinations. With the increase of the outflow rate, the spectral contribution of the outflow also increases. In Fig. 3d, the outflow spectra have been plotted for $t = 14.25$ s (black-solid), $t = 14.45$ s (red-dotted), and $t = 14.75$ s (green-dot-dashed).

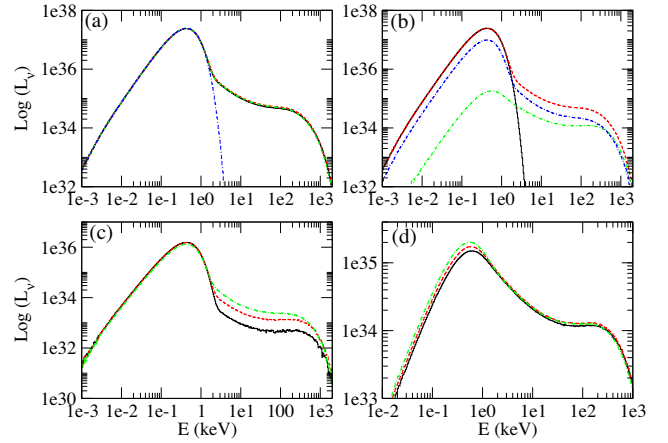


Figure 3. Spectra obtained after Comptonization. (a) The spectra at $t = 14.25$ s (black-solid), $t = 14.45$ s (red-dotted), and $t = 14.75$ s (green-dot-dashed) and injected (Blue-dot-dot-dashed) spectrum. (b) Injected spectrum (black-solid), composite spectrum (red-dashed), Reflected Spectrum (blue-dot-dot-dashed), and Outflow spectrum (green-dot-dashed). (c) Inclination dependence of reflected spectra. The inclination bins are $0-20^\circ$ (black-solid), $40-60^\circ$ (red-dotted), $70-90^\circ$ (green-dot-dashed). (d) Outflow spectra at $t = 14.25$ s (black-solid), $t = 14.45$ s (red-dotted), and $t = 14.75$ s (green-dot-dashed). Note that the outflow spectra changed significantly due to the increase in the outflow rate. Disc accretion rate $\dot{m}_d = 0.1$ is kept fixed throughout the simulations.

4.1 Cooling process

During Comptonization, the photons either gain or lose energy. In the context of accretion physics, the electrons in the CENBOL region are at much higher temperature than that of the photons coming from a Keplerian disc. So, mostly inverse Compton scattering dominates and the signature of that is prominent in the emergent spectra. We supply a steady state hydrodynamical configuration obtained from TVD code, where the density and temperature of the cloud are defined in each grid. After the scattering, the electron from a particular grid may lose or gain ΔE amount of energy which is transferred to the photon via Compton mechanism. In a particular state, after all the scatterings by the injected photons, the energy of the entire grid is modified to generate a new hydrodynamical state where the loss or gain of energy from each grid has been accounted. By this process the final temperature of the cloud steadily decreases. The final temperature of the electron cloud in a particular grid is expressed as

$$k_B T_{\text{new}}(ir, iz) = k_B T_{\text{old}}(ir, iz) - \frac{\Delta E}{3dN_e(ir, iz)}, \quad (9)$$

where T_{new} and T_{old} are updated and old temperature of electron cloud, respectively. $dN_e(ir, iz)$ is the number density of the electron in a particular grid denoted by ir and iz and k_B is the Boltzmann constant. Details of this process of cooling are earlier presented in Ghosh et al. (2011), Garain, Ghosh & Chakrabarti (2012a), and Garain et al. (2014).

In Fig. 4, we see that the shock location moves inward as the Compton cooling starts to affect the post-shock region. Panels (a) and (d) show the CENBOL region which has almost the same electron density and temperature as Fig. 1(a). We have considered this particular data file to see the effect of Compton cooling. Fig. 4(a) is presented when cooling just started to work. Then, after a few iterations cooling starts to reduce the CENBOL size. Spectra

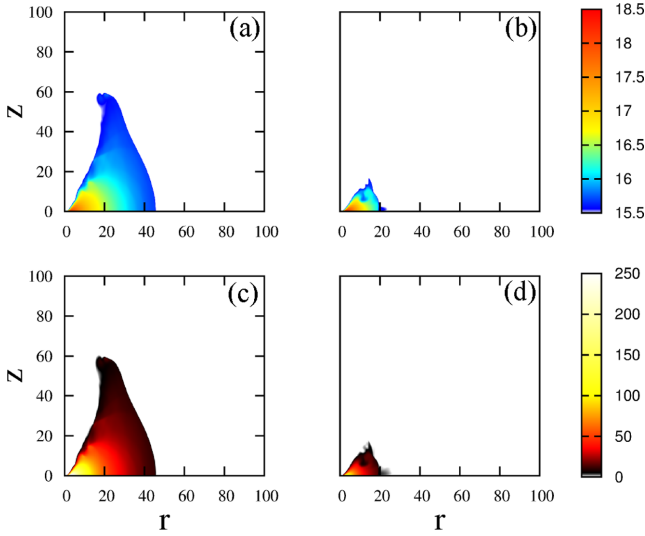


Figure 4. (a–c) Electron number density (in per cm^3 , upper panel), and (b–d) temperature (in the units of keV, lower panel) distribution in the post-shock region at $t = 14.25$ s. Compton cooling collapses the post-shock region (a, c): initial stage of cooling, (b, d): after the cooling process is completed. The shock moves inward due to loss of post-shock pressure. Disc rates $\dot{m}_d = 0.1$ and $\dot{m}_h = 0.1$ are kept constant throughout.

softens as the size of hotter Compton cloud becomes smaller. Finally, the Compton cloud reduces more which makes the accretion disc spectra to be in soft state. The corresponding spectra of Panels (b) and (d) suggest the CENBOL region is so small that a very few numbers of soft photons are intercepted by the cloud. Thus, the spectrum softens and spectral slope increases. The effect of cooling on the outflows is also visible from Fig. 4. With the inclusion of cooling, the amount of matter outflow rate (R_m) reduces drastically.

Fig. 6 represents the outflows spectra for two different CENBOLs. Black solid one is same as the spectra presented for $t = 14.25$ s in Fig. 3d. This is the input `TVB` file on which Compton cooling started to act. The CENBOL reduces its size due to cooling and outflows component vanishes with cooling. The outflow spectra shrink more and more and then it softens. Basically, all photons in the outflows spectra for the particular case come from the base of the jet, which is CENBOL itself. In this paper, our goal is to show the variation of images and spectra, when cooling is included. A very detailed study of the effect of cooling on outflows has already been reported in Garain et al. 2012.

5 RAY-TRACING PROCESS

In Schwarzschild geometry, the non-vanishing components Christoffel symbol yield geodesic equations from the field equation

$$\frac{d^2 x^\mu}{dp^2} + \Gamma_{\nu\lambda}^\mu \frac{dx^\nu}{dp} \frac{dx^\lambda}{dp} = 0, \quad (10)$$

where $\mu = [0, 1, 2, 3]$; $x^0 = t$, $x^1 = r$, $x^2 = \theta$, and $x^3 = \varphi$, p is the Affine parameter. The four coupled, second-order differential equations for photons can be reduced to three using energy $P_t = E = (1 - \frac{1}{r}) \frac{dt}{dp}$ and angular momentum $P_\phi = L = r^2 \sin^2 \theta \frac{d\phi}{dp}$ definitions (Chandrasekhar 1985). So, three geodesic equations

which dictate the trajectory of photons can be written as

$$\begin{aligned} \frac{d^2 r}{dt^2} + \frac{3}{2r(r-1)} \left(\frac{dr}{dt} \right)^2 - (r-1) \left(\frac{d\theta}{dt} \right)^2 \\ - (r-1)r \sin^2 \theta \left(\frac{d\phi}{dt} \right)^2 + \frac{r-1}{2r^3} = 0, \\ \frac{d^2 \theta}{dt^2} + \frac{2r-3}{r(r-1)} \left(\frac{d\theta}{dt} \right) \left(\frac{dr}{dt} \right) - \sin \theta \cos \theta \left(\frac{d\phi}{dt} \right)^2 = 0 \quad \text{and} \\ \frac{d^2 \phi}{dt^2} + \frac{2r-3}{r(r-1)} \left(\frac{d\phi}{dt} \right) \left(\frac{dr}{dt} \right) + 2 \cot \theta \left(\frac{d\theta}{dt} \right) \left(\frac{d\phi}{dt} \right) = 0. \end{aligned} \quad (11)$$

Velocity components are derived using Tetrad formalism (Park 2006) and are given by

$$\begin{aligned} v^{\hat{r}} = \frac{dr}{dt} = \frac{r}{(r-1)} \frac{dr}{dt}, \quad v^{\hat{\theta}} = \frac{d\theta}{dt} = \frac{r\sqrt{r}}{\sqrt{(r-1)}} \frac{d\theta}{dt} \\ \text{and} \quad v^{\hat{\phi}} = \frac{d\phi}{dt} = \frac{r\sqrt{r} \sin \theta}{\sqrt{(r-1)}} \frac{d\phi}{dt}. \end{aligned} \quad (12)$$

The redshift factor added to connect the source and the observer frame is given by the relation

$$1+z = \frac{E_{\text{em}}}{E_{\text{obs}}} = \frac{(P_\alpha u^\alpha)^{\text{em}}}{(P_\alpha u^\alpha)^{\text{obs}}}, \quad (13)$$

where E_{em} and E_{obs} are the energy of emitted and observed photons, respectively. The observed and emitted fluxes are related via the fourth power of redshift factor

$$F_k^{\text{obs}} = \frac{F_k^{\text{disc}}}{(1+z)^4}. \quad (14)$$

This relationship between the observed flux and the observed temperature can be expressed as

$$T_k^{\text{obs}} = \left(\frac{F_k^{\text{obs}}}{\sigma} \right)^{1/4}. \quad (15)$$

Doppler boosting is added during the emitter-observer frame transformation. The Ray-Tracing process and formation of images are done in the same way as reported earlier in CCG17a and CCG17b.

6 RESULTS

6.1 Time-dependent images without cooling

The static images of TCAF in presence of Comptonization were presented in CCG17a. The Compton cloud was modelled using natural angular momentum description of General Relativistic thick discs given by Chakrabarti (1985). The variations in images and spectra were presented for various inclination angles and disc accretion rates. They show that due to steep variation of density and temperature inside the CENBOL, the image does not have sharp edges. On the top of that, for an increasing optical depth of the CENBOL region, the photons from the Keplerian disc region located opposite to the observer are blocked. In this case, where `TVB` code was used to create the CENBOL and the outflow self-consistently, the optical depth of the CENBOL medium allows a fraction of photons from the Keplerian disc to pass through the Compton cloud without getting scattered. This can be seen in each panel presented in Figs 7 and 8.

Fig. 7 is drawn at (a) $t = 14.25$ s and (b) $t = 14.75$ s. In panel (a), the amount of outflow is much less as compared to panel (b). The

photons emitted by the base of the lower jet are mostly blocked by the optically thick Keplerian disc. In Fig. 7a, a very low number of photons are visible from the lower jet. Due to reflection symmetry of the outflows, these photons are also visible in the upper part of the outflows. However, due to Lorentz boosting in the upper jet, its observed temperature is enhanced and it is overall brighter. The sonic surface of the outflow is the cap like curved bright surface. The flow is subsonic and hotter below this surface. We include the photons in this plot which are emitted from the regions with optical depth greater than unity. This creates a special shape for the base of the outflow.

Fig. 8 shows the dynamical evolution of outflows at $t = 14.25$ s, $t = 14.35$ s, $t = 14.45$ s, $t = 14.55$ s, $t = 14.65$ s, and $t = 14.75$ s. The mass outflow increases with time in this case. The images with gravitationally bent rays have been drawn from the perspective of an observer placed at an inclination angle of 80° . The asymmetry of outflows becomes prominent in this figure also. The energy range is chosen in such a way that the observer temperature variation of the Keplerian disc can be found easily.

Fig. 9 shows energy-dependent images of TCAF. With a higher energy detector which works best in the energy range of 1.0–100 keV, the Keplerian disc might not be seen for disc rates lower than 0.1. With increasing \dot{m}_d , the disc may reappear. So, for an outbursting source, whose accretion rates change on a daily basis, the images will change for any particular energy range of detector. These pictorial changes will be due to the physical variations of accretion disc geometry, which will be triggered by the thermodynamical properties (see CT95 for details) of the disc.

In Panel (b), we see the top part of the outflows is also missing in a high-energy observation. This is due to the velocity profile of the outflowing electrons. Most of the photons suffer downscattering during their collision in the outflowing electrons. In Panel 3d, we see that the Jet spectra extends up to 1000 keV. But, in reality, in our model of non-magnetic jets, maximum hard photons in the Jet spectra are contributed only from the base of the jet. The photons that are coming from below the equatorial plane are mostly reflected or absorbed by optically thick Keplerian disc. So, up to 70° , visibility of the base of a lower jet is screened by the Keplerian disc itself. But, at 80° , as in Fig. 8, both sides become visible to the observer.

6.2 Observed spectra

The source spectrum as presented in Fig. 3a has a peak at 0.6 keV. This has been modified due to photon bending and shifted to a lower value for an observer at higher inclination angle. In the observed spectrum of Keplerian disc has a peak at 0.3 keV for 80° inclination angle (Fig. 10). The corresponding characteristic temperature ranges from $10^{6.0}$ to $10^{6.6}$ K and this is visible in Fig. 9a for 70° . The observed spectral variation of the blackbody part with an inclination angle remains almost the same as reported in CCG17a.

From Fig. 10, we see that the contribution of hard photons increases with inclination angle. But, the spectral slope is less for low inclination angle. This is to be contrasted with our earlier studies (see CCG17a) where the outflow component has not been added. Without the outflows, the hard photons in power-law component increase and the slope of the spectrum decreases with increasing inclination angle. With the inclusion of outflows, the spectral slope increases with an increasing inclination angle. But, the contribution of hard photons increases more significantly when the observer moves to a higher inclination angle.

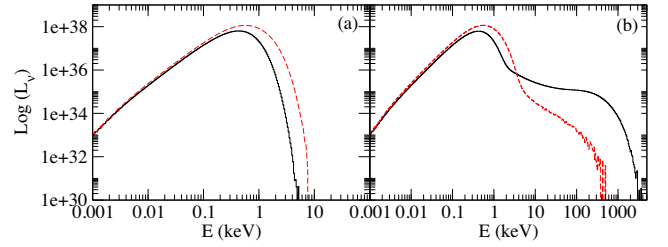


Figure 5. Variation of spectrum due to Compton cooling. Panel (a) corresponds to the injected soft spectrum where solid-black is the same spectrum as injected to the without cooling cases. Dotted-red corresponds to the injected spectra at final stage (Figs. 4b, d). The corresponding emergent Comptonized spectra are presented in Panel (b) where solid-black (same as Fig. 3) and dotted-red (for CENBOL represented in Figs. 4b, d) curves showing the spectral softening due to Compton cooling.

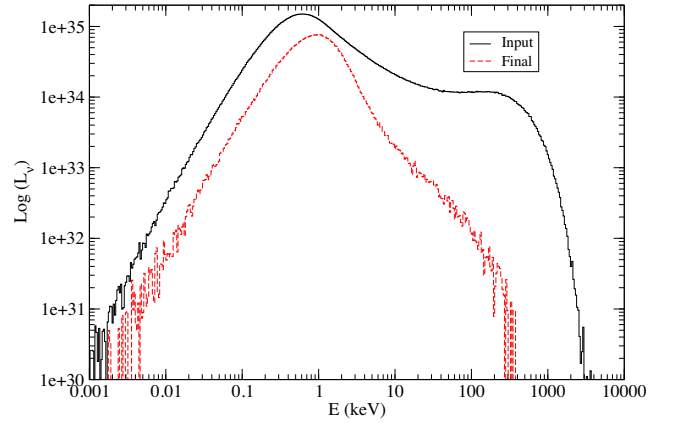


Figure 6. Emergent outflow spectra for corresponding density and temperature distribution shown in Fig. 4. Black-Solid curve shows the outflows spectrum directly from the output of the rvd code (same as the black curve shown in Fig. 3d). Red-dashed represents outflow spectra of the TCAF after final stage of cooling.

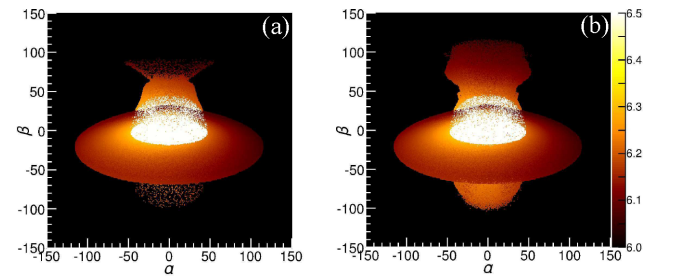


Figure 7. Images of TCAF (in $\log(T_{\text{obs}})$ scale) seen from an inclination angle of 70° in presence of outflows at time $t = 14.25$ s and $t = 14.75$ s. Colourbar is in the range $10^{6.0}$ – $10^{6.5}$ K.

6.3 Images with cooling

To demonstrate the effects of Compton cooling, we consider first the data file from hydrodynamic simulations at $t = 14.25$ s. The electron number density and temperature profile of the post-shock region after Compton cooling are presented in Fig. 4. Corresponding spectrum in Fig. 5 shows the spectral softening due to cooling. From Fig. 11, we see that the size of the CENBOL region is substantially reduced due to cooling effect and the optical depth of CENBOL region is so low that very few soft photons from

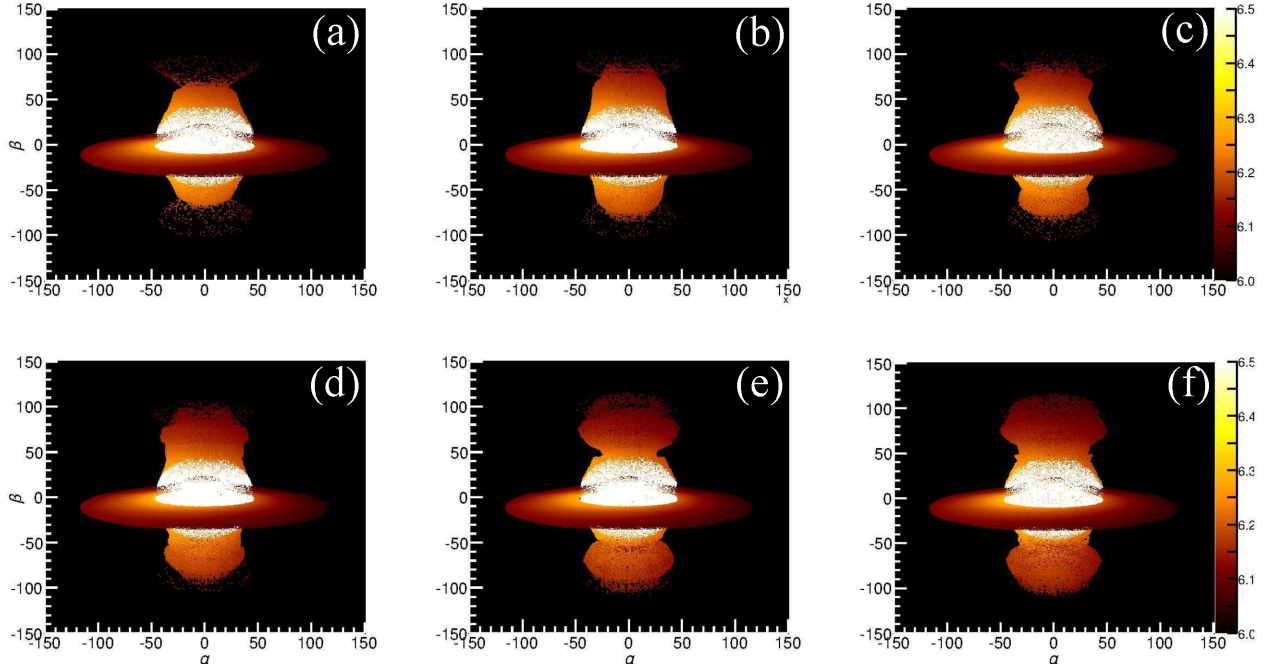


Figure 8. Images of TCAF (in $\log(T_{\text{obs}})$ scale) in presence of outflows at $t = 14.25$ s, $t = 14.35$ s, $t = 14.45$ s, $t = 14.55$ s, $t = 14.65$ s, and $t = 14.75$ s. Colourbar is in the range $10^{6.0} - 10^{6.5}$ K and inclination of the observer is at 80° . Asymmetry in the shape is due to difference in Doppler shifts in upper and lower jet components.

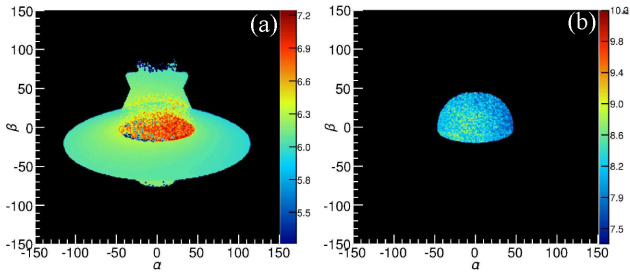


Figure 9. Grid averaged images of TCAF (in $\log(T_{\text{obs}})$ scale) in presence of outflows at time $t = 14.65$ s. Colourbar is in the range $10^{5.4} - 10^{6.7}$ K for panel (a) and $10^{6.7} - 10^{10.3}$ K for panel (b). The observer is placed at 70° . For higher energy range detectors, the Keplerian disc and upper part of jet becomes completely invisible to an observer.

Keplerian disc are intercepted in the Compton cloud. This is the reason of spectral softening. With the inclusion of Compton cooling the outflow drastically reduces its size. This effect can also be visible in the images of TCAF. As the CENBOL size is reduced, the inner edge of the truncated Keplerian disc moves radially inwards.

6.4 Realistic observed image with a finite beamwidth

Fig. 12, panel (a) shows a convolved image of a Keplerian disc with the inner edge at 3 and outer edge at 50. Convolution is over a beamwidth of (a) $\sim 20r_g$ and (d) $\sim 5r_g$. Corresponding spectrum for this image gives a pure soft spectrum (see CT95 and SS73 for further details). The intensity variation due to the disc and the black hole can be seen. In Panels (b,e), the images show an accretion disc with CENBOL (same as fig. 9(i) of CCG17a). In this case, the spectrum is hard and the CENBOL surface completely screens the inner region close to the black hole. This case was studied in CCG17a. In Panels (c, f) outflows are strong as in an intermediate states and

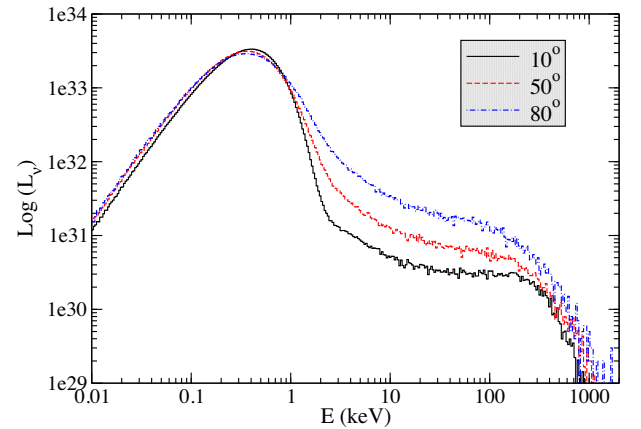


Figure 10. Observed spectra of TCAF seen from inclination angles 10° , 50° , and 80° in presence of Outflows. Increase in the hard photon contribution with inclination angle is visible. However, with the inclusion of outflows, the spectral slope increases with increasing the inclination angle.

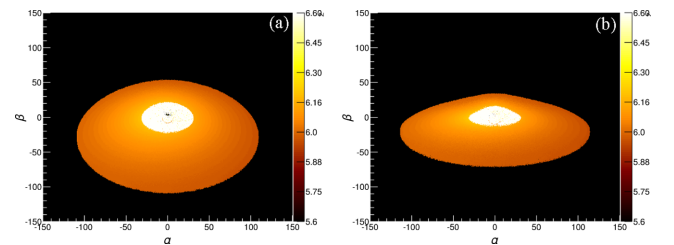


Figure 11. Images of TCAF (in $\log(T_{\text{obs}})$ scale) as seen from the inclination angle of 50° , 70° in presence of Compton cooling. Colourbar is in the range $10^{5.6} - 10^{6.6}$ K. If the outflows are absent, dark central region of black hole just starts to appear to an observer at 50° and becomes clearer if the onlooker moves to a lower inclination angle.

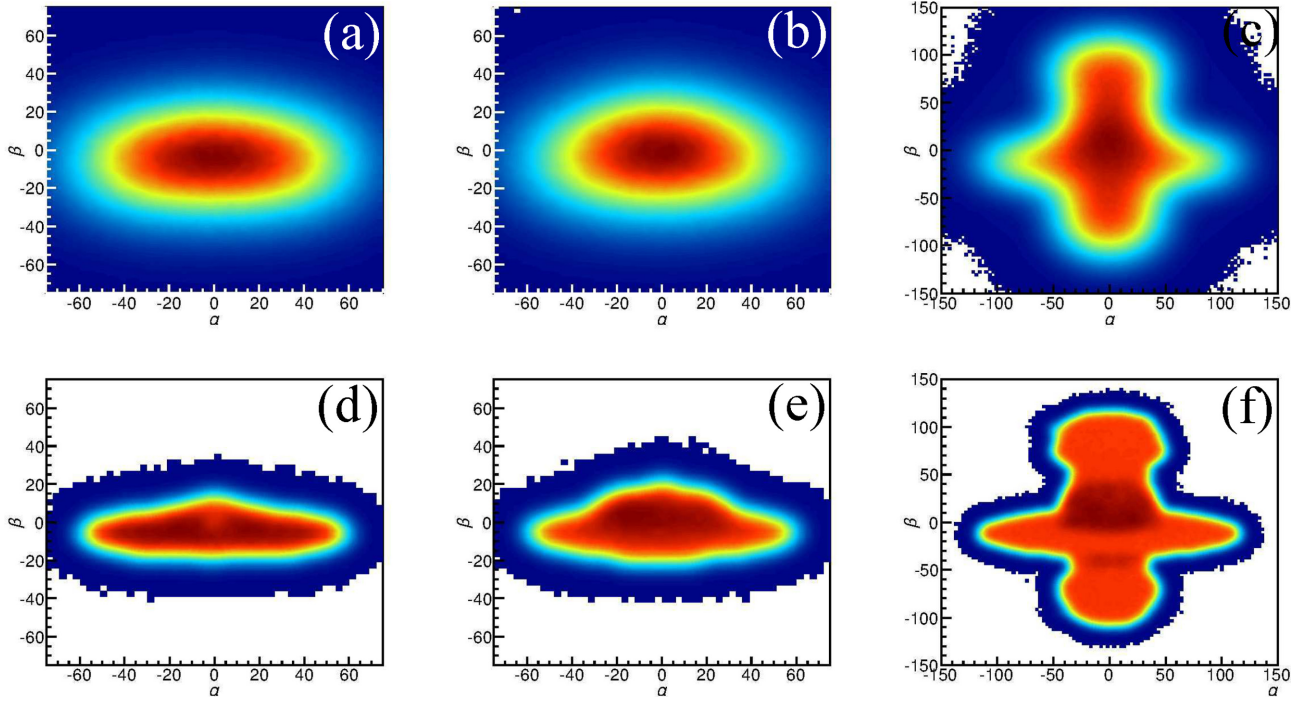


Figure 12. Convolved images of TCAF at different states. 2D Gaussian distribution is considered for convolution with spreading diameter (hypothetical beamwidth of the instrument) of $20r_g$ (for upper panel) and $5r_g$ for lower panel. Panels (a, d) are for a pure Keplerian disc (hard state with weak outflow) with outer edge at $50r_g$. Panels (b, e) show images of TCAF without outflows with outer edge at $50r_g$ (Soft state). Panels (c, f) depict images of TCAF with outflows having outer edges up to $100r_g$. Inclination angle is 80° for all panels. Intensity variation close to the black hole could be seen only when the beamwidth is around $5r_g$.

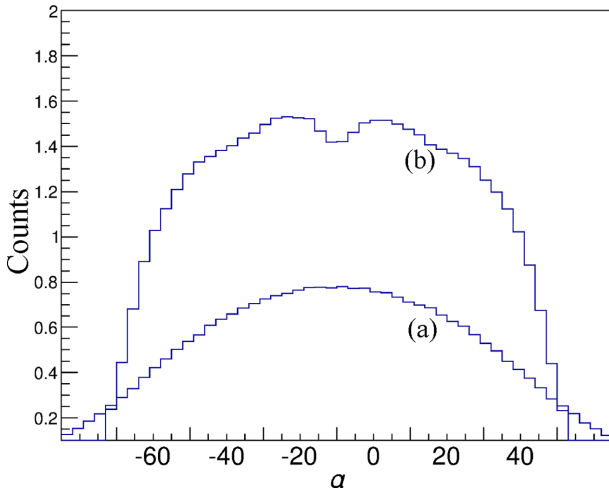


Figure 13. Difference in counts from central region received by detectors with spatial resolution of $20r_g$ (curve a) and of $5r_g$ (curve b). This is case is for a pure Keplerian disc presented in Fig. 12(a, d).

the convolutions show complex structure. If the inclination angle is very low, then possibly the black hole horizon could be seen, but in the present case of high inclination angle, the identification of horizon would be difficult, especially if integrated over time. The images we just presented are relevant for our galactic centre under a sudden high-mass inflow induced by tidal disruption. Various measurements (Markoff et al. 2007; and references therein) suggested that the jet from the Sgr A* is around 75° away from our line of sight. Assuming this is along the perpendicular direction with respect to

the accretion plane, and assuming radio intensity roughly tracks the disc/jet as discussed above, the Galactic Centre should also show images similar to the panels Fig. 12(a–f). Our conclusion is that we may not be able to discern the horizon itself, but some features as in our Fig. 12(a–f) should be visible. Similarly, the presence and absence of accretion disc with change in accretion rates should be seen.

The convolved images presented in Fig. 12 suggest a low spatial resolution of about $5r_g$ will be able to distinguish features in the hole–disc system, unless the object has a very low inclination. Integrated counts from the central region are plotted in Fig. 13b for the Keplerian disc only. A spatial resolution of $5r_g$ shows double-peaked curve in just resolved condition. Fig. 13a shows unresolved count distribution when $20r_g$ is used instead. Considering the possibilities of high-mass inflow on to Sgr A*, in its flaring state, we expect an X-Ray image similar to Fig. 8, which is difficult to resolve at a high inclination. Thus, probing inner region to capture the event horizon can be difficult in this state. Future projects which aim at capturing the image of an event horizon will not be able to detect the difference in the intensity of disc to that of the hole, unless the object is in the soft state. Otherwise, only low-inclination discs would show the even horizon in any spectral states, even in presence of outflows. Of course, this conclusion is valid only if the radio emission roughly tracks the inflow–outflow features as observed in our numerical simulations.

7 DISCUSSIONS AND CONCLUSION

Recent study of the effects of the photon bending on standard and two-component accretion disc spectra and images was made by CCG17a. Earlier, static images of TCAFs were presented where

we discussed how the Keplerian disc (valid for soft states only) and the two-component discs with its intrinsic Compton cloud at the inner edge would look like from various angles. However, a realistic advective flow will be time dependent and also will have outflows as the outflow and inflow solutions are always treated with a common footing (Chakrabarti 1989). This outflow must come out from the post-shock region or CENBOL which behaves as the Compton cloud around a black hole or a neutron star. This has also been shown by numerical simulations through numerous works referred to in the earlier sections. The outflow could be time varying and the image of the disc would, in general, vary with time.

In this paper, we have employed numerical simulations of hydrodynamic process to generate time-dependent accretion disc configurations. At every time step, Monte Carlo simulation was done to compute the effects of Comptonization and to obtain the spectrum. We produced images of the CENBOL with the jet as well as the composite spectra, when the effects of cooling due to Comptonization was turned off or turned on. We find that with cooling effects the CENBOL collapses to a smaller size and the outflow is also reduced as in a soft state. In presence of outflows, we draw images of an accretion disc where the Compton cloud or CENBOL is showing temporal variation of its physical shape and internal properties. With these, the images depict the outflows which are self-consistently produced from the inflowing matter. We showed the combined effects of the disc and the jet and particularly how they would look like in different X-ray energies. Doppler shift was seen to break the symmetry as expected, but the photon bending made it even more asymmetric.

Though our simulations are carried out in regions which emit X-rays, and this will remain true for higher mass black holes also, but the efforts are on to observe the event horizon in radio waves which have higher spatial resolutions. On the other hand, both the radio and the Comptonization track the high-energy electron distribution. So we believe that our simulations will have relevance for radio observations as well. However, intensity of synchrotron photon in radio band depends on the magnetic field strength and is expected to be primarily originated from the post-shock region. Thus, our X-ray images without the cooler Keplerian disc can be treated as radio images. We showed, using two different spatial resolutions, that one requires a resolution of $\sim 5r_g$ in order to separate distinct features in disc-jet systems when high-inclination objects are observed, unless the object is in a soft state and only has the Keplerian disc. However, the obscuration of the central region is not done when viewed at low inclination angle and the event horizon should be observable at any spectral state if the beamwidth is low enough. The images are of particular interest where the mass accretion rate on to central engine is high enough, such as RL Quasars, Seyfert 1 galaxies. The current findings would hold for LLAGNs, like Sgr A*, for high-mass inflow induced by tidal disruptions. Considering the future possibilities of observing such Galactic black holes and SMBHs, we present an example of a high-inclination system, to impress how the inclination basically removes the chances of observing the horizon, especially, when the jets are active. In the absence of jets and outflows, however, the horizon can be discerned. This result will be of importance for future missions which could exclude high inclination Seyfert 2 AGNs and GBHs such as GRS 1915+105, H 1743-322, and GRO 1655-40.

In the spectral study using Monte Carlo simulations, we see effects of photon bending in the multicolour soft photon component. The peak was seen at a lower energy due to redshifts at an inclination of 70° . We also find that the spectra harden at higher inclination angles. Of course, a major result, though expected, in our simulation

is that when the cooling is included, the CENBOL itself shrinks and consequently the spectra become softer. Further simulations including the effects of synchrotron radiation, Compton cooling, and viscosity are being carried out and will be reported elsewhere.

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